I. V. Savelyev

Questions and Problems in General PHYSICS

Mir Publishers
Moscow

Questions and Problems in General Physics has been compiled with a view to the author's three-volume *Physics. A General Course*. It contains about 1200 questions and problems, most of which are original. Problems of a varying degree of difficulty have been included. For this reason, the book can be used with equal success at higher schools with either a normal or an extended syllabus in physics. In compiling the book, the author gave preference to real problems from everyday life, science, and engineering instead of to abstract ones. The problems are arranged in a logical sequence and in the order of increasing difficulty. Consequently, work on the preceding problems prepares the student for solving the following ones. The book is divided into three sections: (1) questions and problems; (2) answers; and (3) hints on solving or the solutions of the most difficult problems. The book has appendices that will be helpful in solving the problems. It is intended for students of higher technical schools.

Igor V. Savelyev, D.Sc. (Physics and Mathematics), has been head of the Department of General Physics at the Moscow Institute of Engineering Physics for over 25 years, after devoting quite a few years to experimental physics. He is the author of about 100 published works in physics, including *Physics. A General Course*, in three volumes, and *Fundamentals of Theoretical Physics*, in two volumes, both of which have been translated into English by Mir Publishers.
His principal interests include instruction in physics at higher technical schools and its improvement. Professor Savelyev holds the title of Honoured Scientist of the RSFSR and is a USSR State Prize Winner.

Mendeleev's Periodic

PERIODS	GROUP OF				
	I	II	III	IV	V
1	(H)				
2	Li 3 6.94_1 Lithium	Be 4 9.01218 Beryllium	5 B 10.81 Boron	6 C 12.011 Carbon	7 14.00 Nitroge
3	Na 11 22.98977 Sodium	Mg 12 24.305 Magnesium	13 Al 26.98154 Aluminium	14 Si 28.08_5 Silicon	15 30.973 Phosphoru
4	K 19 39.09_8 Potassium	Ca 20 40.08 Calcium	Sc 21 44.9559 Scandium	Ti 22 47.90 Titanium	V 23 50.941_4 Vanadium
	29 Cu 63.54_6 Copper	30 Zn 65.38 Zinc	31 Ga 69.72 Gallium	32 Ge 72.5_9 Germanium	33 A 74.92 Arsen
5	Rb 37 85.467_8 Rubidium	Sr 38 87.62 Strontium	Y 39 88.9059 Yttrium	Zr 40 91.22 Zirconium	Nb 41 92.9064 Niobium
	47 Ag 107.868 Silver	48 Cd 112.41 Cadmium	49 In 114.82 Indium	50 Sn 118.6_9 Tin	51 S 121. Antimo
6	Cs 55 132.9054 Cesium	Ba 56 137.3_4 Barium	La* 57 138.905_5 Lanthanum	Hf 72 178.4_9 Hafnium	Ta 73 180.947_9 Tantalum
	79 Au 196.9665 Gold	80 Hg 200.5_9 Mercury	81 Tl 204.3_7 Thallium	82 Pb 207.2 Lead	83 E 208.98 Bismu
7	Fr 87 [223] Francium	Ra 88 226.0254 Radium	Ac** 89 [227] Actinium	Ku 104 [261] Kurchatovium	105

* LANTH

Ce 58 140.12 Cerium	Pr 59 140.9077 Praseodymium	Nd 60 144.2_4 Neodymium	Pm 61 [145] Promethium	Sm 62 150.4 Samarium	Eu 63 151.96_5 Europium	Gd 64 157.2_5 Gadolinium

** AC

Th 90 232.0381 Thorium	Pa 91 231.0359 Protactinium	U 92 238.02_9 Uranium	Np 93 237.0482 Neptunium	Pu 94 [244] Plutonium	Am 95 [243] Americium	Cm 9 [247] Curium

Table of the Elements

ELEMENTS

VI	VII	VIII		
	1 H 1.0079 Hydrogen	2 He 4.00260 Helium		
8 O 15.999_4 Oxygen	9 F 18.998403 Fluorine	10 Ne 20.17_9 Neon		
16 S 32.06 Sulphur	17 Cl 35.453 Chlorine	18 Ar 39.94_8 Argon		
Cr 24 1.996 hromium	Mn 25 54.9380 Manganese	Fe 26 55.84_7 Iron	Co 27 58.9332 Cobalt	Ni 28 58.70 Nickel
34 Se 78.9_6 Selenium	35 Br 79.904 Bromine	36 Kr 83.80 Krypton		
Mo 42 5.9_4 olybdenum	Tc 43 98.9062 Technetium	Ru 44 101.0_7 Ruthenium	Rh 45 102.9055 Rhodium	Pd 46 106.4 Palladium
52 Te 127.6_0 Tellurium	53 I 126.9045 Iodine	54 Xe 131.30 Xenon		
W 74 83.8_5 ungsten	Re 75 186.207 Rhenium	Os 76 190.2 Osmium	Ir 77 192.2_2 Iridium	Pt 78 195.0_9 Platinum
84 Po [209] Polonium	85 At [210] Astatine	86 Rn [222] Radon		

Symbol of element

Atomic number

Atomic mass

Li	3 6.94_1
Lithium	

An integer in brackets is the mass number of the most stable radioactive isotope

IDES

Tb 65 58.9254 erbium	Dy 66 162.5_0 Dysprosium	Ho 67 164.9304 Holmium	Er 68 167.2_6 Erbium	Tm 69 168.9342 Thulium	Yb 70 173.0_4 Ytterbium	Lu 71 174.96_7 Lutetium

IDES

Bk 97 247] erkelium	Cf 98 [251] Californium	Es 99 [254] Einsteinium	Fm 100 [257] Fermium	Md 101 [258] Mendelevium	(No) 102 [255] (Nobelium)	(Lr) 103 [256] (Lawrencium)

ИЗДАТЕЛЬСТВО · МИР ·

И. В. САВЕЛЬЕВ

СБОРНИК ВОПРОСОВ И ЗАДАЧ ПО ОБЩЕЙ ФИЗИКЕ

«НАУКА» МОСКВА

I. V. SAVELYEV

QUESTIONS AND PROBLEMS IN GENERAL PHYSICS

Translated from the Russian by G. LEIB

MIR
PUBLISHERS
MOSCOW

First published 1984
Revised from the 1982 Russian edition

На английском языке

PREFACE

The present book has been compiled with a view to the author's three-volume *Physics. A General Course* (Mir Publishers, 1980, 1980, 1981). It contains about 1200 questions and problems. Most of them are original. Problems of a varying degree of difficulty have been included. For this reason, the book can be used with equal success at higher schools having either a normal or an extended syllabus in physics.

In compiling the book, I gave preference to real problems from everyday life, science, and engineering instead of to abstract ones. In addition, my aim was to include more problems in solving which a student would experience impatient curiosity to see what the result being sought is.

Wherever possible, the problems are arranged in a logical sequence and in the order of increasing difficulty. Consequently, work on the preceding problems prepares a student for solving the following ones.

The book is divided into three sections. The first contains the problems, the second—the answers to them, and the third—hints on how to solve the more difficult problems or their solutions. I have separated the hints from the answers to allow students to continue their attempts to solve a problem by themselves after obtaining a wrong answer. The appendices at the end of the book, in addition to tables of physical quantities, contain information (the values of selected integrals and the like) that may be of assistance in working on the problems.

The initial data and the answers to the problems are given with a view to the accuracy of the relevant quantities and the rules of operating with approximate numbers. As far as I know, this was done for the first time in a book I edited, *Sbornik zadach po obshchei fizike* (Collection of

Problems in General] Physics) (the authors were N.N. Vzorov, O.I. Zamsha, I.E. Irodov, and I.V. Savelyev) published in 1968 by Nauka, Moscow.

It must be borne in mind that work on the problems will yield the maximum returns when the recommendations given in the Introduction to the book are followed. It is therefore necessary to begin with attentive reading of the Introduction and to reread it from time to time.

I am greatly indebted to my colleagues and friends whose advice I took advantage of in working on the book. I am especially grateful to V.I. Gervids and N.V. Maslennikova who checked the answers to many problems and attracted my attention to a number of inaccuracies. I would like to thank Associate Professor V.B. Zernov, who thoroughly acquainted himself with the manuscript, delved deeply into its intentions, and made many helpful remarks.

Moscow, May, 1981 *Igor Savelyev*

CONTENTS

Before beginning to solve the problems, read this Introduction attentively and acquaint yourself with the Appendices at the end of the book!

INTRODUCTION

I. Very Briefly on the Psychology of Scientific Work

The solving of problems will yield the maximum returns only if a student does this *by himself*. It is often not easy to solve a problem without any aid or prompting, and this is not always successful. But even unsuccessful attempts to find a solution, if they were undertaken with sufficient persistence, will give noticeable returns because they develop thinking and strengthen one's will power. It must be borne in mind that the decisive role in working on problems, as in general in studying, is played by will power and diligence.

One must never be confused by the fact that some problems do not lend themselves to prompt solution. It has been authentically established that the process of creative work in the field of the exact sciences (and the solution of problems is a kind of creative work) occurs according to the following scheme*. First comes the preparatory stage during which a scientist persistently seeks the solution to a problem. If he does not succeed in finding the solution and sets the problem aside, the second stage sets in (the stage of incubation)—the scientist does not think about the problem and occupies himself with other matters. But the latent work of the mind continues subconsciously, and it often leads in the long run to the third stage—sudden enlightenment and the obtaining of the required solution. It must be had in view that the stage of incubation does not appear

* See the book: Jacque Adamar, *Issledovanie psikhologii protsessa izobreteniya v oblasti matematiki* (Investigating the Psychology of the Inventive Process in the Field of Mathematics). Moscow, Sov. Radio (1970).

by itself—to start the machinery of subconsciousness, persistent intensive work is needed in the preparatory stage.

The solution of problems, as we have already noted, is also a form of creative work and obeys the same laws as the work of a scientist on a scientific problem. True, the second stage—that of incubation—may sometimes be expressed so slightly that it remains unnoticed.

It follows from the above that the solution of problems must never be postponed to the last evening before lessons, which, unfortunately, is done quite often by students. In this case, the more involved and, at the same time, the most interesting and useful problems clearly cannot be solved. One must begin to think over the problems assigned for homework as early as possible, creating conditions for realizing the incubation stage.

If the conditions of a problem contain numerical data, do not be lazy and continue the solution up to a numerical answer. To obtain a correct numerical answer, one must have a good knowledge of the units of physical quantities and be able to perform calculations accurately and reliably. Both can be achieved only by prolonged practice. Special attention must be given to the proper determination of the order of the quantity being sought. A surprising delusion is often encountered among students—they consider that an error in the order of magnitude of a quantity (even of several orders) is less appreciable than an error in the significant digits. The unfoundedness of such an opinion is easily detected in the following example. The error consisting in that we have obtained 7 instead of 5 is 40%, whereas an error of only one order (say, we have obtained 10^5 instead of 10^4) is 900%!

The section following the answers contains hints on how to solve the more involved problems. Revert to them only after several attempts to solve a problem have been unsuccessful.

Finally, it must be borne in mind that in a number of cases the problems are arranged in a logical sequence and in the order of increasing difficulty. Therefore, acquaintance with several preceding problems may be an impetus to solving a given problem.

II. Methodological Instructions

It is good practice to observe the following rules in solving problems.

1. First of all, thoroughly examine the conditions of a problem. If permitted by the nature of the problem, make a drawing explaining its essence.

2. With rare exceptions, every problem must first be solved in a general form (i.e. using letter symbols, and not numbers), the required quantity being expressed in terms of the given ones. After obtaining the solution in a general form, check whether its dimension is correct. If possible, study the behaviour of the solution in extreme cases. For example, in considering the motion of a body thrown at an angle to the horizontal (Problem 1.28) we are given the magnitude of the initial velocity v_0 and the angle α at which the body was thrown; we also know the acceleration g. For the maximum height h of the body and the range l, we obtain the values

$$h=\frac{v_0^2 \sin^2 \alpha}{2g}, \qquad l=\frac{v_0^2 \sin 2\alpha}{g}$$

Give attention to the fact that both expressions include only the given quantities v_0, α, and g. The expression for l can be written in the form $l = (v_0 \cos \alpha)\, \tau$, where τ is the time of flight. But this expression cannot be considered as a solution because τ does not belong to the given quantities, it itself is a function of v_0 and α. Checking shows that both expressions, as should be the case, have the dimension of length. For $\alpha = \pi/2$, we obtain $h = v_0^2/2g$, which coincides with the known expression for the maximum height of a body thrown vertically. For l, we obtain the correct value—zero.

When in the course of finding the required quantities, we have to solve a system of several cumbersome equations (as often happens in finding the currents flowing in intricate branched circuits), it is good practice to first introduce the numerical values of the coefficients into these equations and only then determine the values of the required quantities.

3. After being convinced in the correctness of the general solution, substitute for each of the symbols the numerical values of the quantities they stand for, naturally taking

all these values in the same system of units. To facilitate the determination of the order of magnitude of the quantity being evaluated, it is helpful to represent the initial quantities in the form of numbers close to unity multiplied by 10 raised to the corresponding power (for example, introduce 2.47×10^2 instead of 247, and the number 0.86×10^{-1} instead of 0.086). After introducing numerical values into a formula and before beginning calculations, check whether it is possible to use the formulas for approximate calculations given in Appendix 11 (see also Problem 1.37).

4. It must be remembered that the numerical values of physical quantities are always approximate. This is why it is necessary to observe the rules of operations with approximate numbers in calculations. Particularly, in the obtained value of a calculated quantity, the significant digits should be retained up to the one, a unit of which exceeds the error in this quantity. All the following significant digits should be discarded.

5. Having obtained a numerical answer, appraise its plausibility. Such an appraisal can often reveal an error in the result obtained. For example, the speed of a body cannot be higher than that of light in a vacuum, the distance covered by a stone which a man has thrown cannot be of the order of 1000 m, and the mass of a molecule cannot be of the order of 1 mg.

III. On Calculations with Approximate Numbers

1. In physics, one is supposed, in addition to the numerical value of a quantity, to indicate the error with which this quantity has been measured or determined. For example, $l = 356 \pm 2$ m signifies that the true value of the length l is confined within the limits from 354 to 358 m. Strictly speaking, one must also indicate the probability of the fact that the statement made is correct (the confidence level). Often, however, in writing the values of a physical quantity, its error (the confidence interval) is not indicated, and only one number is given, for example $l = 467$ m. In this case, the error in the quantity should be considered to be within one unit of the last significant digit (in our example this is 1 m). Consequently, all the significant digits of a number expressing the value of a physical quantity except

for the last one should be considered authentic; the last digit should be considered as doubtful (the true value of the digit may differ from the indicated one by unity).

Recall that by significant digits are meant all the digits in a decimal number except for the zeros at the beginning of the number. For example, in the number 0.03040, the first two zeros are not significant. Their purpose is only to show the smallness of the number. The zero between 3 and 4 is significant. The zero after 4 is also significant.

When dealing with large integers ending in zeros (for example, 123 000), we do not know whether the zeros indicate significant digits or only show the places of the other digits. To avoid this ambiguity, such numbers should be written in the form of 1.23×10^5 if they have three significant digits, or in the form of 1.230×10^5 if they have four significant digits, etc.

2. The absolute error of the approximate number a is defined to be the quantity

$$\Delta a = |A - a|$$

where A is the exact value of the same number.

3. The relative error of the approximate number a is defined to be the quantity

$$\delta a = \frac{\Delta a}{|A|}$$

In physics, we usually have to do in calculations with numbers whose exact values remain unknown. This is why the following formula is used in practice to determine the relative error:

$$\delta a = \frac{\Delta a}{|a|}$$

The error introduced here is not large because usually $A \approx a$.

4. If the quantity u is a function of the quantities x_1, x_2, . . ., x_n, that is

$$u = f(x_1, x_2, \ldots, x_n)$$

the maximum absolute error in the value of u is determined by the formula

$$\Delta u = \sum_{i=1}^{n} \left| \frac{\partial u}{\partial x_i} \right| \Delta x_i$$

where Δx_i are the absolute errors in the values of x_i.

5. Dividing Δu by $|u|$, we obtain the maximum relative error in the value of u:

$$\delta u = \frac{\Delta u}{|u|} = \sum_{i=1}^{n} \left| \frac{\partial}{\partial x_i} \ln u \right| \Delta x_i$$

6. Table 1 gives expressions for the maximum absolute Δu and relative δu errors of selected functions. By Δx_i and δx_i are meant the absolute and relative errors in the value of x_i.

7. Let us consider an example of determining the error in the results of calculations. We shall take the problem mentioned above on the motion of a body thrown at an angle to the horizontal. The maximum height of the body is given by the formula

$$h = \frac{v_0^2 \sin^2 \alpha}{2g}$$

Using the formulas in Table 1, we find for the maximum relative error in the value of h:

$$\delta h = 2\delta v_0 + 2\delta (\sin \alpha) + \delta g = 2\delta v_0 + 2\,|\cot \alpha|\,\Delta\alpha + \delta g$$

(the number 2 in the denominator is exact, its error is zero).

Assume that $v_0 = 95$ m/s and $\alpha = 45$ deg; for g we assume a value of 9.81 m/s^2. Hence, $\Delta v_0 = 1$ m/s (a unit of the last significant digit), $\delta v_0 = 1/95$, $\Delta\alpha = 1$ deg $= 1/57$ radian, $\delta g = 1/981 \approx 0.001$. We substitute these values into the formula for δh ($\cot 45° = 1$):

$$\delta h = \frac{2}{95} + \frac{2}{57} + 0.001 \approx \frac{1}{20} \text{ or } 5\%$$

We shall note that there was no need of taking the acceleration g with an accuracy to the third digit. If we assume that $g = 9.8$ m/s^2 and thus increase the relative error δg to 2/980, the relative accuracy of the result will not virtually change, but the calculations will be simpler.

Now we evaluate h:

$$h = \frac{v_0^2 \sin^2 \alpha}{2g} = \frac{95^2 \times 0.707^2}{2 \times 9.8} = 2.3 \times 10^2 \text{ m}$$

Table 1

Form of function	Maximum absolute error Δu	Maximum relative error δu
$u=\sum_{i=1}^{n} x_i$	$\Delta u=\sum_{i=1}^{n} \Delta x_i$	—
$u=x_1-x_2$	$\Delta u=\Delta x_1+\Delta x_2$	$\delta u=\dfrac{\Delta x_1+\Delta x_2}{\lvert x_1-x_2\rvert}$
$u=x_1x_2\ldots x_n$	$\Delta u=\lvert u\rvert\delta u$	$\delta u=\sum_{i=1}^{n}\delta x_i$
$u=\dfrac{x}{y}$	—	$\delta u=\delta x+\delta y$
$u=\dfrac{x_1x_2\ldots x_n}{y_1y_2\ldots y_m}$	—	$\delta u=\sum_{i=1}^{n}\delta x_i+\sum_{i=1}^{m}\delta y_i$
$u=x^m$	$\Delta u=m\lvert x^{m-1}\rvert\,\Delta x$	$\delta u=m\,\delta x$
$u=\sqrt[m]{x}$	—	$\delta u=\dfrac{1}{m}\delta x$
$u=\ln x$	$\Delta u=\dfrac{\Delta x}{x}=\delta x$	$\delta u=\dfrac{\delta x}{\lvert\ln x\rvert}$
$u=\log x$	$\Delta u=\dfrac{1}{2.30}\dfrac{\Delta x}{x}=\dfrac{\delta x}{2.30}$	$\delta u=\dfrac{1}{2.30}\dfrac{\delta x}{\lvert\log x\rvert}$
$u=e^{\pm\alpha x}\ (\alpha>0)$	$\Delta u=\alpha e^{\pm\alpha x}\Delta x$	$\delta u=\alpha\,\Delta x$
$u=e^{f(x)}$	$\Delta u=e^{f(x)}\left\lvert\dfrac{df}{dx}\right\rvert\Delta x$	$\delta u=\left\lvert\dfrac{df}{dx}\right\rvert\Delta x$
$u=\sin mx$	$\Delta u=m\,\lvert\cos mx\rvert\,\Delta x$	$\delta u=m\,\lvert\cot mx\rvert\,\Delta x$
$u=\cos mx$	$\Delta u=m\,\lvert\sin mx\rvert\,\Delta x$	$\delta u=m\,\lvert\tan mx\rvert\,\Delta x$
$u=\tan mx$	$\Delta u=\dfrac{m}{\cos^2 mx}\Delta x$	$\delta u=\dfrac{2m}{\lvert\sin 2mx\rvert}\Delta x$

We may not write the result in the form of 230 m because this would mean that the error in the found value of h does not exceed 1 m. Actually, as we have ascertained, with the given accuracy of v_0 and α, the height h cannot be evaluated with a relative accuracy exceeding 5%, i.e. with an absolute error less than 10 m.

IV. Explanations in Connection with the Numerical Coefficients in the Conditions of Problems

Expressions of the kind $\mathbf{r} = 1\mathbf{e}_x + 2t\mathbf{e}_y + 3t^2\mathbf{e}_z$ or $U = 1x + 2y^2 + 3z^3$ and so on should be considered as an abbreviated conditional writing of the expressions $\mathbf{r} = (1\text{ m})\,\mathbf{e}_x + (2\text{ m/s})\,t\mathbf{e}_y + (3\text{ m/s}^2)\,t^2\mathbf{e}_z$ or $U = (1\text{ J/m})\,x + (2\text{ J/m}^2)\,y^2 + (3\text{ J/m}^3)\,z^3$ and so on, i.e. the proper dimensions must be assigned to the numerical coefficients.

PART 1

THE PHYSICAL FUNDAMENTALS OF MECHANICS

SYMBOLS

A	amplitude; work
$\mathbf{a}$	acceleration
$\mathbf{B}$	magnetic induction
C	centre of mass
c	speed of light in a vacuum
d	diameter
$\mathbf{E}$	unit tensor
E	energy
E_k	kinetic energy
E_p	potential energy
$\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$	unit vectors of coordinate axes
e	base of natural logarithms; elementary charge
$\mathbf{F}$	force
f	coefficient of friction
G	gravitational constant
$\mathbf{g}$	acceleration of free fall
h	height
I	moment of inertia
K	reference frame
k	force constant
$\mathbf{L}$	angular momentum
l	length
$\mathbf{M}$	moment of force (torque)
M	mass
M_z	moment of force relative to z-axis
m	mass
N	number of revolutions
n	number of revolutions per unit time
P	power
$\mathbf{p}$	momentum
p	pressure
Q	flow of liquid; quality of oscillating system
q	electric charge
$\mathbf{R}$	position (radius) vector
R	distance; radius; radius of curvature
Re	Reynolds number
$\mathbf{r}$	position (radius) vector
r	distance; radius
S	area; surface
s	distance
$\mathbf{T}$	tensor
T	period of oscillations
t	time
$\mathbf{u}$	velocity
V	volume
$\mathbf{v}$	velocity
$\boldsymbol{\alpha}$	angular acceleration
α	angle
β	angle; damping factor
γ	angle
η	viscosity (dynamic)

λ	linear mass (mass per unit length); logarithmic decrement	τ	time
μ	reduced mass	φ	angle; latitude of locality
ρ	density	Ω	solid angle
σ	surface density (mass per unit surface area); surface tension	ω	angular velocity
		ω	cyclic frequency

1.1. Kinematics

1.1. What does the integral $\int_{t_1}^{t_2} \mathbf{v}\, dt$ determine?

1.2. What does the integral $\int_{t_1}^{t_2} v_x\, dt$ determine?

1.3. Can the increment of the magnitude of a vector Δa be larger than the magnitude of the increment of the vector $|\Delta \mathbf{a}|$?

1.4. Can the increment of the magnitude of a vector Δa be equal to the magnitude of the increment of the vector $|\Delta \mathbf{a}|$?

1.5. What is the relation between the increment of the magnitude of a vector Δa and the magnitude of the increment of the vector $|\Delta \mathbf{a}|$ if the vectors $\mathbf{a}$ and $\Delta \mathbf{a}$ are directed oppositely?

1.6. The direction of the vector $\mathbf{a}$ was reversed. Find: $\Delta \mathbf{a}$, $|\Delta \mathbf{a}|$, Δa.

1.7. The vector $\mathbf{a}$ turned without a change in its "length" through the small angle $\delta\varphi$. (a) Write an approximate expression for $|\Delta \mathbf{a}|$. (b) What does Δa equal?

1.8. The initial value of the velocity is $\mathbf{v}_1 = 1\mathbf{e}_x + 3\mathbf{e}_y + 5\mathbf{e}_z$ (m/s), and its final value is $\mathbf{v}_2 = 2\mathbf{e}_x + 4\mathbf{e}_y + 6\mathbf{e}_z$ (m/s). Find: (a) the increment of the velocity $\Delta \mathbf{v}$; (b) the magnitude of the increment of the velocity $|\Delta \mathbf{v}|$; (c) the increment of the magnitude of the velocity Δv.

1.9. Write an expression for the cosine of the angle α between vectors with the components a_x, a_y, a_z and b_x, b_y, b_z.

1.10. The components of one vector are (1, 3, 5), and of another one are (6, 4, 2). Find the angle α between the vectors.

1.11. Transform the expression $\mathbf{a}\,[\mathbf{bc}]$, in which the vec-

tors **a** and **c** are mutually perpendicular and the vector **b** makes the angle α with a normal to the plane containing the vectors **a** and **c**, to a form containing only the magnitudes of the vectors and the angle α.

1.12. Proceeding from the definition of the mean value of a function, prove that:

(a) the mean value of the velocity $\langle \mathbf{v} \rangle$ of a point during the time τ equals the displacement $\Delta \mathbf{r}$ of the point during this time divided by τ;

(b) the mean value of the acceleration $\langle \mathbf{a} \rangle$ of a point during the time τ equals the increment of the velocity $\Delta \mathbf{v}$ during this time divided by τ.

1.13. A particle is moving uniformly around a circle of radius R clockwise, completing one revolution during the time τ. The circle is in the coordinate plane x, y, and its centre coincides with the origin of coordinates. At the instant $t = 0$, the particle is at a point with the coordinates $x = 0$ and $y = R$. Find the mean value of the velocity of the point during the time interval: (a) from 0 to $\tau/4$; (b) from 0 to $\tau/2$; (c) from 0 to $3\tau/4$; (d) from 0 to τ; (e) from $\tau/4$ to $3\tau/4$.

1.14. A particle travelled three-fourths of a circle at a mean value of the speed equal to $\langle v \rangle$ during a certain time. Find the magnitude of the mean velocity of the particle $|\langle \mathbf{v} \rangle|$ during the same time.

1.15. A particle initially at rest completed 1.5 revolutions around a circle of radius $R = 5.00$ m at a constant tangential acceleration during the time $\tau = 10.0$ s. Calculate the values of the following quantities corresponding to this time interval:

(a) the mean value of the speed $\langle v \rangle$;

(b) the magnitude of the mean velocity $|\langle \mathbf{v} \rangle|$;

(c) the magnitude of the mean acceleration $|\langle \mathbf{a} \rangle|$.

1.16. A vector **a** of constant magnitude turns uniformly counterclockwise in the plane x, y and during the time t passes from the position in which it coincides in direction with the x-axis to the position in which it coincides in direction with the y-axis. Find the mean value of the vector **a** during the time t and the magnitude of this mean value.

1.17. The position vector **r** of a point changes: (a) only in magnitude; (b) only in direction. What can be said about the point's trajectory?

1.18. The position vector of a particle is determined by the expression $\mathbf{r} = 3t^2\mathbf{e}_x + 4t^2\mathbf{e}_y + 7\mathbf{e}_z$ (m). Evaluate:

(a) the distance s travelled by the particle during the first 10 s of its motion;

(b) the magnitude of the displacement $|\Delta\mathbf{r}|$ during the same time;

(c) explain the obtained results.

1.19. The position vector of a particle changes in time according to the law $\mathbf{r} = 3t^2\mathbf{e}_x + 2t\mathbf{e}_y + 1\mathbf{e}_z$ (m). Find:

(a) the velocity $\mathbf{v}$ and the acceleration $\mathbf{a}$ of the particle;

(b) the speed v at the instant $t = 1$ s;

(c) the approximate value of the distance s travelled by the particle during the 11th second of its motion.

1.20. A particle is travelling at the velocity $\mathbf{v} = 1\mathbf{e}_x + 2t\mathbf{e}_y + 3t^2\mathbf{e}_z$ (m/s). Find:

(a) the displacement $\Delta\mathbf{r}$ of the particle during the first two seconds of its motion;

(b) the speed at the instant $t = 2$ s.

1.21. A particle is travelling at the velocity $\mathbf{v} = bt \times (2\mathbf{e}_x + 3\mathbf{e}_y + 4\mathbf{e}_z)$ ($b = 1.00$ m/s²). Find:

(a) the speed v of the particle at the instant $t = 1.00$ s;

(b) the acceleration $\mathbf{a}$ of the particle and its magnitude a;

(c) the distance s travelled by the particle from the instant $t_1 = 2.00$ s to the instant $t_2 = 3.00$ s;

(d) what the nature of the particle's motion is.

1.22. A particle travels uniformly along a curvilinear trajectory. Its speed is v. Find the radius of curvature R of the trajectory at the point where the magnitude of the particle's acceleration is a.

1.23. What is the trajectory of a particle if $a_\tau = 0$ and $a_n = \text{const}$?

1.24. At a certain instant t, the components of a particle's velocity $\mathbf{v}$ have the values (1.00, 2.00, —3.00) (m/s), and the components of the acceleration $\mathbf{a}$, the values (—3.00, 2.00, 1.00) (m/s²). Find:

(a) the value of the expression dv/dt at the instant t;

(b) the radius of curvature R of the trajectory at the point where the particle is at the instant t.

1.25. A point is travelling along the x-axis, the coordinate x varying according to the law $x = A \cos \frac{2\pi}{T} t$. Find:

(a) expressions for the projections onto the x-axis of the velocity $\mathbf{v}$ and the acceleration $\mathbf{a}$ of the point;

(b) the distance s_1 travelled by the point during the time interval from $t = 0$ to $t = T/8$;

(c) the distance s_2 travelled by the point during the time interval from $t = T/8$ to $t = T/4$;

(d) the distance s travelled by the point during the time interval from $t = 0$ to $t = T$.

1.26. The components of a particle's velocity change in time according to the laws

$$v_x = A \cos \omega t, \quad v_y = A \sin \omega t, \quad v_z = 0$$

where A and ω are constants. Find the magnitudes of the velocity $\mathbf{v}$ and the acceleration $\mathbf{a}$, and also the angle α between the vectors $\mathbf{v}$ and $\mathbf{a}$. Use the results obtained to determine the nature of the particle's motion.

1.27. The time dependence of a particle's coordinates has the form

$x = A \cos \omega t,\ y = A \sin \omega t, z = 0$

(A and ω are constants)

(a) Determine the position vector $\mathbf{r}$, the velocity $\mathbf{v}$ and the acceleration $\mathbf{a}$ of the particle, and also their magnitudes.

(b) Evaluate the scalar product of the vectors $\mathbf{r}$ and $\mathbf{v}$. What does the result obtained signify?

(c) Evaluate the scalar product of the vectors $\mathbf{r}$ and $\mathbf{a}$. What does the result obtained signify?

(d) Find the equation of the particle's trajectory.

(e) In what direction does the particle move along its trajectory?

(f) Characterize the motion of the particle.

(g) How will the particle's motion change if we reverse the sign in the expression for y?

1.28. A small body (a point particle) is thrown at point O at the angle α to the horizontal with the initial velocity $\mathbf{v}_0$ (Fig. 1.1). Disregarding air resistance, find:

(a) the range l;

(b) the maximum height h of the body;

(c) the time of flight τ;

(d) the equation of the body's trajectory in the coordinates x', y';

(e) the values of $|d\mathbf{v}/dt|$ and $d|\mathbf{v}|/dt$ at the apex of the trajectory;

(f) the radius of curvature R of the trajectory at points O and O'.

Consider the points of throwing and falling to be at the same level.

1.29. A body is thrown at an angle to the horizontal with the initial velocity $\mathbf{v}_0$. Disregarding air resistance, find the mean value of the velocity $\mathbf{v}$ during the first τ seconds of flight.

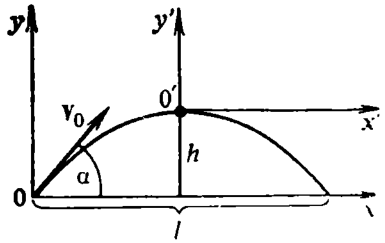

Fig. 1.1

1.30. At what angle α to the horizontal must the barrel of a gun be arranged to hit a target at a range of $l = 10.0$ km if the initial speed of the projectile $v_0 = 500$ m/s? Disregard air resistance.

1.31. We know: (1) the function $f(s)$ determining the dependence of the derivative dv/dt on the distance s travelled by a particle; (2) the value of the speed v_0 at the beginning of this distance. Write an expression for $v(s)$—the particle's speed after travelling the distance s.

1.32. Knowing the function $v(s)$ showing how a particle's speed depends on the distance s it has travelled, write an expression for the time t spent by the particle to cover the distance s.

1.33. The dependence of a particle's speed v on the distance s it has travelled is determined by the function $v(s) = v_0 - bs$.

(a) Find how s depends on the time t.

(b) Determine the dependence of v on t.

(c) Write approximate expressions for $s(t)$ and $v(t)$ holding for $t \ll 1/b$.

1.34. A particle's speed varies with time according to the law $v = v_0 e^{-bt}$. What is the physical meaning of the constant b?

1.35. A boat crosses a river at a velocity $\mathbf{v}$ that is constant relative to the water and perpendicular to the banks. The magnitude of the velocity is 0.300 m/s. The width of the river is $b = 63.0$ m. The speed of the current varies according to the parabolic law

$$u = u_0 - 4\frac{u_0}{b^2}\left(x - \frac{b}{2}\right)^2$$

where x is the distance from the bank, and u_0 is a constant equal to 5.00 m/s. Find how far downstream (s) the boat will land from the point directly opposite its place of starting.

1.36. The x-axis in Fig. 1.2 is the boundary between an area covered with grass and one covered with loose sand. A pedestrian wants to go from A to B. He can walk over the grass at a speed of $v_1 = 5.00$ km/h and over the sand at a speed of $v_2 = 3.00$ km/h. To complete his trip in a shorter time, the pedestrian chooses the broken path AOB. At what ratio between the sines of the angles α_1 and α_2 will the pedestrian's trip from A to B take the shortest time?

1.37. Below are given approximate expressions for selected functions that hold when $x \ll 1$:

(a) $\frac{1}{1 \pm x} \approx 1 \mp x$; (b) $\sqrt{1 \pm x} \approx 1 \pm \frac{x}{2}$; (c) $e^{\pm x} \approx \approx 1 \pm x$; (d) $\ln(1 \pm x) \approx \pm x$; (e) $\sin x \approx x$; (f) $\cos x \approx \approx 1 - \frac{1}{2}x^2$.

Determine for $x = 0.1$ the relative error of the values of these functions found according to the formulas for approximate calculations.

1.38. A motor vehicle is travelling at the constant speed $u = 20.0$ m/s along the straight road AB (Fig. 1.3). A cannon at point C at a distance of $l = 2000$ m from AB fires a shell at the instant when the vehicle reaches a point on the perpendicular from C to AB. Assuming that the shell flies rectilinearly at the constant speed of $v = 200$ m/s, determine:

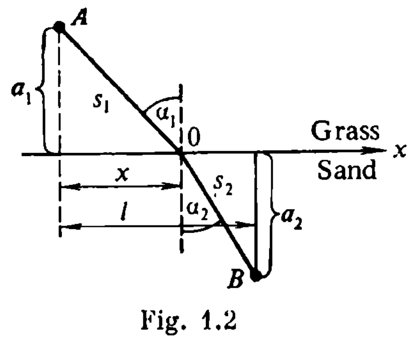

Fig. 1.2

(a) the angle α through which the cannon's barrel must be turned to destroy the vehicle;

(b) the time t of flight of the shell;

(c) the distance s travelled by the vehicle during the time t.

1.39. Two motor boats develop a speed of $v = 5.00$ m/s relative to the water. The speed of the river current is identical over its entire width and is $u = 0.500$ m/s. The

width of the river is $l = 1.000$ km. Two piles C and D have been driven into the river bottom at its middle at a distance from each other equal to the width l of the river (Fig. 1.4). One boat is to cross the river strictly in a transverse direction from point A to B and back. The second boat is to travel from pile C to pile D and back.

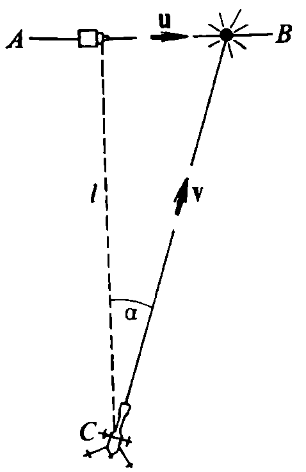

Fig. 1.3

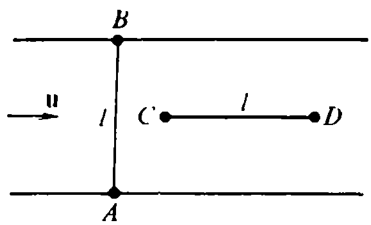

Fig. 1.4

(a) How must the first boat travel relative to the water to move along straight line AB relative to the banks?

(b) Find the times t_1 and t_2 spent on covering the distance $2l$ by the first and second boats.

(c) Obtain approximate expressions for t_1 and t_2 that hold when $u \ll v$. Use these expressions to find the values of t_1 and t_2; compare them with the exact values.

1.40. An aeroplane is flying at a constant speed of $u = 100.0$ m/s along a straight line at an altitude of $h = 5000$ m. At the instant when it is over an antiaircraft battery, a shot is fired (Fig. 1.5). The initial speed of the projectile is $v_0 = 500.0$ m/s. Disregarding air resistance, find:

(a) at what angle α to the horizontal the barrel of a gun must be positioned for the projectile and the aeroplane to reach the point of intersection of their trajectories simultaneously;

(b) to what flight time t the detonator must be set for

the projectile to explode at the point of meeting with the target;

(c) at what distance s along the horizontal the meeting point is from the battery.

1.41. A clock is two minutes slow every day. What does the angular acceleration α of the minute hand equal?

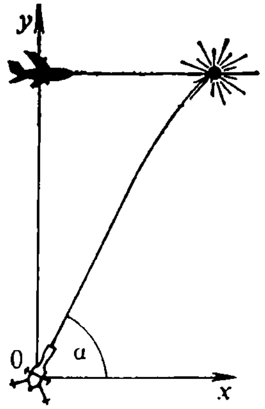

Fig. 1.5

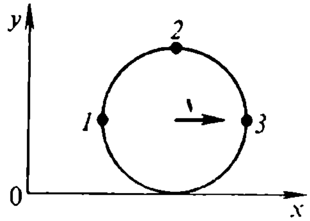

Fig. 1.6

1.42. A vector **a** of constant magnitude rotates at a constant angular velocity $\boldsymbol{\omega}$ about a fixed axis perpendicular to it. Express the derivatives $\dot{\mathbf{a}}$ and $\ddot{\mathbf{a}}$ in terms of the vectors **a** and $\boldsymbol{\omega}$.

1.43. A cylinder rolls without slipping at the velocity **v** (Fig. 1.6). Find the velocities of points *1*, *2*, and *3*. Express them in terms of the unit vectors of the coordinate axes.

1.44. A body participates in two rotations at the velocities $\boldsymbol{\omega}_1 = bt^2\mathbf{e}_x$ and $\boldsymbol{\omega}_2 = 2bt^2\mathbf{e}_y$ ($b = 1.00$ rad/s^3).

(a) Through what angle φ will the body turn during the first 3.00 s?

(b) About what axis will this rotation occur?

1.45. The rotor of an electric motor that had rotated at $n = 50$ rps moved after the current was switched off with uniform retardation and completed $N = 1680$ revolutions before stopping. Find the angular acceleration α of the rotor.

1.46. Before its brakes were applied, a motor vehicle had a speed of $v_0 = 60$ km/h. After application of its brakes, it moved rectilinearly with a varying acceleration and

stopped in $t = 3.00$ s. During this time it travelled the distance $s = 20.0$ m. Determine the mean angular speed $\langle\omega\rangle$ and the mean angular acceleration $\langle\alpha\rangle$ of a wheel of the vehicle during the braking time. The radius of the wheel is $R = 0.23$ m.

1.47. A particle moves along the radius of a rotating disk at a speed of $v = 3.00$ m/s. At the initial instant, the particle is at the centre of the disk. The angular speed of the disk is $\omega = 20.0$ rad/s. Find the approximate value of the distance s travelled by the particle in a stationary reference frame during the time from the instant $t_1 = 9.00$ s to the instant $t_2 = 10.00$ s.

1.2. Dynamics of a Point Particle and of Translational Motion of a Body. Work and Power

Instructions. When solving problems in dynamics, proceed as follows.

1. Establish the bodies which the body being considered interacts with. Accordingly establish the forces acting on this body.

2. Write an equation of motion of the body in the vector form. If the system whose motion is being considered consists of several interconnected bodies, such an equation must be written for each of the bodies separately.

3. Go over in each equation from vectors to their projections onto the appropriately chosen direction. These directions may differ for equations relating to different bodies. If the direction of a vector is known beforehand, its projection must be expressed in terms of the magnitude of the vector taken with the appropriate sign.

4. Solve the system of scalar equations obtained.

We shall explain what has been said above by the following example. A system consists of bodies *1* and *2* connected by a massless* inextensible string (Fig. 1.7). The mass of body *1* is m_1, and of body *2* is m_2. The string can slide without friction over a guide groove. There is no friction between body *1* and the plane on which it lies. The plane

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* Here and in the following by a massless string is meant a string with a negligibly small mass.

makes the angle α with the horizontal. Find the accelerations of bodies *1* and *2*.

1. The system whose motion is being considered consists of bodies *1* and *2* and the string connecting them. Body *1* interacts with the string, the plane, and with the Earth. Accordingly, it experiences the string tensioning force $\mathbf{F}_1$, the force of the plane's reaction $\mathbf{F}_r$, and the force of gravity $m_1\mathbf{g}$. Body *2* interacts with the string and the Earth. Accordingly, it experiences the string tensioning force $\mathbf{F}_2$ and the force of gravity $m_2\mathbf{g}$. The string is massless, therefore it interacts only with bodies *1* and *2*, and also with the groove. The relevant forces are $-\mathbf{F}_1$ and $-\mathbf{F}_2$ (these forces are not shown in the figure; they are opposite in direction to the forces $\mathbf{F}_1$ and $\mathbf{F}_2$); there is also a force distributed over the entire length of the groove. All the elements of this force are perpendicular to the string and do not affect its motion in a longitudinal direction.

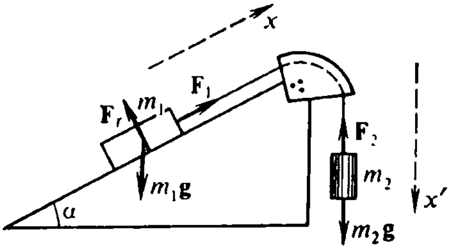

Fig. 1.7

2. The equations of motion of bodies *1* and *2* have the form

$$m_1\mathbf{a}_1 = \mathbf{F}_1 + \mathbf{F}_r + m_1\mathbf{g}, \quad m_2\mathbf{a}_2 = \mathbf{F}_2 + m_2\mathbf{g}$$

The mass of the string is zero. Therefore the equation of motion for the string is $0 = 0$. Writing it has no meaning.

It follows from the masslessness of the string that the magnitudes of the forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are identical. Let us designate this identical magnitude by F ($F_1 = F_2 = F$). It follows from the string being inextensible that the displacements and, consequently, the velocities and accelerations of bodies *1* and *2* are identical in magnitude.

3. It is good to "project" the equation for body *1* onto the x-axis, that for body *2*—onto the x'-axis (see Fig. 1.7). In doing this, it must be taken into account that (a) the projection of the force $\mathbf{F}_r$ onto the x-axis is zero; (b) the projection of the force $m_1\mathbf{g}$ onto the x-axis is $-m_1 g \sin\alpha$; (c) the projection of the force $\mathbf{F}_1$ onto the x-axis is $F_1 = F$; (d) the projection of the force $\mathbf{F}_2$ onto the x'-axis is $-F_2 =$

$= -F$; (e) the projection of the force $m_2\mathbf{g}$ onto the x'-axis is $m_2 g$; (f) the projection of the acceleration $\mathbf{a}_1$ of body *1* onto the x-axis (i.e. $a_{1,x}$) and that of the acceleration $\mathbf{a}_2$ of body *2* onto the x'-axis (i.e. $a_{2,x'}$) are identical in both magnitude and sign. We therefore introduce the symbol $a_x = a_{1,x} = a_{2,x'}$.

4. With a view to everything said above, the equations of motion in projections can be written as follows:

$$m_1 a_{1,x} = F_1 - m_1 g \sin\alpha \quad \text{or} \quad m_1 a_x = F - m_1 g \sin\alpha$$
$$m_2 a_{2,x'} = m_2 g - F_2 \quad \text{or} \quad m_2 a_x = m_2 g - F$$

By solving this system of two equations, we obtain the following value for a_x:

$$a_x = \frac{m_2 - m_1 \sin\alpha}{m_1 + m_2} g$$

Depending on the relation between the masses m_1 and m_2, and also on the value of the angle α, the projection of the acceleration a_x may be either positive or negative, or be equal to zero. When $a_x > 0$, the acceleration of body *1* is directed to the right along the x-axis, and that of body *2* is directed downward. When $a_x < 0$, the directions of the accelerations are the opposite. Finally, when $a_x = 0$, the system is either at rest or moves uniformly in the direction of the velocity communicated to it.

1.48. To determine the coefficient of friction f between wooden surfaces, a block was placed on a board and one end of the latter was lifted until the block began to slide down it. This occurred when the board was inclined at an angle of $\alpha = 14°$. What does f equal?

1.49. Two blocks in contact with each other are on a horizontal table along which they can slide without friction. The mass of the first block is $m_1 = 2.00$ kg, and that of the second block is $m_2 = 3.00$ kg. One of the blocks is pushed with a force $F_0 = 10.0$ N (Fig. 1.8).

1. Find the force F with which the blocks press on each other if the force F_0 is applied (a) to block *1*; (b) to block *2*.
2. What is noteworthy in the results obtained?

1.50. Solve the preceding problem assuming that the coefficient of friction between a block and the table is $f_1 = 0.100$ for block *1* and $f_2 = 0.200$ for block *2*.

1.51. Solve the preceding problem assuming that $f_1 = 0.200$ and $f_2 = 0.100$.

Compare the results of Problems 1.49, 1.50, and the given problem.

1.52. Two blocks in contact with each other are sliding down an inclined board (Fig. 1.9). The mass of the first block is $m_1 = 2.00$ kg, and that of the second one is $m_2 = 3.00$ kg. The coefficient of friction between a block and the board is $f_1 = 0.100$ for block *1* and $f_2 = 0.200$ for block *2*. The angle of inclination of the board is $\alpha = 45°$.

1. Determine: (a) the acceleration a of the blocks; (b) the force F with which the blocks press against each other.

2. What would happen if $f_1 > f_2$?

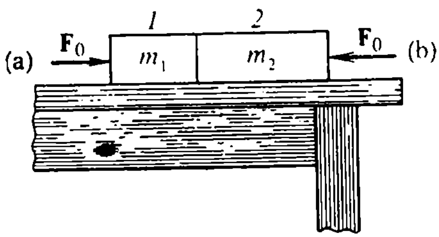

Fig. 1.8

1.53. Two bodies each of mass $M = 1.000$ kg are on a horizontal table (Fig. 1.10). The bodies are connected by a massless inextensible string. A similar string connects body *2* to a weight of mass $m = 0.500$ kg. The string can slide without friction over a bent groove fastened to the edge of the table. The coefficient of friction of the first body with the table is $f_1 = 0.100$, and of the second body is $f_2 = 0.150$. Find:

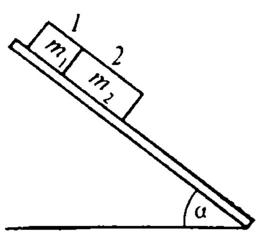

Fig. 1.9

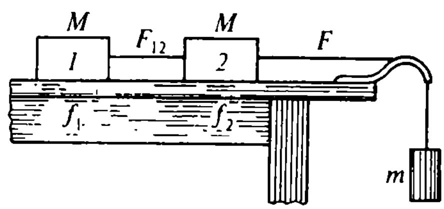

Fig. 1.10

(a) the acceleration a of the bodies;

(b) the tension F_{12} of the string connecting bodies *1* and *2*;

(c) the tension F of the string on which the weight is suspended.

1.54. An overpass at a highway intersection has a radius of curvature of $R = 1000$ m. The pavement of the top part

of the overpass contains transmitters registering the pressure force acting on it. The instrument registering this force is graduated in kgf (1 kgf = 9.81 N). What force F does the instrument show at the instant when an automobile with a mass of $m = 1.000$ tonne rides over the overpass at a speed of $v = 60.0$ km/h?

1.55. A charged particle moving in a magnetic field experiences the magnetic force $\mathbf{F} = q[\mathbf{vB}]$ (q is the charge of the particle, $\mathbf{v}$ is its velocity, and $\mathbf{B}$ is a characteristic of the field known as its magnetic induction). Find the equation of the trajectory along which the particle will move in the homogeneous magnetic field (i.e. a field at all of whose points $\mathbf{B}$ is identical in magnitude and direction) if at the initial instant the vector $\mathbf{v}$ is perpendicular to $\mathbf{B}$. There are no forces except for the magnetic one. Consider that the mass m, charge q, and speed v of the particle, and also the magnetic induction B of the field are known. Take the plane in which the particle is moving as the coordinate plane x, y.

1.56. A ball of mass $m = 0.200$ kg is tied to a string with a length of $l = 3.00$ m whose other end is fixed in place. The ball rotates in a horizontal plane along a circle of radius $R = 1.00$ m. Find:

(a) the number of revolutions n of the ball per minute;

(b) the tension F of the string.

1.57. A horizontal disk rotates about a vertical axis passing through its centre at a speed of $n = 10.0$ rpm. At what distance r from the centre of the disk can a small body placed on it remain in place if the coefficient of friction $f = 0.200$?

1.58. An initial momentum is imparted to a small body as a result of which it begins to move translationally without friction up an inclined plane at a speed of $v_0 = 3.00$ m/s. The plane makes the angle $\alpha = 20.0°$ with the horizontal. Determine:

(a) the height h which the body will rise to;

(b) the time t_1 during which the body will move upward until it stops;

(c) the time t_2 needed for the body to slide down to its initial position;

(d) the body's speed v at the instant when it returns to its initial position.

1.59. Solve the preceding problem assuming that the coefficient of friction between the body and the plane is $f = 0.100$. The mass of the body is $m = 1.00$ kg. In addition to the quantities indicated above in Problem 1.58, determine:

(e) the work A done by the force of friction over the entire upward and downward path.

Compare the results obtained with the answer to the preceding problem.

1.60. A ball of mass m is placed in a tall vessel containing a liquid and is released without a push. The density of the liquid is $1/\eta$ of that of the ball. When the ball moves, a force of resistance of the medium appears that is proportional to the speed: $F = -kv$.

(a) Describe the nature of the ball's motion qualitatively.

(b) Find the time dependence of the ball's speed, i.e. the function $v = f(t)$.

1.61. A thin steel chain with very small links having a length of $l = 1.000$ m and a mass of $m = 10.0$ g is on a horizontal table. The chain is stretched out into a straight line perpendicular to the table's edge. The end of the chain hangs down from the table's edge. When the length of the hanging part is $\eta = 0.275$ of the entire length, the chain begins to slide down from the table. Considering the chain to be homogeneous in length, find:

(a) the coefficient of friction f between the chain and the table;

(b) the work A of the forces of friction of the chain against the table during the time of sliding;

(c) the speed v of the chain at the end of sliding.

1.62. A thin steel chain with very small links is hanging vertically and its bottom end is touching a table. The mass of the chain is m and its length is l. At the instant $t = 0$, the chain is released. Considering the chain to be homogeneous in length, find:

(a) the instantaneous value $F(t)$ of the force with which the chain acts on the table;

(b) the mean value of this force $\langle F \rangle$ during the time of falling.

1.63. A force acting on a particle has the form $\mathbf{F} = b\mathbf{e}_x$ (N), where b is a constant. Evaluate the work A done by this force on the particle along the path from a

point with the coordinates (1, 2, 3) (m) to a point with the coordinates (7, 8, 9) (m).

1.64. A particle travels uniformly in a circle. What is the work A done by the resultant of all the forces acting on the particle: (a) during one revolution; (b) during half a revolution; (c) during a quarter of a revolution?

1.65. A particle travels in a circle of radius r under the action of a central force $\mathbf{F}$. The centre of the circle coincides with the force centre. What is the work A done by the force $\mathbf{F}$ over the distance s?

1.66. The tangential acceleration a_τ of a particle moving along a curvilinear trajectory changes with the distance s measured along the trajectory from a certain initial position of the particle according to the law $a_\tau = a_\tau(s)$. The mass of the particle is m. Write an expression for the work A done on the particle by all the forces acting on it on the portion of the trajectory from s_1 to s_2.

1.67. A body of mass $m = 1.00$ kg falls from a height of $h = 20.0$ m. Disregarding air resistance, find:

(a) the mean power in time $\langle P \rangle$ developed by the force of gravity along the path h; (b) the instantaneous power P at the height $h/2$.

1.68. A stone of mass m when thrown rises above the level of the point of throwing to the height h. At the top point of the stone's trajectory, its speed is v. The force of air resistance does the work A_{res} on the stone along the path from the point of throwing to the top of the trajectory. What is the work A of throwing the stone?

1.69. A body of mass m is thrown at the angle α to the horizontal with the initial speed v_0. Disregarding air resistance, find:

(a) the instantaneous power $P(t)$ developed in the body's flight by the force applied to it;

(b) the value of the power P at the top of the trajectory;

(c) the mean value of the power $\langle P \rangle_{\text{rise}}$ during the time the body rises;

(d) the mean value of the power $\langle P \rangle_{\text{fl}}$ during the entire time of flight (the point of throwing and that of falling are at the same level).

1.70. A body of mass m begins to move under the action of the force $\mathbf{F} = 2t\mathbf{e}_x + 3t^2\mathbf{e}_y$. Find the power $P(t)$ developed by the force at the instant t.

1.3. Energy

1.71. Find the increment of the energy ΔE if (a) $E_1 = 2$ J, $E_2 = 5$ J; (b) $E_1 = 10$ J, $E_2 = 8$ J.

1.72. For the values of the initial E_1 and final E_2 energies indicated in the preceding problem, find the decrement of the energy $-\Delta E$.

1.73. A particle initially at rest under the action of the force $\mathbf{F} = 1\mathbf{e}_x + 2\mathbf{e}_y + 3\mathbf{e}_z$ (N) moved from the point (2, 4, 6) (m) to the point (3, 6, 9) (m). Find the particle's kinetic energy E_k at the terminal point.

1.74. Being under the action of a constant force with the components (3, 10, 8) (N), a particle moved from point *1* with the coordinates (1, 2, 3) (m) to point *2* with the coordinates (3, 2, 1) (m).

(a) What work A is done here?

(b) How did the kinetic energy of the particle change?

(c) What can be said about the value of the particle's kinetic energy at point *1*?

1.75. Prove the relation

$$E_{k,l} = E_{k,C} + \frac{mV_C^2}{2}$$

where $E_{k,l}$ is the kinetic energy of a system of point particles determined in a laboratory reference frame (an l-frame), $E_{k,C}$ is the kinetic energy determined in a centre-of-mass frame (a c.m.-frame), m is the total mass of the system, and V_C is the speed of the centre of mass in the l-frame.

1.76. The potential energy of a particle in a force field is determined by the expression $E_p = 1.00x + 2.00y^2 + 3.00z^3$ (E_p is in J, the coordinates are in m). Find the work A done on the particle by the forces of the field in a transition from a point with the coordinates (1.00, 1.00, 1.00) to a point with the coordinates (2.00, 2.00, 2.00).

1.77. The potential energy of a particle is determined by the expression $E_p = a(x^2 + y^2 + z^2)$, where a is a positive constant. The particle begins to move from a point with the coordinates (3.00, 3.00, 3.00) (m). Find its kinetic energy E_k at the instant when the particle is at a point with the coordinates (1.00, 1.00, 1.00) (m).

1.78. Two bodies slide without friction and without an initial velocity down inclined planes *1* and *2* (Fig. 1.11).

(a) Compare the speeds v_1 and v_2 of the bodies at the end of sliding.

(b) Are the sliding times t_1 and t_2 the same?

1.79. Two inclined planes coincide with the chords of the same circle of radius R (Fig. 1.12). A small body slides down each of them without friction and without an initial velocity. For which of the planes is the time of sliding greater?

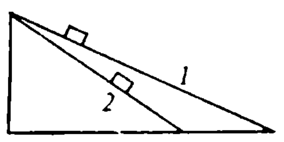

Fig. 1.11

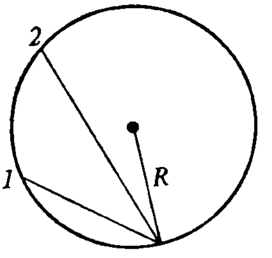

Fig. 1.12

1.80. A small body of mass m is placed at the top of an inclined plane and the initial speed v_0 is imparted to it, as a result of which it begins to slide down the plane (Fig. 1.13). The surface of the plane is not uniform, therefore the sliding speed varies arbitrarily. But at the bottom of the plane, the speed has its initial value v_0. The difference in height between the top and the bottom of the plane is h. What work A do the forces of friction do over the entire path of the body?

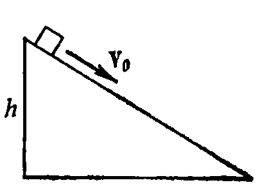

Fig. 1.13

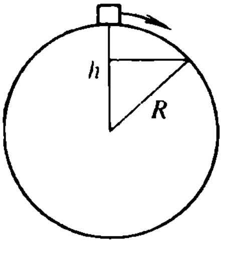

Fig. 1.14

1.81. A small body begins to slide down from the top of a sphere (Fig. 1.14) without friction. At what height h

above the centre of the sphere will the body diverge from the sphere's surface and fly freely? The sphere's radius is R.

1.82. A small body (a point particle) begins to slide without friction from the height h along a groove having the shape shown in Fig. 1.15 (the horizontal portion of the groove is shifted relative to the inclined one in a direction perpendicular to the figure).

(a) At what minimum value of the height h will the body make a complete loop without losing contact with the groove?

(b) With what force F does the body press on the groove at point A at this value of h?

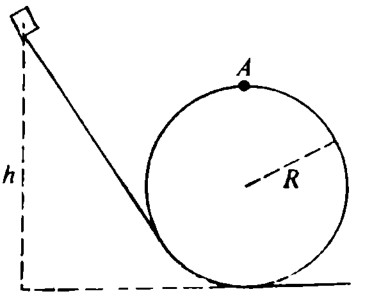

Fig. 1.15

1.83. The gradient of a scalar function φ at a point P is a vector whose direction coincides with that of $\mathbf{l}$ along which the function φ, increasing in magnitude, changes at point P at the greatest rate. The magnitude of this vector equals the value of $d\varphi/dl$ at point P. This can be written analytically as follows:

$$\nabla\varphi = \frac{d\varphi}{dl}\mathbf{e}_l$$

1. Proceeding from this definition, find expressions for: (a) ∇r; (b) $\nabla\left(\frac{1}{r}\right)$; (c) $\nabla f(r)$, where r is the magnitude of the position vector of point P.

2. Convince yourself that the same expressions are obtained with the aid of the formula

$$\nabla\varphi = \frac{\partial\varphi}{\partial x}\mathbf{e}_x + \frac{\partial\varphi}{\partial y}\mathbf{e}_y + \frac{\partial\varphi}{\partial z}\mathbf{e}_z$$

1.84. The potential energy of a particle has the form: (a) $E_p = \alpha/r$; (b) $E_p = \frac{1}{2}kr^2$, where r is the magnitude of the particle's position vector $\mathbf{r}$, and α and k are constants ($k > 0$). Find the force $\mathbf{F}$ acting on the particle and the work A done on the particle when it passed from the point (1, 2, 3) to the point (2, 3, 4).

1.85. The potential energy of a particle has the form

$$E_p = a\left(\frac{x}{y} - \frac{y}{z}\right)$$

where a is a constant. Find:

(a) the force **F** acting on the particle;

(b) the work A done on the particle by the forces of the field when the particle passed from the point (1, 1, 1) to the point (2, 2, 3).

1.86. The potential energy of a particle in a centrally symmetrical force field has the form

$$E_p = \frac{a}{r^3} - \frac{b}{r^2}$$

where a and b are positive constants.

(a) Does the particle have a position of stable equilibrium with respect to displacements in a radial direction?

(b) Draw approximately the curve showing how E_p depends on r.

1.87. A particle moves in a circle in the field of a central force inversely proportional to the square of the distance from the force centre. What are the relations in this case between the kinetic E_k, potential E_p, and total E energies of the particle?

1.88. A particle of mass m is in a force field of the kind $\mathbf{F} = -(\alpha/r^2)\,\mathbf{e}_r$ (where α is a positive constant, r is the magnitude, and $\mathbf{e}_r$ is the unit vector of the particle's position vector). The particle was placed at a point with the position vector $\mathbf{r}_0$, and the initial velocity $\mathbf{v}_0$ perpendicular to $\mathbf{r}_0$ was imparted to it. What will the particle's trajectory be?

1.89. In what condition will the trajectory of the particle from the preceding problem be a circle?

1.90. A massless inextensible string can slide without friction over a grooved member (Fig. 1.16). Bodies *1* and *2* of mass $m_1 = 3.00$ kg and $m_2 = 1.00$ kg are attached to the ends of the string. Body *1* is lifted until body *2* touches the floor and is then released. The height $h_1 = 1.00$ m. What height h_2 above the floor will body *2* rise to after body *1* falls onto the floor?

1.91. An automobile with a mass of $m = 1.00$ tonne rode for a certain time along a level section of a highway at a constant speed of $v = 80$ km/h. When it drove onto an

ascending grade making the angle $\alpha = 10.0°$ with the horizontal, its driver, to retain its previous speed, had to "step on the gas" and increase the torque applied to the driving wheels $\eta = 6.20$ times. Considering the force F of air resistance to the automobile's motion proportional to the square of the speed, determine the coefficient k in the formula $F = kv^2$. Disregard the friction in the tyres.

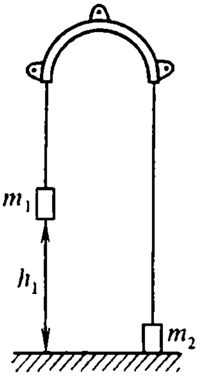

Fig. 1.16

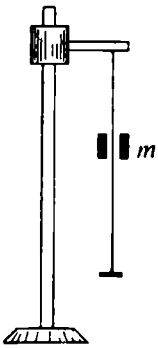

Fig. 1.17

1.92. A sleeve with a mass of $m = 0.300$ kg can slide with friction independent of the speed along a rubber cord, one end of which is fastened to an arm (Fig. 1.17). The magnitude of the friction is characterized by the force $F = 0.294$ N. The length of the undeformed cord is $l_0 = 1.00$ m, the proportionality factor between the elastic force and the elongation of the cord is $k = 560$ N/m. There is a stop on the lower end of the cord. The sleeve is lifted to its extreme top position and released. Disregarding the internal friction in the cord, the dimensions of the sleeve, and also the masses of the cord and the stop, determine:

(a) the elongation Δl of the cord at the instant when the sleeve reaches the stop;

(b) the speed v of the sleeve at this instant;

(c) the maximum elongation Δl_{max} of the cord.

1.4. Momentum

1.93. A system consists of particle *1* with a mass of 0.100 g, particle *2* with a mass of 0.200 g, and particle *3* with a mass of 0.300 g. Particle *1* is placed at a point with the coordinates (1.00, 2.00, 3.00), particle *2* at a point with

the coordinates (2.00, 3.00, 1.00), and particle *3* at a point with the coordinates (3.00, 1.00, 2.00) (the values of the coordinates are in metres). Find the position vector $\mathbf{r}_C$ of the centre of mass of the system.

1.94. A homogeneous circular cone has an altitude of h. At what distance l from its vertex is its centre of mass?

1.95. What does the momentum $\mathbf{p}$ of a system of particles equal in the reference frame of their centre of mass?

1.96. How does the centre of mass behave if the total momentum of a system of particles is zero?

1.97. A system of interacting bodies is in the field of gravity forces near the Earth's surface. How does the centre of mass of the system behave? Disregard air resistance.

1.98. A body of mass m was thrown with the initial velocity $\mathbf{v}_0$ at an angle to the horizontal. After the time τ elapsed, the body fell onto the Earth. Disregarding air resistance, find:

(a) the increment of the body's momentum $\Delta\mathbf{p}$ during the time of flight;

(b) the mean value of the momentum $\langle\mathbf{p}\rangle$ during the time τ.

1.99. A particle of mass m moves in the plane x, y under the action of a force $\mathbf{F}$ of constant magnitude rotating clockwise in this plane at a constant angular speed ω. At the initial instant, the force is directed along the x-axis, and the particle's velocity is $\mathbf{v}_0$. Find the particle's momentum at the instant t.

1.100. Two spheres move toward each other along the straight line passing through their centres. The mass and speed of the first sphere are 4.00 kg and 8.00 m/s, and of the second sphere are 6.00 kg and 2.00 m/s. How will the spheres move after a completely inelastic collision?

1.101. Two spheres experience a central completely inelastic collision. Before it, the sphere of mass m_2 was stationary, and that of mass m_1 moved at a certain speed. What part η of the initial kinetic energy is lost during the collision if: (a) $m_1 = m_2$; (b) $m_1 = 0.1m_2$; (c) $m_1 = 10m_2$?

1.102. A sphere of mass m_1 participates in a central perfectly elastic collision with a sphere of mass m_2 at rest.

(a) At what relation between the masses m_1 and m_2 will the first sphere fly after the collision in the opposite direction?

(b) What will happen to the first sphere if both spheres have the same mass?

(c) What will happen to the first sphere if $m_1 \ll m_2$?

1.103. Two balls move toward each other along the x-axis. The mass of the first ball is $m_1 = 0.200$ kg, and that of the second is $m_2 = 0.300$ kg. Before the collision, the projections of the balls' velocities onto the axis are $v_{10} = 1.00$ and $v_{20} = -1.00$ m/s.

(a) Find the projections of the balls' velocities v_{1x} and v_{2x} after their central perfectly elastic collision.

(b) Having obtained the numerical values of the projections of the velocities, verify the observance of the laws of energy and momentum conservation.

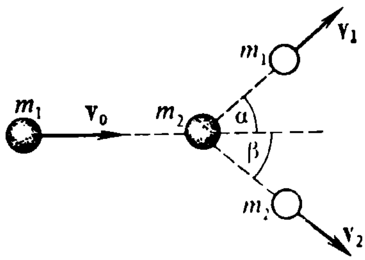

Fig. 1.18

1.104. Ball *1* of mass m_1 travelling at the velocity $\mathbf{v}_0$ collides with stationary ball *2* of mass m_2. After a perfectly elastic collision, the balls fly at the velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ in the directions shown in Fig. 1.18.

1. At what relation between the masses m_1 and m_2 are the following cases possible: (a) $\alpha = \pi/2$; (b) $\alpha = \beta \neq 0$; (c) $\alpha = \beta = 0$; (d) $\alpha = \pi$, $\beta = 0$?

2. Is the case $\beta = \pi/2$ possible?

3. What does the extreme possible value of the angle β equal when $\alpha = \pi/2$?

4. What relative fraction η of its kinetic energy will the first ball transfer to the second one in the cases: (a) $\alpha = \pi/2$; (b) $\alpha = \beta \neq 0$; (c) $\alpha = \beta = 0$; (d) $\alpha = \pi$, $\beta = 0$?

5. Compare the results of 4a, 4b, 4c, and 4d.

6. What does the extreme value of η equal in case 4b?

7. At what values of m_1, m_2, and β will ball *1* be at rest after the collision?

8. Find the angle β if: (a) $\alpha = \pi/2$ and $m_1 = 0.99 m_2$; (b) $\alpha = \beta \neq 0$ and $m_1 = m_2$.

9. Compare the angle of divergence of the balls (i.e. $\alpha + \beta$) in cases 8a and 8b.

10. Prove that when m_1 m_2 at any value of α (within the limits from 0 to $\pi/2$) the angle of divergence of the balls is $\pi/2$.

1.105. Two identical spheres experience a central collision. Before the collision, the second sphere is stationary, and the first one travels at a speed of $v_{10} = 1.000$ m/s. The nature of the collision is such that the loss of energy is exactly half of what it would be in a completely inelastic collision. Determine the speeds v_1 and v_2 of the spheres after the collision. Compare the results with what would be obtained for a perfectly elastic and a completely inelastic collision.

1.106. Solve the preceding problem provided that the loss of energy (a) is one-tenth, and (b) is nine-tenths of what would occur in a completely inelastic collision.

1.107. A child when playing inadvertently threw a ball at a lorry after the latter had driven past him. At what speed v will the ball rebound from the rear wall of the lorry if the speed of the latter is $u = 7.0$ m/s, the velocity $\mathbf{v}_0$ of the ball directly before the impact had a magnitude of 15.0 m/s and was directed along a normal to the wall's surface. Consider that the impact is perfectly elastic.

1.108. A proton begins to move in a direction toward an alpha particle freely at rest from "infinity" (i.e. from a distance at which the interaction between the proton and the particle is negligibly small) at a speed of $v_0 = 1.00 \times 10^6$ m/s. Considering the "collision" to be central, determine the minimum distance $r_{\min}$ to which the particles will approach each other.

When solving the problem, take into account that the mutual potential energy of two point charges q_1 and q_2 at a distance r from each other is $E_p = k\frac{q_1q_2}{r}$ (compare with the expression $E_p = -G\frac{m_1m_2}{r}$ for the mutual potential energy of two point masses in gravitational attraction). In the SI, the numerical value of the proportionality factor k is 9×10^9. The charge of a proton is $+e$, and that of an alpha particle is $+2e$, where e is the elementary charge. The mass of a proton is $m_p = 1.67 \times 10^{-27}$ kg, and that of an alpha particle is $m_\alpha = 6.64 \times 10^{-27}$ kg.

1.109. The water-jet engine of a motorboat takes in water from the river and discharges it at a speed of $u = 10.0$ m/s backward relative to the boat. The mass of the boat is $M = 1000$ kg. The mass of the water discharged every sec-

ond is constant and is $m = 10.0$ kg/s. Disregarding the resistance to the motion of the boat, determine:

(a) the boat's speed v in $t = 1.00$ min after the beginning of motion;

(b) the maximum speed v_{max} which the boat can attain.

1.5. Angular Momentum

1.110. A force with the components (3, 4, 5) (N) is applied to a point with the coordinates (4, 2, 3) (m). Find:

(a) the moment **M** of the force relative to the origin of coordinates;

(b) the magnitude of the vector **M**;

(c) the moment M_z of the force relative to the z-axis.

1.111. Rotation from an automobile's engine is transmitted to its driving wheels via a number of devices, one of which, called a clutch, makes it possible when required to disengage the engine from the other devices. One type of clutch in general use consists in principle of two identical friction members pressed against each other by strong springs. In the "Lada" automobile, the friction members are rings with an inner diameter of $d_1 = 142$ mm and an outer one of $d_2 = 203$ mm. The coefficient of friction of one member over the other one is $f = 0.35$. Find the smallest force F with which the members must be pressed together to transmit a torque of $M = 100$ N·m.

1.112. A body of mass $m = 1.00$ kg is thrown vertically upward at an initial speed of $v_0 = 10.0$ m/s from a point with the coordinates (0, 2, 0) (m). Find the increment of the angular momentum $\Delta\mathbf{L}$ relative to the origin of coordinates during the entire time of flight of the body (until it returns to its initial point). The z-axis is directed upward. Disregard air resistance.

1.113. A body of mass m is thrown at the initial velocity $\mathbf{v}_0$ making the angle α with the horizontal. Taking the plane in which the body moves as the plane x, y and directing the y-axis upward and the x-axis in the direction of motion, find the vector of the angular momentum of the body relative to the point of throwing at the instant when the body is at the top point of its trajectory. Disregard air resistance.

1.114. Two particles are moving uniformly in opposite directions along parallel rectilinear trajectories (Fig. 1.19). The distance between the trajectories is l. In the figure, $\mathbf{n}$ designates a normal to the plane in which the trajectories of the particles are. It is directed away from the viewer.

Find: (a) the total momentum $\mathbf{p}$ of the particles; (b) the values $\mathbf{L}_1$ and $\mathbf{L}_2$ of the total angular momenta of the particles taken relative to points O_1 and O_2 shown in the figure.

Consider two cases: 1. The momenta of the particles differ in magnitude. 2. The magnitudes of the momenta of the particles are the same: $p_1 = p_2 = p$.

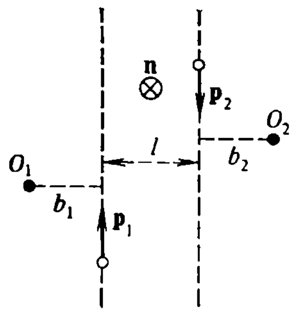

Fig. 1.19

1.115. A closed system consists of n interacting particles. Owing to the interaction between the particles, their momenta $\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n$ are functions of time: $\mathbf{p}_i = \mathbf{p}_i(t)$. But since the system is closed, we have $\sum \mathbf{p}_i = \text{const}$. Prove that when the total momentum of the system is zero, the angular momentum of the system does not depend on the choice of the point relative to which it is determined.

1.116. Prove the relation

$$\mathbf{L}_O = \mathbf{L}_C + [\mathbf{R}_C \mathbf{p}]$$

where $\mathbf{L}_O$ is the angular momentum of a system of point particles relative to the origin O of a laboratory reference frame (an l-frame), $\mathbf{L}_C$ is the angular momentum relative to the centre of mass C (the intrinsic angular momentum), $\mathbf{R}_C$ is the position vector of the centre of mass in the l-frame, and $\mathbf{p}$ is the total momentum of the system of particles determined in the l-frame.

1.117. A small body (a point particle) of mass m begins to slide without friction from the top of an inclined plane (Fig. 1.20). The letter $\mathbf{n}$ in the figure designates a normal directed away from the viewer. Find expressions for:

(a) the moment $\mathbf{M}$ of the resultant force acting on the body relative to point O;

(b) the angular momentum $\mathbf{L}(t)$ of the body relative to point O.

1.118. A point particle of mass m is thrown at an angle α to the horizontal at the initial velocity $\mathbf{v}_0$. The trajectory of the particle's flight is in plane x, y (Fig. 1.21; the z-axis is directed toward the viewer). Disregarding air resistance, find the time dependence of:

(a) the moment $\mathbf{M}$ of the force acting on the particle;

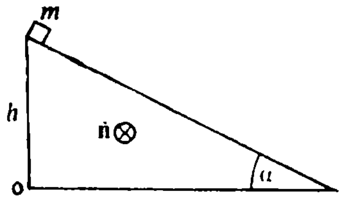

Fig. 1.20

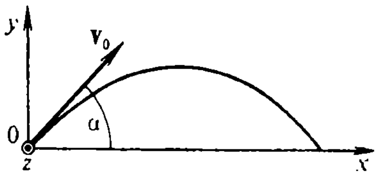

Fig. 1.21

(b) the angular momentum $\mathbf{L}$ of the particle.

Evaluate both quantities relative to the point of throwing.

1.119. A body of mass $m = 0.100$ kg is thrown from a certain height in a horizontal direction at the speed $v_0 = 20.0$ m/s. Find the magnitude of the increment of the body's angular momentum $|\Delta\mathbf{L}|$ relative to the point of throwing during the first $\tau = 5.00$ s. Disregard air resistance.

1.120. Four identical spheres each with a mass of $m = 0.300$ kg are joined in pairs with the aid of massless rods having a length each of $l = 1.000$ m into two dumbbells. The size of the spheres is much smaller than l, therefore they can be considered as point particles. The dumbbells move translationally toward each other at the same speed $v = 1.000$ m/s (Fig. 1.22). Considering the impact of the spheres to be instantaneous and perfectly elastic,

(a) characterize the motion of the dumbbells after the collision;

(b) find the angular speed ω of rotation of the dumbbells;

(c) determine the time τ during which this rotation occurs;

(d) characterize the motion of the dumbbells after the time τ elapses.

1.121. Solve the preceding problem, considering the impact to be completely inelastic.

(a) Characterize the motion of the dumbbells after the impact.

(b) Find the speed v_C at which the centres of the dumbbells are moving.

(c) Calculate the angular speed ω of rotation of the dumbbells.

(d) Determine how the mechanical energy of the system changes.

1.122. There is a system of two dumbbells similar to that described in Problem 1.120. Initially, the left-hand dumbbell is at rest, while the right-hand one moves translationally at a speed of $2v$ (Fig. 1.23). Answer the questions (a)-(d) of Problem 1.120.

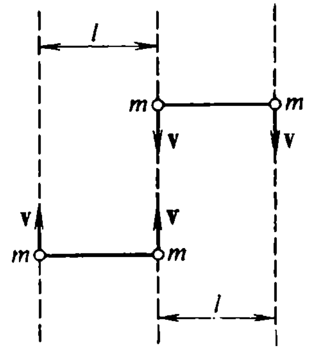

Fig. 1.22

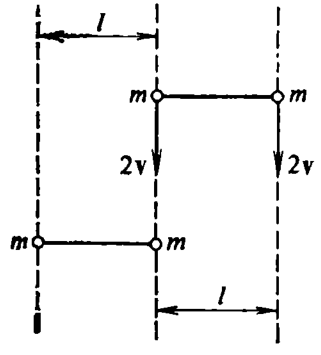

Fig. 1.23

1.123. Solve a problem similar to Problem 1.121, the only difference being that initially the left-hand dumbbell is at rest, while the right-hand one moves translationally at a speed of $2v$.

1.124. The maximum distance from the Sun to the Earth is $R_{max} = 1.52 \times 10^{11}$ m, the minimum is $R_{min} = 1.47 \times 10^{11}$ m, and the mean distance is $R = 1.495 \times 10^{11}$ m. Proceeding from these data, find the mean $\langle v \rangle$, maximum v_{max}, and minimum v_{min} speeds of the Earth in its orbit. Compare the maximum and minimum speeds with the mean one.

1.125. What is the reduced mass μ of a system of two particles having the same mass m?

1.126. Find the approximate value of the reduced mass μ of particles having the masses m and M for the case when $m \ll M$.

1.6. Non-Inertial Reference Frames

1.127. A horizontal disk rotates at the angular speed ω_0. A body on the disk is at rest relative to it. The mass of the body is m, and its distance from the axis of rotation is r.

(a) What forces act on the body in a stationary reference frame?

(b) In what frame is only the centrifugal force of inertia added to the preceding forces?

(c) In what frame does a Coriolis force also appear?

1.128. What power P does the Coriolis force develop?

1.129. What work A does the Coriolis force do on a particle when the latter moves relative to a rotating reference frame from point *1* to point *2*?

1.130. The motion of a particle of mass $m = 10.0$ g is considered in a reference frame rotating relative to an inertial frame at the angular speed of $\omega = 10.0$ rad/s. What work A do the forces of inertia do on the particle when it moves from a point at a distance of $R_1 = 1.00$ m from the axis of rotation to a point at a distance of $R_2 = 2.00$ m from it?

1.131. A small body falls without an initial velocity onto the Earth at the equator from a height of $h = 10.0$ m. In what direction and over what distance x will the body deflect from the vertical during its time τ of falling? Disregard air resistance. Compare the found value of x with the difference Δs of the distances that will be travelled owing to the Earth's rotation during the time τ by a point at the height h and by one on the Earth's surface.

1.132. The muzzle of a horizontally arranged rifle is at the axis of a vertical cylinder of radius R (Fig. 1.24). The cylinder rotates at an angular speed of ω.

(a) Considering that a bullet fired from the rifle flies horizontally at the constant speed v, find the displacement s of point B of the cylinder hit by the bullet relative to point A that was opposite the muzzle at the instant when the shot was fired. Solve the problem in two ways: in a sta-

tionary reference frame and in a frame associated with the cylinder.

(b) Does the result depend on whether the rifle rotates together with the cylinder or is stationary?

1.133. A shot was fired at a latitude of $\varphi = 45°$ from a rifle fastened horizontally in the plane of the meridian at a target at a distance of $l = 100.0$ m from the rifle's muzzle. The centre of the target is on the centre line of the rifle's barrel. Considering that the bullet flies horizontally at a constant speed of $v = 500$ m/s, determine over what distance and in what direction the bullet will deviate from the centre of the target if the rifle is fired in a direction (a) to the north; (b) to the south.

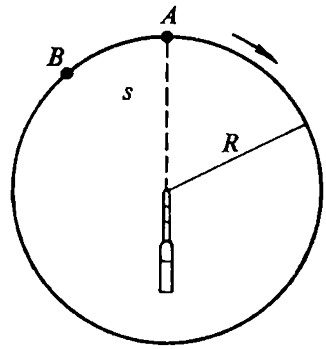

Fig. 1.24

1.134. An electric locomotive of mass $m = 184 \times 10^3$ kg moves along a meridian at a speed of $v = 20.0$ m/s (72 km/h) at a latitude of $\varphi = 45°$. Determine the horizontal component F of the force with which the locomotive presses on the rails.

1.135. A horizontal disk rotates about an axis passing through its centre at an angular velocity of ω. A particle moves along the disk uniformly at a constant distance from the axis of rotation. Find the instantaneous value:

(a) of the velocity $\mathbf{v}'$ of the particle relative to the disk at which the Coriolis force will be balanced by the centrifugal force of inertia. Express $\mathbf{v}'$ in terms of the instantaneous value of the position vector $\mathbf{r}$ drawn to the particle from the centre of the disk;

(b) of the velocity $\mathbf{v}$ of the particle relative to a stationary reference frame in the same conditions.

1.136. A horizontal rod rotates about a vertical axis passing through its end at the angular speed of $\omega = 1.00$ rad/s. The distance from the axis to the other end of the rod is $l = 1.00$ m. The rod carries a sleeve of mass $m = 0.100$ kg. The sleeve is fastened with the aid of a thread at a distance of $l_0 = 0.100$ m from the axis of rotation. At the instant

$t = 0$, the thread is burned, and the sleeve begins to slide along the rod virtually without friction. Find:

(a) the time τ after which the sleeve flies off the rod;

(b) the force F with which the rod acts on the sleeve at the instant τ;

(c) the work A that is done on the sleeve during the time τ in a stationary reference frame.

1.137. A horizontal disk rotates at an angular velocity of ω. A particle moves along the radius of the disk, and the distance from it to the centre of the disk changes with time according to the law $r = at$ (a is a constant). Find the resultant moment $\mathbf{M}$ of the forces acting on the particle in the reference frame associated with the disk. What is meant is the moment relative to the centre of the disk.

1.138. A reference frame rotates relative to an inertial frame about the z-axis at a constant angular velocity of ω. From point O on the z-axis, a particle of mass m flies out in a perpendicular direction to the axis and flies rectilinearly relative to the inertial frame at a constant speed of v. Find the angular momentum $\mathbf{L}(t)$ of the particle relative to point O observed in the rotating reference frame. Show that the appearance of $\mathbf{L}(t)$ is due to the action of a Coriolis force.

1.7. Mechanics of a Rigid Body

1.139. A body performs plane motion in the plane x, y. The centre of mass C of the body moves along the x-axis at the constant velocity $\mathbf{v}_0$. At the instant $t = 0$, the centre of mass coincided with the origin of coordinates O. The body simultaneously rotates in the direction indicated in Fig. 1.25 at the velocity ω. Write an expression for the position vector $\mathbf{r}$ of the point of intersection of the body's instantaneous axis of rotation with the plane x, y.

1.140. A girder of mass $m = 300$ kg and length $l = 8.00$ m lies on two supports (Fig. 1.26). The distances from the ends of the girder to the supports are $l_1 = 2.00$ m and $l_2 = 1.00$ m. Find the forces F_1 and F_2 with which the girder acts on its supports.

1.141. A ladder of length $l = 5.00$ m and mass $m = 11.2$ kg is leaned against a smooth wall at an angle of

$\alpha = 70°$ to the floor (Fig. 1.27). The coefficient of friction between the ladder and the floor is $f = 0.290$. Find:

(a) the force $\mathbf{F}_1$ with which the ladder acts on the wall;

(b) the limiting value of the angle α_0 at which the ladder begins to slip.

1.142. An extended body of an arbitrary shape is thrown at an angle to the horizontal. How will the centre of mass

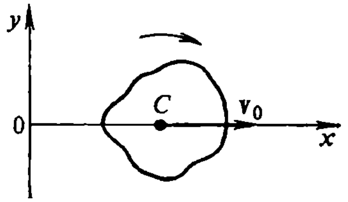

Fig. 1.25

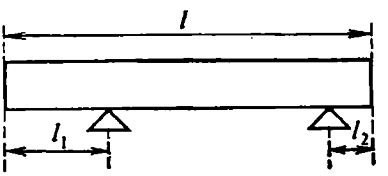

Fig. 1.26

of the body move if air resistance may be disregarded?

1.143. A massless inextensible thread slides without friction over a grooved member fastened to a wall (Fig. 1.28) under the action of weights of $m_1 = 1.00$ kg and $m_2 = 2.00$ kg. With what acceleration $\mathbf{a}_C$ will the centre of mass of the weights move?

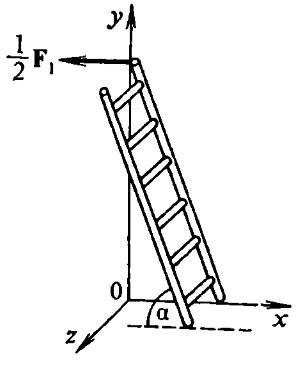

Fig. 1.27

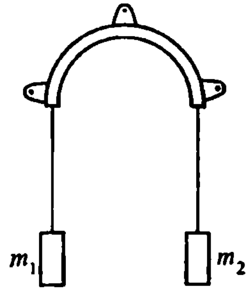

Fig. 1.28

1.144. Figure 1.29 shows two particles joined by a rigid rod. Can the particles have the velocities shown in the figure? The particles and the velocities are in the plane of the figure.

1.145. Two point particles of masses m_1 and m_2 are con-

nected by a rigid massless rod of length l. Find the moment of inertia I of this system relative to an axis perpendicular to the rod and passing through the centre of mass.

1.146. Find the moment of inertia of a homogeneous round right cylinder of mass m and radius R relative to the cylinder's axis.

1.147. The density of a cylinder of length $l = 0.100$ m and radius $R = 0.0500$ m varies linearly with the distance from the axis from the value $\rho_1 = 500$ kg/m^3 to the value $\rho_2 = 3\rho_1 = 1500$ kg/m^3. Find:

(a) the mean density $\langle\rho\rangle_V$ over the volume of the cylinder; compare it with the mean density $\langle\rho\rangle_r$ over the radius;

(b) the moment of inertia I of the cylinder relative to the axis; compare it with the moment of inertia I' of a homogeneous cylinder having the same mass and dimensions.

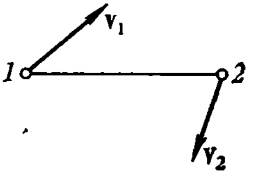

Fig. 1.29

1.148. Find the moment of inertia of a homogeneous sphere of radius R and mass m relative to an axis passing through the centre of the sphere.

1.149. A right round homogeneous cone has a mass of m and a radius of its base of R. Find the moment of inertia of the cone relative to its axis.

1.150. Find the moment of inertia of a thin homogeneous rod of length l and mass m:

(a) relative to an axis perpendicular to the rod and passing through its centre of mass;

(b) relative to an axis perpendicular to the rod and passing through its end.

1.151. Find the moment of inertia of a homogeneous rectangular plate of mass m, length a, and width b relative to an axis perpendicular to it and passing:

(a) through the centre of the plate;

(b) through one of the vertices of the plate.

Compare the results obtained with the answer to the preceding problem.

1.152. Find the moment of inertia of a homogeneous cube relative to an axis passing through the centres of opposite faces. The mass of the cube is m, and the length of its edge is a.

1.153. It can be proved that the moment of inertia of any body evaluated relative to any axis passing through the centre of mass of the body is related to the principal moments of inertia I_x, I_y, I_z (i.e. to the moments of inertia relative to the principal axes) by the expression

$$I = I_x \cos^2 \alpha + I_y \cos^2 \beta + I_z \cos^2 \gamma$$

where α, β, γ are the angles made by the given axis with the axes x, y, z. On this basis, show that the moment of inertia of a homogeneous cube relative to any axis passing through its centre is the same (as in a sphere!).

1.154. Find the moment of inertia of a homogeneous pyramid whose base is a square with a side of a relative to an axis passing through its vertex and the centre of the base. The mass of the pyramid is m.

1.155. Find the ratio of the moments of inertia:

(a) of a pyramid (with a square base) and a cone of an identical height, density, and mass;

(b) of a cube and a sphere of the same density and mass (in a cube, as in a sphere, the moment of inertia relative to any axis passing through its centre is the same; see Problem 1.153).

What is meant are axes passing through the vertex and the centre of the base in case (a), and passing through the centre in case (b).

1.156. Find the principal moments of inertia of a thin homogeneous disk of mass m and radius R. Bear in mind that it is expedient to perform the calculations in the polar coordinates r and φ.

1.157. Calculate the moment of inertia of a homogeneous round right cylinder relative to an axis perpendicular to the axis of symmetry of the cylinder and passing through its centre. The mass of the cylinder is m, its radius is R, and its height is h. Compare the result obtained with the answers to Problems 1.150 and 1.156. Consider the extreme cases: $R \ll h$ and $h \ll R$.

1.158. In a homogeneous right round cylinder, at what ratio of the cylinder's height h to its radius R will all three principal moments of inertia be the same?

1.159. Find the moment of inertia of a homogeneous body having the shape of a disk with a square window cut out of it. One of the corners of the window coincides with the

centre of the disk. The radius of the disk $R = 20.0$ cm, a side of the square is $a = 10.0$ cm, and the mass of the body is $m = 5.00$ kg. What is meant is the moment relative to an axis perpendicular to the disk and passing through its centre.

1.160. A homogeneous plate has a length of $a = 20.0$ cm, a width of $b = 10.0$ cm, and a thickness of $c = 5.00$ cm. The mass of the plate is $m = 2.70$ kg. The origin of coordinates is at the centre of the plate, the x-axis is parallel to side a, the y-axis is parallel to side b, and the z-axis is parallel to side c. Find the coordinates of the components of the inertia tensor of the plate relative to this system of coordinates and write the tensor itself.

1.161. We have a vector **a** with the components $a_x = 1$, $a_y = 2$, $a_z = 3$ and a tensor **T** of rank two all of whose components are the same and equal $T_{ik} = 1$. Find the components of the vector **b** obtained as a result of multiplying the vector **a** by the tensor **T** (**b** = **Ta**).

1.162. We have an arbitrary vector **a** with the components a_x, a_y, a_z and a tensor E of rank two determined by the table

$$\mathbf{E} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(such a tensor is known as a unit one). Find the vector **b** obtained as a result of multiplying the vector **a** by the tensor **E** (**b** = **Ea**).

1.163. Calculate the components of an inertia tensor and write the tensor itself for a homogeneous sphere of radius $R = 10.0$ cm and of mass $m = 25.0$ kg for the case when the origin of coordinates is at the centre of the sphere.

1.164. In what cases are the angular momentum **L** and the angular velocity $\boldsymbol{\omega}$ of a rotating body collinear?

1.165. In what cases can the equation of dynamics of rotational motion be represented in the form $I\dot{\boldsymbol{\omega}} = \mathbf{M}$?

1.166. In what cases is the kinetic energy of a rotating body determined by the expression $E_k = \frac{1}{2}I\omega^2$?

1.167. The plate of Problem 1.160 rotates about an axis passing through its centre. The components of the angular velocity are $\omega_x = \omega_y = \omega_z = 1.00$ rad/s. Find:

(a) the magnitude of the plate's angular momentum L and the angle α between the vectors $\boldsymbol{\omega}$ and $\mathbf{L}$;

(b) the kinetic energy E_k of the plate.

1.168. Two particles of the same mass m constantly located at opposite ends of a diameter (Fig. 1.30) move along a circle of radius r at the velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ identical in magnitude $[v_1 = v_2 = v\,(t)]$.

(a) Determine the total angular momentum $\mathbf{L}$ of the particles relative to arbitrary point O (not necessarily

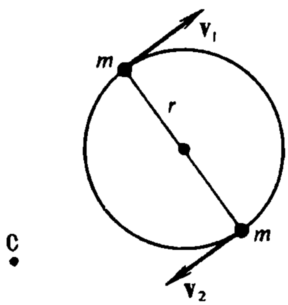

Fig. 1.30

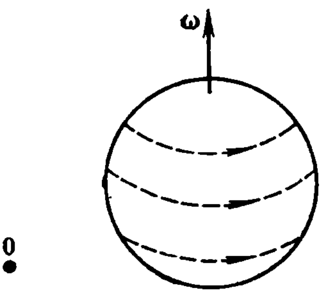

Fig. 1.31

in the plane of the circle). Express $\mathbf{L}$ in terms of the angular velocity $\boldsymbol{\omega}\,(t)$ at which the diameter joining the particles turns.

(b) Does $\mathbf{L}$ depend on the choice of point O?

1.169. A homogeneous sphere of radius R and mass m rotates at the angular velocity $\boldsymbol{\omega}$ about an axis passing through its centre. What is the angular momentum $\mathbf{L}$ of the sphere relative to arbitrary point O (Fig. 1.31)?

1.170. A body of an arbitrary shape drops, while rotating, in the homogeneous field of forces of gravity. There is no resistance of the medium. How does the intrinsic angular momentum of the body behave? (See Problem 1.116.)

1.171. A homogeneous cylinder of mass m and radius R rolls without sliding along a horizontal plane. The centre of the cylinder moves at the velocity $\mathbf{v}_0$ (Fig. 1.32). Find the magnitude of the cylinder's angular momentum relative to points *1*, *2*, and *3* that are in a plane perpendicular to the cylinder and passing through its centre.

1.172. Calculate the Earth's angular momentum L_0 due to its rotation about its axis. Compare it with the angular

momentum L due to the Earth's motion about the Sun. Consider the Earth to be a homogeneous sphere, and its orbit—a circle.

1.173. A horizontal homogeneous cylinder of radius R rotates without friction about an axis coinciding with one of its generatrices.

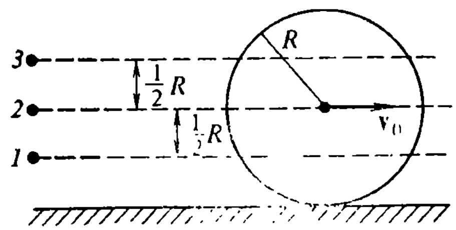

Fig. 1.32

(a) Indicate the positions of the cylinder in which the magnitude of its angular acceleration α is maximum and minimum.

(b) Find the maximum and minimum values of α.

1.174. Two bodies lie on a horizontal table and can slide over it without friction. The bodies are joined to each other

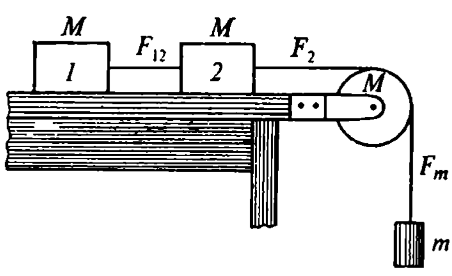

Fig. 1.33

by a massless inextensible thread (Fig. 1.33). A similar thread passed over a pulley joins body *2* to a weight of mass $m = 0.500$ kg. The pulley is a homogeneous solid cylinder. The mass of the bodies and the pulley is the same and equals $M = 1.00$ kg.

(a) Considering that the pulley rotates without friction, and that the thread does not slide over the pulley, find the acceleration a of the bodies, the tension F_{12} of the thread

joining bodies *1* and *2*, the tension F_2 of the thread on the section from body *2* to the pulley, and the tension F_m of the thread on the section from the pulley to weight m.

(b) Determine the same quantities assuming that the pulley does not rotate, but the thread slides over it without friction. Compare the results obtained.

1.175. A thin rod of length $l = 1.00$ m and mass $m = 0.600$ kg can revolve without friction about a horizontal axle perpendicular to it and at a distance of $a = 0.100$ m from the centre of the rod. The rod is brought into a horizontal position and is released without a push with a zero initial velocity. Determine:

(a) the angular acceleration α_0 of the rod and the force F_0 pressing on the axle at the initial instant;

(b) the angular speed ω and the force F pressing on the axle at the instant when the rod passes through its position of equilibrium.

1.176. A thin rod of mass $m = 0.200$ kg and length $l = 1.00$ m can rotate in a vertical plane about a horizontal axle passing through its end. The friction in the axle produces the torque M constant in magnitude. We choose as the coordinate determining the position of the rod the angle φ between it and the vertical measured from the top position of the rod. At a value of this angle equal to $\varphi_0 = 10.0°$, the rod begins to turn spontaneously. Find:

(a) the angular speed ω of the rod at the instant when it passes through its bottom position;

(b) the magnitude of the angular momentum L of the rod at this instant.

1.177. A pole with a height of $h = 3.00$ m and a mass of $m = 50.0$ kg falls from a vertical position onto the Earth. Determine the magnitude of the pole's angular momentum L relative to its point of support and the speed v of the pole's top end when it strikes the Earth.

1.178. A rule of mass $m = 0.1200$ kg and length $l = 1.000$ m lies on a smooth table. A blow is struck at a point at a distance from the centre of the rule of $a = 40.0$ cm (Fig. 1.34) upon which the rule receives a momentum of $p = 7.50 \times 10^{-2}$ kg·m/s. Considering the blow to be instantaneous and disregarding friction,

(a) find the distance x from the centre of the rule to point O that will not "feel" the blow;

(b) determine how the rule moves directly after the blow.

1.179. A homogeneous ball is placed on a plane making the angle $\alpha = 30°$ with the horizontal (Fig. 1.35).

(a) At what values of the coefficient of friction f will the ball roll down the plane without slipping?

(b) Assuming that $f = 0.100$, determine the nature of the ball's motion.

(c) Find the values of the speeds of points A, B, and C of the ball in $t = 1.00$ s after the beginning of motion.

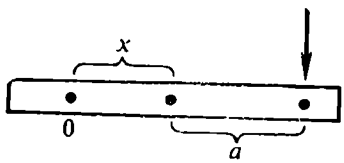

Fig. 1.34

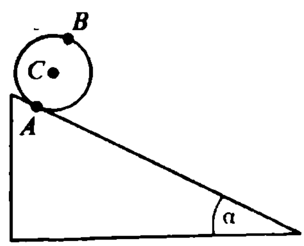

Fig. 1.35

1.180. An initial momentum is imparted to a homogeneous cylinder as a result of which it begins to roll without slipping up an inclined plane at a speed of $v_0 = 3.00$ m/s. The plane makes the angle $\alpha = 20°$ with the horizontal.

(a) What height h will the cylinder rise to?

(b) How long (t_1) will the cylinder move until it stops?

(c) How long (t_2) will it take the cylinder to roll down to its initial position?

(d) What speed v will the cylinder have at the instant when it returns to its initial position?

Compare the results obtained with the answer to Problem 1.58.

1.181. Solve the preceding problem assuming that the cylinder experiences a moment of the force of rolling friction constant in magnitude: $M_{fr} = 0.100$ N·m. The mass of the cylinder is $m = 1.00$ kg, and its radius is $R = 0.100$ m. In addition to the quantities indicated in Problem 1.180, determine:

(e) what work A the force of rolling friction does along the entire upward and downward path,

Compare the results obtained with the answers to Problems 1.180 and 1.59.

1.182. A spool whose mass $m = 50.0$ g and whose moment of inertia relative to its axis is $I = 5.00 \times 10^{-6}$ kg·m^2 lies on a horizontal plane. A virtually massless and inextensible thread is wound onto the spool (Fig. 1.36). The radius of the outermost layer of the turns is $r = 2.00$ cm, and that of the spool ends is $R = 3.00$ cm. The coefficient of friction

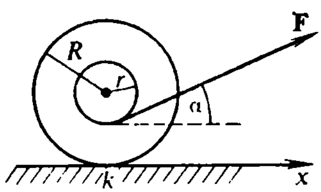

Fig. 1.36

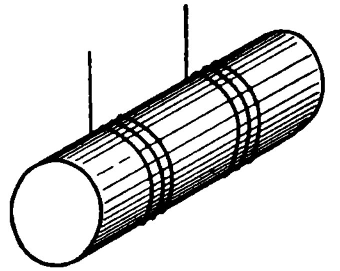

Fig. 1.37

between the spool and the plane is $f = 0.200$. How will the spool behave if the force F with which the thread is pulled and the angle α have the values: (a) $F = 0.128$ N and $\alpha = 30°$; (b) $F = 0.100$ N and $\alpha = 48.2°$; (c) $F = 0.100$ N and $\alpha = 30°$; and (d) $F = 0.100$ N and $\alpha = 60°$.

For all the cases, determine a_x—the projection of the acceleration of the spool's axis onto the x-axis.

1.183. A homogeneous solid cylinder of mass $m = 1.00$ kg is suspended horizontally on two massless strings wound onto it (Fig. 1.37). The cylinder is released without a push.

(a) How long (t) will it take the cylinder to lower over a distance of $h = 50.0$ cm?

(b) What tension F will each of the strings experience when the cylinder lowers?

1.184. A pulley of radius R can rotate about its axis with friction characterized by the torque M_{fr} that does not depend on the pulley's speed. A virtually massless inextensible string is attached with one end to the pulley and wound onto it. A weight of mass m (Fig. 1.38) is suspended on the other end of the string. The weight is released without a push and it begins to lower, turning the pulley. Find the angular momentum L (t) of this system of bodies relative

to the pulley axis at the instant t after the beginning of motion.

1.185. Find the angular momentum L relative to the pulley axis and the kinetic energy E_k of the system of the preceding problem when the speed of the weight m is v. Assume that the moment of inertia of the pulley is I.

1.186. One of two identical homogeneous disks can rotate without friction about a vertical fixed axis passing through its centre. This disk is initially stationary. The second disk is rotated until its angular speed is ω_0 and is dropped in a horizontal position onto the first disk so that the edge of one of the disks coincides with the centre of the other. Upon coming into contact, the disks instantaneously become glued together. Determine:

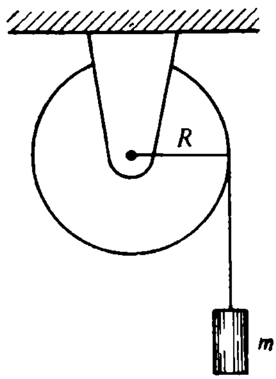

Fig. 1.38

(a) the angular speed ω at which the system formed will rotate;

(b) how the kinetic energy of the disks will change.

1.187. A horizontal wooden rod of mass $m = 0.800$ kg and length $l = 1.80$ m can rotate about a vertical axis that passes through its centre. A bullet of mass $m' = 3.00$ g hits the end of the rod and gets stuck in it. The bullet was flying horizontally and at right angles to the rod at a speed of $v = 50.0$ m/s. Determine the angular speed ω at which the rod begins to rotate.

1.188. Solve the preceding problem, replacing the bullet with a plastic sphere of the same mass and travelling at the same speed, but rebounding from the rod. Consider the impact to be perfectly elastic. Determine:

(a) the angular speed ω of the rod;

(b) the speed v' of the sphere after the impact.

Compare the result obtained for ω with the answer to the preceding problem.

1.189. A horizontal disk of mass m and radius R can rotate about a vertical axis passing through its centre. A man of mass m' is standing on the edge of the disk. First

the disk and the man are stationary. Next the man begins to walk along the edge of the disk at a speed of v' relative to the disk. At what speed ω does the disk rotate relative to a stationary reference frame? Disregard the size of the man in comparison with R.

1.190. A body rotates about the z-axis at an angular speed of $\omega = \omega(t)$. The moment $M_z = M_z(t)$ of forces acts on the body. Write an expression for the work done by the forces applied to the body during the time interval from t_1 to t_2.

1.191. A horizontally arranged homogeneous round cylinder of mass $m = 10.0$ kg rotates without friction about its axis under the action of a weight of mass $m' = 1.00$ kg fastened to a light inextensible string wound on the cylinder. Find the kinetic energy E_k of the system in $\tau = 3.53$ s after the beginning of motion.

1.192. A bucket with water pulled out of a well was dropped accidentally, and it started to lower, accelerating the windlass. The friction in the bearings of the windlass produces a constant torque of $M = 0.170$ N·m. The mass of the bucket with water is $m = 13.2$ kg. The mass of the windlass is $m' = 43.1$ kg, and its radius is $r = 12.8$ cm. The distance from the edge of the well's top to the surface of the water in the well is $h = 7.0$ m. Find:

(a) the law observed by the change in time of the angular speed ω of the windlass;

(b) the tension F of the rope when the bucket is falling;

(c) the time τ that elapses until the bucket touches the water in the well;

(d) the speed v of the bucket at the end of its fall;

(e) the work A done by the forces of friction during the time the bucket falls.

Consider the windlass to be a solid homogeneous cylinder. Disregard the mass and thickness of the rope, the mass of the windlass crane, and also air resistance.

1.193. A horizontal homogeneous cylinder of radius R can rotate about an axle coinciding with its geometric axis. The friction in the axle is produced by the torque M_{fr} not depending on the speed of rotation. A point particle of mass m' (Fig. 1.39) is secured to the cylinder. The latter is set with this particle at the level of the axle and released without a push. Find the value of m' at which

(a) the cylinder will begin to move;

(b) after completing one-fourth of a revolution, the cylinder will stop.

1.194. A disk of mass m and radius R initially rotates about its axis at the angular speed ω. External forces cause the disk to stop. What is the work A of the external forces?

1.195. A homogeneous cylinder of mass m and radius R rotates about its axis. The angular speed of the cylinder changes during the time t from the value ω_1 to ω_2. What mean power $\langle P \rangle$ do the forces acting on the cylinder develop?

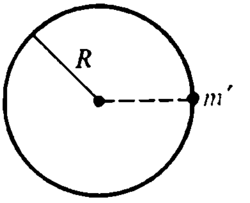

Fig. 1.39

1.196. The rotor of a contrivance is provided with a disk brake. The latter consists of two disks of radius $R = 150$ mm, one of which is fastened on the end of the rotor axle, while the other, deprived of the possibility of rotating, can be pressed against the first one with a force of $F = 100$ N. The brake is applied at the instant when the rotor is rotating by inertia at a speed of $\omega = 50.0$ rad/s (the friction in the bearings may be ignored). The moment of inertia of the rotor together with the brake disk fastened to it is $I = 0.628\ \text{kg}\cdot\text{m}^2$. The coefficient of friction between the surfaces of the disks does not depend on their relative speed and is $f = 0.25$. Considering that the force F is uniformly distributed over the surface of the disks, determine the number of revolutions N the rotor will have time to complete before it stops.

1.8. Universal Gravitation

1.197. In an experiment similar to the one by means of which H. Cavendish in 1798 determined the value of the gravitational constant G, the masses of small and large lead spheres were respectively $m = 0.729$ kg and $M = 158$ kg. The small spheres were secured on a light balance beam suspended on a steel wire and having a length measured between the centres of the spheres of $l = 216$ cm. The diameter of the wire was 0.6 mm, and its length was about

one metre. At a distance of r between the centres of a small and the corresponding large spheres equal to 300 mm, the wire carrying the balance beam with the small spheres twisted through an angle of $\alpha = 39.6''$. The experimentally determined proportionality factor k between the angle of torsion of the wire and the applied torque is $1.04 \times \times 10^3$ rad/(N·m). Find the value of G.

1.198. With what force F do two identical homogeneous spheres of mass $m = 1.00$ kg each attract each other if their centres are $r = 1.00$ m apart?

1.199. Two identical homogeneous spheres in contact attract each other with a force of F. How will the force change if the mass of the spheres is increased n times? The material which the spheres are made from is assumed to be the same.

1.200. In a very thin homogeneous straight rod of length l and mass M, a particle of mass m is on the straight line perpendicular to the axis of the rod and passing through its centre, at a distance of b from the rod.

(a) Find the magnitude F of the force with which the rod acts on the particle if $b = l = 2a$.

(b) Investigate the case $b \gg l$.

(c) Compare the force F with the force F' with which point particles having the masses M and m at a distance of $b = 2a$ from each other would interact.

1.201. Solve the preceding problem considering that the particle is on the axis of the rod at a distance of $b = l = = 2a$ from its centre.

1.202. In a very thin homogeneous ring of mass M and radius R, a particle of mass m is on the straight line perpendicular to the plane of the ring and passing through its centre, at a distance of x from the centre. Find:

(a) the mutual potential energy $E_p(x)$ of the particle and the ring;

(b) the force F_x exerted on the particle from the side of the ring. Evaluate the force in two ways: (1) by summation of the elementary forces, and (2) by using the expression for $E_p(x)$.

(c) Investigate the case $x \gg R$.

1.203. A very thin homogeneous disk of radius R has a surface density (mass of a unit area) of σ. A particle of mass m is at a distance of b from the disk on the straight

line perpendicular to it and passing through its centre. Determine:

(a) the force F with which the disk acts on the particle;

(b) in what condition the force F differs from its extreme value F_∞ obtained at $R \to \infty$ by not over 1%.

1.204. An infinite very thin homogeneous straight rod has a linear density (mass per unit length) of λ. A particle of mass m is at a distance of b from its axis.

(a) Find the magnitude F of the force with which the rod acts on the particle.

(b) A particle of what mass M at a distance of b from m would act on it with the same force?

1.205. A very thin infinite homogeneous plate has a surface density of σ. A particle of mass m is at a distance of b from it.

(a) Find the magnitude F of the force with which the plate acts on the particle.

(b) What is a feature of the expression for F?

(c) How will the result change if a plate with a negligibly small thickness is replaced with one of finite thickness d made from a substance of volume density ρ?

1.206. An infinite homogeneous plate of thickness $d = 0.100$ m has a density of $\rho = 10.0$ g/cm^3. With what force F does this plate act on a body of mass $m = 1.00$ kg near it?

1.207. With what force F (calculated per unit area) do two parallel infinite homogeneous plates attract each other? Each plate has a density of $\rho = 10.0$ g/cm^3 and a thickness of $d = 0.100$ m.

1.208. What is the relation between the solid angle $d\Omega$ and the surface dS cut out by it on a sphere of radius R whose centre coincides with the apex of the solid angle?

1.209. Express the surface element dS of a sphere of radius R whose centre is at the origin of coordinates in spherical coordinates.

1.210. Express the elementary solid angle $d\Omega$ whose apex is at the origin of coordinates in spherical coordinates.

1.211. Determine the gravitational force F experienced by a point particle inside a homogeneous spherical layer.

1.212. Inside one homogeneous spherical layer there is another homogeneous spherical layer of smaller dimensions. The centres of the layers do not coincide. What is the force F of interaction between the layers?

1.213. A very thin homogeneous layer has the form of a hemisphere of radius R and mass M. A particle of mass m is at the centre of the hemisphere. Find the magnitude F of the force with which the layer acts on the particle.

1.214. Find the mutual potential energy $E_p(r)$ of a very thin homogeneous spherical layer and a particle of mass m at a distance of r from the centre of the layer. The mass of the layer is M, and its radius is R. Consider the cases: (a) $r < R$, and (b) $r > R$.

1.215. Using the result of the preceding problem, find the mutual potential energy $E_p(r)$ of a thick spherical layer and a particle of mass m at a distance of r from the centre of the layer. The mass of the layer is M, its inner radius is R_1, and its outer radius is R_2.

1. Consider the cases: (a) $r < R_1$, and (b) $r > R_2$.

2. What conclusion on the force F acting on the particle from the side of the layer can be made on the basis of the answer to item 1 (a)?

1.216. What data must be available to determine the mass: (a) of the Earth, and (b) of the Sun?

1.217. Using the values of astronomical quantities and physical constants, calculate the mass m and mean density $\langle\rho\rangle$: (a) of the Earth, and (b) of the Sun.

1.218. Find the force F with which (a) the Earth and the Sun attract each other; (b) the Moon and the Earth attract each other. Compare these forces.

1.219. Considering the Earth to travel in a circular orbit, find the acceleration a imparted to the Earth by the Sun. Compare a with g.

1.220. Find the orbital (first cosmic) velocity v_1 for the Earth, i.e. the velocity that must be imparted to a body for it to become a satellite of the Earth.

1.221. Find the escape (second cosmic) velocity v_2 for the Earth, i.e. the smallest velocity that must be imparted to a body for it to overcome the action of the Earth's attraction and leave the Earth forever. Compare v_2 with the orbital velocity v_1.

1.222. In which case will a body travel a greater distance from the Earth: (a) when launched vertically upward at a speed of 10 km/s, or (b) when launched at an angle to the horizontal of 5° at a speed of 12 km/s? Disregard the resistance of the air.

1.223. Determine at what radius R of its orbit (in metres) a satellite can travel in the plane of the equator so as to constantly be over the same point on the Earth's surface. Compare R with the Earth's radius R_E.

1.224. A planet moves in a circular orbit. Find the relation between the radius R of the orbit and the period T of revolution of the planet about the Sun.

1.225. Proceeding from the facts that the radius of the Earth's orbit is $R_E = 149.5 \times 10^6$ km, and that of Mars's orbit is $R_M = 227.8 \times 10^6$ km, find the period T_M of revolution of Mars about the Sun (express it in years).

1.226. Considering the Earth to be a homogeneous sphere and disregarding the Earth's rotation:

(a) find the acceleration of free fall $g(h)$ as a function of the altitude h;

(b) determine the values of this acceleration for altitudes h of 100, 1000, and 10 000 km. Express the found values in terms of g—the acceleration near the Earth's surface.

1.227. (a) Find the potential energy E_p of a body of mass m at an altitude of h from the Earth's surface. Consider the potential energy at $h = 0$ to be zero.

(b) Obtain an approximate expression for E_p that holds for $h \ll R$ (R is the Earth's radius).

1.228. A body is launched from the Earth's surface at an angle of $\alpha = 45°$ to the horizontal at a speed of $v_0 = 3.68 \times 10^3$ m/s. Disregarding air resistance and the Earth's rotation, determine:

(a) the height h to which the body will rise;

(b) the speed v of the body at the top point of its trajectory;

(c) the radius of curvature R_c of the trajectory at its top point.

1.229. Considering the Earth to be a homogeneous sphere and disregarding air resistance, determine how a small body will move if it is dropped into a narrow channel drilled along the Earth's axis.

1.230. For the conditions of the preceding problem, find:

(a) the magnitude of the body's acceleration $a(r)$ as a function of the distance r from the Earth's centre;

(b) the magnitude of the body's speed $v(r)$ as a function of r;

(c) the body's speed $v(0)$ at the instant when it reaches

the centre of the Earth; compare $v(0)$ with the orbital (first cosmic) speed v_1 (see Problem 1.220);

(d) the time τ after which the body returns to the initial point; compare τ with the time t_1 during which a body travelling at the orbital speed will fly around the Earth;

(e) the mean (in time) speed $\langle v \rangle$ of the body; compare it with $v(0)$.

1.231. For the body from Problem 1.229, find:

(a) the potential energy $E_p(r)$ as a function of the distance r from the Earth's centre (assume that the potential energy of the body at an infinitely large distance from the Earth is zero);

(b) the potential energy $E_p(0)$ that a body has at the Earth's centre; compare $E_p(0)$ with the potential energy of the body near the Earth's surface $E_p(R)$.

1.232. Let us introduce a rotating reference frame whose axis passes through the Sun's centre and is perpendicular to the plane of the Earth's orbit. The frame rotates in the same direction as the Earth at an angular speed double that of the Earth.

(a) What forces must be taken into account when considering the Earth's motion relative to the Sun in this frame?

(b) Calculate the value and indicate the direction of these forces. Compare them with the force F_g of gravitational attraction of the Earth to the Sun.

1.233. Determine the force F with which a small body of mass m on the equator not far from the Earth's surface attracts the Earth. Consider the acceleration of free fall at the equator to be known and equal to g_{eq}.

1.9. Oscillatory Motion

1.234. A particle oscillates along the x-axis according to the law $x = A \cos \omega t$. Plot graphs:

(a) of the functions x, $\dot{x}$, and $\ddot{x}$ versus t;

(b) of the functions $\dot{x}$ and $\ddot{x}$ versus x.

1.235. A particle executes harmonic oscillations with the amplitude A and the period T. Find:

(a) the time t_1 during which the displacement of the particle changes from 0 to $A/2$;

(b) the time t_2 during which the displacement of the particle changes from $A/2$ to A.

1.236. A particle oscillates along the x-axis according to the law $x = 0.100 \sin 6.28t$ (m). Find the mean value of the particle's speed $\langle v \rangle$: (a) during the period T of oscillations; (b) during the first one-eighth of T; (c) during the second one-eighth of T. Compare the obtained results.

1.237. For the particle from the preceding problem, find the mean value of the velocity $\langle \mathbf{v} \rangle$: (a) during the period T of oscillations; (b) during the first quarter of T; (c) during the second quarter of T.

1.238. How can we find the frequency ω of harmonic oscillations if we know the amplitude A of the displacement and that of the speed v_m?

1.239. How can we find the amplitude A and the frequency ω of harmonic oscillations if we know the amplitude of the speed v_m and that of the acceleration a_m?

1.240. A horizontal platform performs the harmonic oscillations $x = A \cos \omega t$ in a vertical direction. A disk of a completely inelastic material is on the platform.

(a) In what condition will the disk separate from the platform?

(b) At what position is the platform and in what direction is it moving when the disk breaks away from it?

(c) To what height h will the disk rise above its position corresponding to the mean position of the platform if $A = 20.0$ cm and $\omega = 10.0\ s^{-1}$?

1.241. Find the mean values of $\sin^2 x$ and $\cos^2 x$ within the interval from α to $\alpha + n\pi$ (α is an arbitrary angle, and n is an integer).

1.242. What is the work A of a quasi-elastic force in harmonic oscillations during the time equal to a period of the oscillations?

1.243. (a) Find an equation relating the value of the momentum $p_x = m\dot{x}$ to the values of the coordinate x of a one-dimensional harmonic oscillator. The mass of the oscillator is m, its frequency is ω, and the oscillation amplitude is A. (b) Plot the curve described by this equation. (c) Express the area S confined by this curve in terms of the energy E of the oscillator.

1.244. Determine the frequency ω of the small-amplitude

oscillations of the particle from Problem 1.86 appearing if the particle is displaced in a radial direction from the position of stable equilibrium. Assume that the mass of the particle is m.

1.245. (a) At what length l will the period of oscillations of a mathematical pendulum be 1 s? (b) What is the period T of oscillations of a mathematical pendulum of length $l = 1$ m?

1.246. The part of a physical pendulum is played by a thin rod suspended by one of its ends.

(a) At what length l of the rod will the period of oscillations of this pendulum be 1 s?

(b) What is the period T of oscillations when the rod is 1 m long?

1.247. At what distance x from the centre must a thin rod of a preset length l be suspended to obtain a physical pendulum oscillating at the maximum frequency? What does the value ω_{max} of this frequency equal?

1.248. Find the law showing how the tension F of the thread of a mathematical pendulum performing the oscillations $\varphi = \varphi_m \cos \omega t$ varies with time. The pendulum's mass is m, and its length is l.

1.249. A pendulum oscillates in a stationary lift. Because of breaking of the rope, the lift begins to fall with the acceleration g. How will the pendulum behave relative to the lift if at the instant when the rope breaks it

(a) was in one of its extreme positions;

(b) was passing through its equilibrium position?

1.250. A pendulum is suspended in a lift, and its period of oscillations, when the lift is stationary, is T_0.

(a) What will the period T of oscillations of the pendulum be if the lift begins to lower with an acceleration equal to $\frac{3}{4}g$?

(b) With what acceleration a must the lift be raised for the period of oscillations of the pendulum to be $\frac{1}{2}T_0$?

1.251. A pendulum is suspended in the cabin of an aeroplane. When the latter flies without acceleration, the pendulum oscillates with the frequency ω_0.

(a) What will the frequency ω of oscillations of the pendulum be if the aeroplane is flying with the acceleration

a whose direction makes the angle α with the downward part of a vertical?

(b) Find ω for the case when $a = g$ and $\alpha = \pi/2$.

1.252. Find the period T of oscillations of a mathematical pendulum, the length l of whose suspension equals the Earth's radius R. Compare the result obtained with the answer to Problem 1.230 (d).

1.253. A physical pendulum is placed with its centre of mass above its point of suspension. From this position, the pendulum begins to move without friction at a zero initial speed. At the instant when it passes through its bottom position, the pendulum's angular speed reaches the value $\dot{\varphi}_{max}$. Find the natural frequency ω_0 of the small-amplitude oscillations of this pendulum.

1.254. A ball of mass $m = 50.0$ g is suspended on a spring with a force constant of $k = 49.0$ N/m. The ball is raised to the position at which the spring is not tensioned and is released without a push. Disregarding friction and the mass of the spring,

(a) find the period T and the amplitude A of the initiated oscillations;

(b) directing the x-axis downward and making the point $x = 0$ coincide with the initial position of the ball, write the equation of its motion.

1.255. Disregarding friction, determine the frequency ω of the small-amplitude oscillations of mercury poured into a U-tube with an inner cross-sectional area of $S = 0.500$ cm^2 (Fig. 1.40). The mass of the mercury is $m = 136$ g.

1.256. A croquet mallet consists of a cylindrical head of radius $R = 4.00$ cm and a handle of length $l = 90.0$ cm. The mass of the head is 0.800 kg, and that of the handle is 0.600 kg. The mallet is placed on two parallel bars (Fig. 1.41). Find the period T of the small-amplitude oscillations of the mallet.

1.257. A log of mass $M = 20.0$ kg hangs on two cords each with a length of $l = 1.00$ m (Fig. 1.42). A bullet of mass $m = 10.0$ g flying at a speed of $v = 500$ m/s strikes the end of the log and gets stuck in it. Find the amplitude φ_m and period T of the initiated oscillations of this system. Disregard friction.

1.258. A sphere of mass $m = 2.00$ kg is suspended from

two springs connected in series (Fig. 1.43). Their force constants are $k_1 = 1000$ N/m and $k_2 = 3000$ N/m. Disregarding the mass of the springs and friction, find:

(a) the frequency ω of the small-amplitude oscillations of the sphere;

(b) the amplitude A of the oscillations initiated if the sphere is placed at the level at which the springs are not tensioned and is released without a push.

1.259. A pulley of the design shown in Fig. 1.44 is a solid homogeneous cylinder that can rotate about its axis without

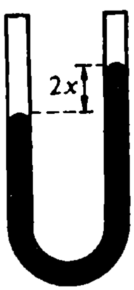

Fig. 1.40

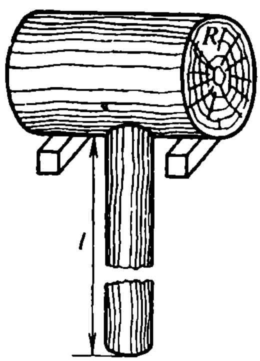

Fig. 1.41

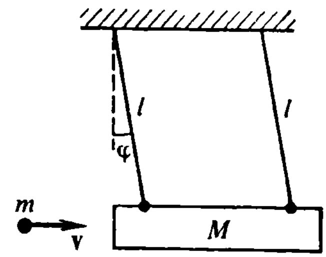

Fig. 1.42

noticeable friction. The mass of the pulley is $M = 5.00$ kg, and its radius is $R = 10.0$ cm. The force constant is $k = 1000$ N/m. The mass of the spring and of the string passed over the pulley may be disregarded. The mass of the weight hanging on the string is $m = 1.00$ kg. Assuming that the string does not slip over the pulley, find:

(a) the frequency ω of the small-amplitude oscillations of the device;

(b) the maximum force tensioning the string at the left (F_{1m}) and at the right (F_{2m}) of the pulley when the amplitude of the oscillations is $A = 5.00$ mm.

1.260. Two spheres with the masses m_1 and m_2 can slide without friction along a thin horizontal rod (Fig. 1.45). The spheres are connected by a massless spring whose force constant is k. The spheres are moved in opposite directions and are then released without a push. Determine:

(a) how the centre of mass of the system behaves;

(b) the frequency ω of the initiated oscillations;

(c) the maximum value of the relative speed v_{max} of the spheres if the initial relative displacement of the spheres is A.

1.261. Two spheres with the masses m_1 and m_2 can slide without friction along a long taut horizontal wire (see Fig. 1.45). The spheres are connected by a massless spring whose force constant is k. The system is initially at rest

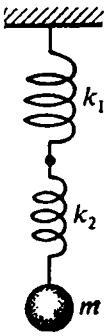

Fig. 1.43

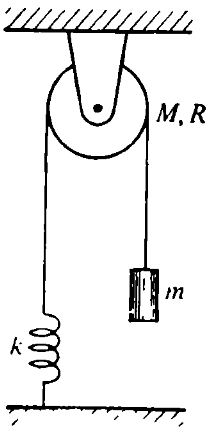

Fig. 1.44

and the spring is not tensioned. A momentum of $p_0 = m_1 v_0$ is imparted to the first sphere. Determine:

(a) the velocity v_C of the centre of mass of the system;

(b) the energy E_{trans} of the translational and E_{osc} of the oscillatory motion of the system;

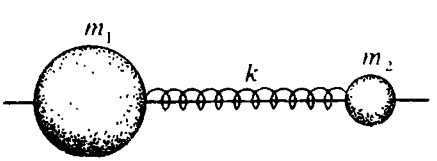

Fig. 1.45

(c) the frequency ω and the amplitude A of the oscillations.

1.262. A homogeneous disk of mass $m = 3.00$ kg and radius $R = 20.0$ cm is fastened to a thin rod whose other end is rigidly fixed (Fig. 1.46). The coefficient of torsion

of the rod (the ratio of the applied torque to the angle of torsion) is $k = 6.00$ N·m/rad. Determine:

(a) the frequency ω of the small-amplitude torsional oscillations of the disk;

(b) the amplitude φ_m and the initial phase α of the oscillations if at the initial instant $\varphi = 0.0600$ rad, and $\dot{\varphi} = 0.800$ rad/s.

1.263. Two disks can rotate without friction about a horizontal axle. The radius of both disks is the same and is

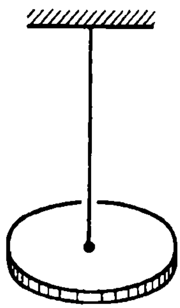

Fig. 1.46

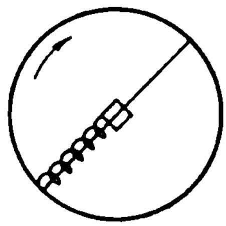

Fig. 1.47

$R = 0.500$ m. The masses of the disks are $m_1 = 2.00$ kg and $m_2 = 3.00$ kg. The disks are connected by a spring for which the constant of proportionality between the appearing torque and the angle of torsion is $k = 5.91$ N·m/rad. The disks are turned in opposite directions and released. What is the period T of the torsional oscillations of the disks? Disregard the diameter of the axle.

1.264. A small sleeve of mass $m = 0.100$ kg can move along the diameter of a horizontal disk, sliding without friction along a guide rod. The sleeve is "tied" to the end of the rod with the aid of a massless spring whose force constant is $k = 10.0$ N/m (Fig. 1.47). If the spring is not tensioned, the sleeve is at the centre of the disk. Find the frequency ω of the small-amplitude oscillations of the sleeve when the disk rotates about its axis at the angular speed $\dot{\varphi}$ equal to (a) 6.00 rad/s; (b) 10.1 rad/s.

1.265. A sphere of mass $m = 5.00$ kg is suspended from the dome of a hall on a light inextensible cord. The length of the suspension is $l = 9.81$ m. The sphere was moved

aside along a certain direction x to a distance of $A = 30.0$ cm, and a momentum of $p = 2.00$ kg·m/s was imparted to it in the direction y perpendicular to x. Disregarding friction, find the equation of the trajectory along which the centre of the sphere will move.

1.266. During 10 s, the amplitude of free oscillations diminished to one-tenth of its initial value. During what time τ will the amplitude diminish to 1/100 of its initial value?

1.267. During 1.00 s, the amplitude of free oscillations was halved. During what time τ will the amplitude diminish to one-tenth of its initial value?

1.268. During the time $t = 16.1$ s, the amplitude of oscillations diminishes $\eta = 5.00$ times.

(a) Find the damping factor β.

(b) In what time τ will the amplitude diminish e times?

1.269. During 100 s, a system performs 100 oscillations. During the same time, the amplitude of the oscillations diminishes 2.718 times. Determine:

(a) the damping factor β;

(b) the logarithmic decrement λ;

(c) the quality factor Q of the system;

(d) the relative decrement of the energy $-\Delta E/E$ of the system during the period of oscillations.

1.270. During the time needed for a system to complete $N = 100$ oscillations, the amplitude diminishes $\eta = 5.00$ times. Find the quality factor Q of the system.

1.271. The quality factor of an oscillatory system is $Q = 2.00$, the frequency of its free oscillations is $\omega = 100\ \text{s}^{-1}$. Determine the natural frequency ω_0 of oscillations of the system.

1.272. Damped oscillations of a particle were produced by displacing it from its equilibrium position over a distance of $A_0 = 1.00$ cm. The logarithmic decrement is $\lambda = 0.0100$. With such slight damping, one may consider with a high degree of accuracy that the maximum deviations from the equilibrium position are achieved at the instants $t_n = (T/2)\,n$ ($n = 0, 1, 2, \ldots$). In this approximation, find the distance s covered by the particle until it stops.

1.273. The frequency of the free oscillations of a system is $\omega = 100.0\ \text{s}^{-1}$, the resonance frequency is $\omega_{res} = 99.0\ \text{s}^{-1}$. Determine the quality factor Q of this system.

1.274. An iron bar attached to a spring when brought out of its equilibrium position performs free oscillations at a frequency of $\omega' = 20.0\ s^{-1}$, the amplitude of the oscillations diminishing $\eta = 2$ times during the time $\tau = 1.11$ s. A coil supplied with alternating current is placed near the lower end of the bar (Fig. 1.48). At a current frequency of $\omega = 11.0\ s^{-1}$, the bar oscillates with an amplitude of $A = 1.50$ mm. At what current frequency ω_{res} will the oscillations of the bar reach their maximum intensity? What will the amplitude A_{res} of the oscillations be at this frequency? The amplitude of the driving force is assumed to be constant. Have in view that the frequency of the driving force equals the double frequency of the changes in the current in the coil.

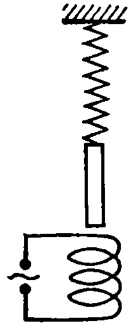

Fig. 1.48

1.275. Under the action of the driving force $F_x = F_m \cos \omega t$, a system performs steady-state oscillations described by the function $x = A \cos (\omega t - \varphi)$.

(a) Find the work A_{dr} of the driving force during a period.

(b) Show that the work of the force of friction during a period $A_{fr} = -A_{dr}$.

1.276. At a constant amplitude of the driving force, the amplitude of forced oscillations at frequencies of $\omega_1 = 100\ s^{-1}$ and $\omega_2 = 300\ s^{-1}$ is the same. Find the resonance frequency ω_{res}.

1.277. At a constant amplitude of the driving force, the amplitude of the velocity at frequencies of $\omega_1 = 100\ s^{-1}$ and $\omega_2 = 300\ s^{-1}$ is the same. Find the frequency ω'_{res} at which the velocity amplitude is maximum.

1.10. Relativistic Mechanics

1.278. Does the notion of a body in the form of a sphere of radius $R = 1.00$ m rotating about its axis at the angular speed of $\omega = 3.30 \times 10^8$ rad/s agree with the principles of the special theory of relativity?

1.279. Does the notion of an electron as of a homogeneous ball of mass $m = 0.911 \times 10^{-30}$ kg (the mass of an electron) and of radius $R = 2.82 \times 10^{-15}$ m (the classical radius of an electron) having an intrinsic angular momentum of $L = 0.913 \times 10^{-34}$ kg·m²/s (following from the quantum theory and experimentally confirmed) argee with the principles of the special theory of relativity?

1.280. Of two identical rods, rod *1* is at rest in the reference frame K_1, and rod *2* is at rest in the frame K_2. The frames move relative to each other along coinciding x-axes. The rods are parallel to these axes. Which rod will be shorter: (a) in the frame K_1; (b) in the frame K_2?

1.281. What longitudinal speed v must be imparted to a rod for its length to become equal to half its length in a state of rest?

1.282. (a) What is the relative increment of the length of a rod $\Delta l/l_0$ if a speed of $v = 0.1c$ is imparted to it in a direction making the angle α with the axis of the rod at rest? (b) Calculate $\Delta l/l_0$ for values of α equal to 0, 45, and 90°.

1.283. Solve the preceding problem for a speed of $v = 0.9c$.

1.284. In the frame K', relative to which it is at rest, a rod has a length of $l' = 1.00$ m and makes the angle $\alpha' = 45°$ with the x'-axis. Determine the length l of the rod in the frame K and the angle α which the rod makes with the x-axis. The relative speed of the frames is $v_0 = 0.500c$.

1.285. A stationary body of an arbitrary shape has the volume V_0. What does the volume V of the same body equal if it travels at a speed of $v = 0.866c$?

1.286. The total surface area of a body at rest having the shape of a cube is S_0. What is the surface area S of the same body if it is moving in the direction of one of its edges at a speed of $v = 0.968c$?

1.287. The relative speed of two reference frames K and K' is not known. A rod parallel to the x'-axis and moving relative to the frame K' at a speed of $v'_x = 0.100c$ has a length of $l' = 1.10$ m in this frame. In the frame K, the length of the rod is $l = 1.00$ m. Find the speed v_x of the rod in the frame K and the relative speed v_0 of the frames.

1.288. The first of two identical clocks (clock *1*) is at rest

in the reference frame K_1, and the second (clock *2*) is at rest in the frame K_2. The frames are moving relative to each other. Which clock runs faster: (a) in the frame K_1; (b) in the frame K_2?

1.289. Identical clocks synchronized with each other are secured to both ends of two rods having a proper length of $l_0 = 10.00$ m (Fig. 1.49). The rods are brought into motion

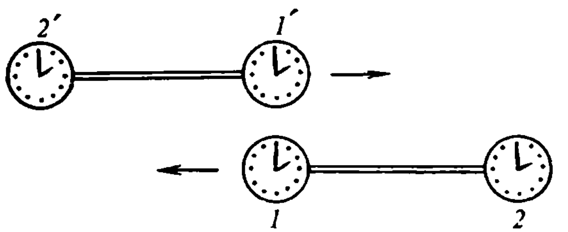

Fig. 1.49

at a relative speed of $v = c/2$. At the instant when the clocks *1* and *1′* are opposite each other, the hands of both clocks show a zero reading. Determine:

(a) the readings τ_1 and τ_2' of the clocks *1* and *2′* at the instant when they come alongside of each other;

(b) the readings τ_2 and τ_1' of the clocks *2* and *1′* at the instant when they come alongside of each other;

(c) the readings τ_2 and τ_2' of the clocks *2* and *2′* at the instant when they come alongside of each other.

1.290. The proper lifetime of a particle is $\tau = 1.00 \times 10^{-6}$ s. What is the interval Δs between the birth and the decay of this particle?

1.291. At what speed v must a particle fly relative to the reference frame K for the interval of proper time $\Delta\tau$ to be one-tenth of the interval Δt measured on a clock of the frame K?

1.292. During a time interval of $\Delta t = 1.00$ s measured on a clock of a reference frame K, a particle moving rectilinearly and uniformly travelled from the origin of coordinates of the frame K to a point with the coordinates $x = y = z = 1.50 \times 10^8$ m. Find the interval of the particle's proper time $\Delta\tau$ during which this displacement occurred.

1.293. The reference frame K' moves relative to the

frame K at a speed of $v_0 = 0.500c$. The velocity of a particle in the frame K' is $\mathbf{v}' = 0.1732\,(\mathbf{e}'_x + \mathbf{e}'_y + \mathbf{e}'_z)$. Find:

(a) the magnitude v' of the velocity $\mathbf{v}'$ and the angle α' made by $\mathbf{v}'$ with the x'-axis;

(b) the velocity $\mathbf{v}$ of the particle in the frame K, the magnitude v of this velocity, and the angle α made by $\mathbf{v}$ with the x-axis;

(c) the ratio v/v' of the magnitudes of the vectors $\mathbf{v}$ and $\mathbf{v}'$.

1.294. Find the speed v of a relativistic particle of mass $m = 0.911 \times 10^{-30}$ kg (the mass of an electron) whose momentum is $p = 1.58 \times 10^{-22}$ kg·m/s.

1.295. The rest energy of a particle is E_0. What is the total energy of the particle in a reference frame in which the particle's momentum is p?

1.296. The momentum of a body of mass m is $p = mc$. What is the kinetic energy E_k of the body?

1.297. At what speed v of a particle does its kinetic energy equal the rest energy?

1.298. Find the momentum p of a relativistic particle of mass m whose kinetic energy is E_k.

1.299. Using the result of the preceding problem, determine the momentum p of a relativistic particle of mass m whose kinetic energy E_k equals the rest energy mc^2 of the particle.

1.300. At a speed v_0 of a particle, its momentum is p_0.

(a) How many times η does the speed of the particle have to be increased for its momentum to double?

(b) Find the values of η for v_0/c equal to 0.1, 0.5, 0.9, and 0.99.

(c) Obtain an approximate expression for η for values of v_0 close to c.

1.301. A particle of mass m begins to move under the action of a force $\mathbf{F}$ constant in magnitude and direction. Find the time dependence of the particle's momentum $\mathbf{p}$ and velocity $\mathbf{v}$.

1.302. The work $A = 8.24 \times 10^{-14}$ J was done on a particle of mass $m = 0.911 \times 10^{-30}$ kg originally travelling at a speed of $v = 0.100c$. How did the speed, momentum, and kinetic energy of the particle change as a result of this? (Find Δv, Δp, and ΔE_k.)

1.303. The relative speed of the reference frames K and K' is $v_0 = 0.800c$. In the frame K', the momentum of a particle is $\mathbf{p}' = 2.30 \times 10^{-18}\,(\mathbf{e}'_x + \mathbf{e}'_y + \mathbf{e}'_z)$ (kg·m/s), and

its energy is $E' = 1.50 \times 10^{-9}$ J. Find the momentum **p** and the energy E of the particle in the frame K.

1.11. Hydrodynamics

1.304. A cylindrical vessel of height H standing on a table is filled to its top with water. Disregarding the viscosity of the water, determine the height h at which a small hole must be made in the vessel for the stream of water flowing out of it to reach the table at the farthest distance from the vessel.

1.305. A cylindrical vessel of height $h = 0.500$ m and radius $R = 10.0$ cm is filled to its top with water. An orifice of radius $r = 1.00$ mm is opened in the bottom of the vessel. Disregarding the viscosity of the water, determine:

(a) the time τ it will take all the water to flow out of the vessel;

(b) the time dependence of the speed v of lowering of the level of the water in the vessel.

1.306. A grease gun used to lubricate hinged joints of an automobile was filled with kerosene for washing. The radius of the gun's piston is $R = 2.00$ cm, and its stroke is $l = 25.0$ cm. The radius of the discharge orifice of the gun is $r = 2.00$ mm. Disregarding the viscosity of the kerosene and the friction of the piston against the walls, determine the time τ during which the kerosene will be displaced from the gun if a constant force of $F = 5.00$ N is applied to the piston. Assume that the density ρ of the kerosene is 0.800 g/cm^3.

1.307. A narrow bent tube is lowered from a bridge crossing a stream into the latter with its open end facing the current (Fig. 1.50). The water rises in the tube to a height of $h = 150$ mm above the water level in the stream. Determine the speed v of the current.

1.308. A device known as a Pitot-Prandtl tube consists of two narrow coaxial tubes (Fig. 1.51). The inner tube is open at its bottom end, and the outer one has side openings. The upper ends of the tubes are connected to a differential manometer (i.e. to a manometer showing the pressure difference Δp). This device can be used to measure the speed of a fluid. For this purpose, it is submerged into the fluid with its open end facing the flow, and Δp is measured. When the tube was submerged into a liquid stream with a density $\rho =$

$= 1.10 \times 10^3$ kg/m^3, a pressure difference of $\Delta p = 4.95 \times \times 10^3$ Pa was detected. Find the speed v of the stream.

1.309. Water flows along a horizontal pipe of radius $R = 12.5$ mm. The rate of flow of the water through the pipe cross section is $Q = 3.00 \times 10^{-5}$ m^3/s. Find:

(a) the nature of the flow;

(b) the pressure difference per unit pipe length dp/dl.

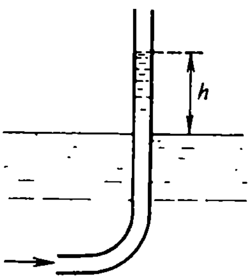

Fig. 1.50

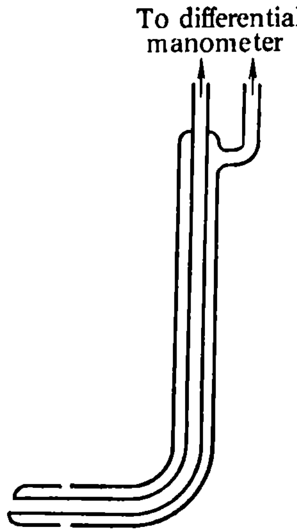

Fig. 1.51

Assume that the viscosity of the water is $\eta = 1.00 \times 10^{-3}$ Pa·s.

1.310. Two identical cylindrical tanks are connected by a narrow tube with a cock at its middle (Fig. 1.52). The radius of a tank is $R = 20.0$ cm, the radius of the tube is $r = = 1.00$ mm. The length of the tube is $l = 1.00$ m. The cross section of the cock opening coincides with that of the tube. Water is poured into one of the tanks to a height of $h = = 50.0$ cm, while the second tank is empty. At the instant $t = 0$, the cock is opened. Determine:

(a) the nature of the flow of the water in the tube during the first seconds;

(b) the time τ after which the difference between the levels of the water in the tanks decreases e times.

Assume that the viscosity of the water is $\eta = 1.00 \times 10^{-3}$ Pa·s.

1.311. A steady flow of air directed vertically upward and having a speed of $u = 20.0$ cm/s set in over a heated section of the Earth's surface. The flow contains a spherical dust particle moving upward at a steady speed of $v = 4.0$ cm/s. The density of the particle is $\rho = 5.00 \times 10^3$ kg/m³, and that of the air is $\rho_0 = 1.29$ kg/m³. The viscosity of the air is $\eta = 1.72 \times 10^{-5}$ Pa·s.

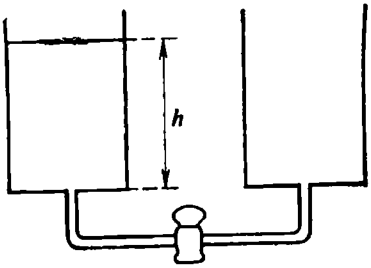

Fig. 1.52

(a) Determine the radius r of the dust particle.

(b) Convince yourself that the flow of the air over the particle is of a laminar nature.

Note. For a sphere, the critical value of the Reynolds number *Re* (i.e. the value at which laminar flow around the sphere transforms into turbulent flow) is 0.250 if we take the radius of the sphere as the characteristic dimension.

1.312. A tall wide vessel contains glycerin (its density is $\rho_0 = 1.21 \times 10^3$ kg/m³, its viscosity is $\eta = 0.350$ Pa·s). A sphere with a radius of $r = 1.00$ mm is submerged into the glycerin far from the walls of the vessel and is released without pushing it. The density of the sphere is $\rho =$ $= 10.0 \times 10^3$ kg/m³. The initial height of the sphere above the vessel's bottom is $h = 0.500$ m.

(a) Determine whether the force of resistance to the sphere's motion may be calculated by the Stokes formula (see the note to the preceding problem).

(b) Convince yourself by substitution that the distance s covered by the sphere during the time t is determined by the formula

$$s = 0.0547t - 0.000\,347\,(1 - e^{-158t})$$

(try to obtain this formula).

(c) Find the time τ during which the sphere will reach the bottom of the vessel.

(d) Determine the time t after which the speed of the sphere differs from the extreme value of 0.0547 m/s by 1%.

PART 2

MOLECULAR PHYSICS AND THERMODYNAMICS

SYMBOLS

A	work
A_r	relative atomic mass
a, b	constants; van der Waals constants
C	heat capacity; molar heat capacity; rate of evacuation
C_p	molar heat capacity at constant pressure
C_V	molar heat capacity at constant volume
c	specific heat capacity
D	diffusion coefficient
D_{12}	interdiffusion coefficient
d	diameter; distance
E	energy
F	force; free energy
g	acceleration of free fall
h	depth; height
k	Boltzmann constant
L	heat of transition
l	length; mean free path of particle
M	molar mass
M_r	relative molecular mass
m	mass
N	number of molecules
N_A	Avogadro constant
n	number of particles per unit volume; polytropic exponent
P	probability
p	pressure
Q	amount of heat
R	gas constant; radius
r	radius
S	area; entropy
S_m	molar entropy
T	absolute temperature
t	temperature, Celsius scale; time
U	internal energy
U_m	molar internal energy
u	speed
V	volume
V_m	molar volume
v	velocity; volume
α	angle
β	damping factor
γ	ratio of heat capacities C_p/C_V
ε	energy of molecule
η	efficiency; viscosity
θ	angle
$\varkappa$	thermal conductivity coefficient (thermal conductivity)
ν	number of moles
ρ	density
τ	time
φ	angle
χ	torsion coefficient
Ω	statistical weight of a state
ω	angular speed

2.1. Molecular-Kinetic Notions. The First Law of Thermodynamics

2.1. How many molecules are contained in a glass of water?

2.2. Using the value of the Avogadro constant, determine the mass: (a) of a hydrogen atom; (b) of an oxygen molecule (O_2); (c) of a uranium atom.

2.3. Calculate the mass M of a mole of electrons.

2.4. Using the value of the Avogadro constant, determine the value of the atomic mass unit (amu).

2.5. Appraise the diameter d of a mercury atom.

2.6. A mole of a gas such as helium, hydrogen, nitrogen, or oxygen occupies a volume equal to 22.4 l at S.T.P. (standard temperature $t = 0$ °C and pressure $p = 1013$ hPa). Find the values of the following quantities in this gas:

(a) the number n of molecules of a gas in unit volume;

(b) the mean distance $\langle a \rangle$ between the molecules. Compare the latter with the diameter d of a molecule.

2.7. How can we find the number n of molecules of a substance in unit volume knowing the density ρ and the molar mass M?

2.8. Among the metals, the largest value of the ratio ρ/A_r belongs to beryllium, and the smallest to potassium. Determine the number n of atoms in unit volume of these metals.

2.9. We have a flux of molecules of mass m flying at the velocity $\mathbf{v}$ identical in magnitude and direction. The number of molecules per unit volume in the flux is n. Find:

(a) the number ν of impacts of the molecules a second against unit surface area of a flat wall, a normal to which makes the angle θ with the direction of $\mathbf{v}$;

(b) the pressure p of the flux of molecules on the wall. Consider that the molecules experience mirror reflection from the wall with no loss of energy.

2.10. A gas expands in identical conditions from the volume V_1 to the volume V_2, one time rapidly, and another time slowly. When is the work done by the gas greater?

2.11. A gas is compressed in identical conditions from the volume V_1 to V_2, one time rapidly, and another time slowly. When is the work done by the gas greater in magnitude?

2.12. At a constant pressure of $p = 3.00 \times 10^5$ Pa, a gas:

(a) expands from the volume $V_1 = 2.00$ l to $V_2 = 4.00$ l;

(b) is compressed from the volume $V_1 = 8.00$ l to $V_2 = 5.00$ l. Find the work A done by the gas and the work A' done on the gas.

2.13. A gas performs a process in the course of which its pressure p varies with the volume V according to the law $p = p_0 \exp [-\alpha (V - V_0)]$, where $p_0 = 6.00 \times 10^5$ Pa,

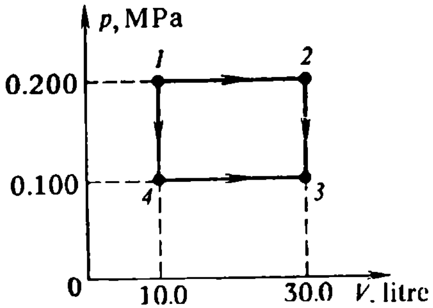

Fig. 2.1

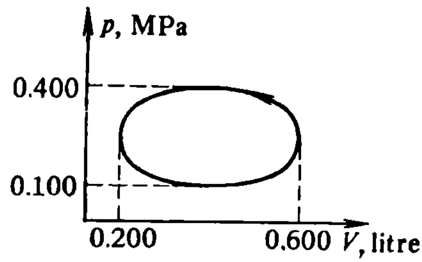

Fig. 2.2

$\alpha = 0.200$ m^{-3}, and $V_0 = 2.00$ m^3. Find the work A done by the gas upon expansion from $V_1 = 3.00$ m^3 to $V_2 = 4.00$ m^3.

2.14. A body with a heat capacity not depending on the temperature and equal to $C = 20.0$ J/K is cooled from $t_1 = 100$ °C to $t_2 = 20$ °C. Determine the amount of heat Q received by the body.

2.15. Within the temperature interval being considered, the heat capacity of a body is determined by the function $C = 10.00 + 2.00 \times 10^{-2}T + 3.00 \times 10^{-5}T^2$ (J/K). Determine the amount of heat Q received by the body when heated from $T_1 = 300$ K to $T_2 = 400$ K.

2.16. A body passes from state *1* to state *3* one time by means of the process *1-2-3* and another time by means of the process *1-4-3* (Fig. 2.1). Using the data indicated in the figure, find the difference $Q_{123} - Q_{143}$ between the amount of heat received by the body during the two processes.

2.17. A cyclic process is depicted in a p-V diagram by the ellipse shown in Fig. 2.2. Using the data given in the figure, determine the amount of heat Q obtained by the working body during one cycle.

2.18. The internal energy of an imaginary gas is determined by the formula $U = a \ln (T/T_0) + b \ln (p/p_0)$, where $a = 3.00$ kJ, $b = 7.00$ kJ, $T_0 = 200$ K, and $p_0 = 10.0$ kPa. The amount of heat $Q = 500$ J is imparted to the gas at a constant pressure of $p = 1.00 \times 10^5$ Pa, as a result of which its volume receives the increment of $\Delta V = 0.500$ l. How will the temperature of the gas change?

2.19. The gas of the preceding problem is heated from $T_1 = 250$ K to $T_2 = 500$ K. In the course of heating, the gas receives the amount of heat $Q = 14.33$ kJ and does the work $A = 4.56$ kJ. How will the pressure of the gas change?

2.20. The internal energy of an imaginary gas is determined by the formula $U = a \ln (T/T_0) + b \ln (V/V_0)$, where $a = 4.00$ kJ, $b = 5.00$ kJ, $T_0 = 200$ K, and $V_0 = 10.0$ l. The gas is initially in a state characterized by the following parameters: $V_1 = 20.0$ l, $p_1 = 1.00 \times 10^5$ Pa, and $T_1 = 300$ K. Next the gas expands isobarically to a volume of $V_2 = 30.0$ l. During its expansion, the gas obtains the heat $Q = 4.00$ kJ. Determine the final temperature T_2 of the gas.

2.2. Ideal Gas

2.21. Determine the number n of air molecules in unit volume (m^3 and cm^3) at a temperature of 0 °C and a pressure of 1.013×10^5 Pa (1 atm).

2.22. Find the mass: (a) of one cubic metre; (b) of one litre of air at 0 °C and 1.013×10^5 Pa (1 atm).

2.23. Near the Earth's surface, 78.08% of the air's molecules fall to the share of nitrogen (N_2), 20.95% to the share of oxygen (O_2), 0.93% to the share of argon (Ar), and 0.04% to the share of other gases.

(a) Assuming the pressure of the air to be 1.013×10^5 Pa, find the partial pressure of nitrogen, oxygen, and argon.

(b) Determine the mean molecular mass M_r of the air.

2.24. A rotary pump during one revolution entrains a volume v of a gas and discharges it into the atmosphere. How many revolutions N must the pump make to lower the pressure of the air in a vessel of volume V from p_0 to p?

2.25. A fore pump connected to a vessel of volume V removes from it during the time dt a gas of volume $dV = C\,dt$ (the constant C is called the rate of evacuation). Considering that during pump operation the pressure of the

gas is the same at all points of the vessel and disregarding the pressure drop in the connecting pipe between the vessel and the pump, find the law $p(t)$ according to which the pressure of the gas in the vessel changes. The initial pressure is p_0. Assume the gas to be ideal.

2.26. Using the result of the preceding problem, determine the time τ needed to lower the pressure from $p_0 = 1.00 \times 10^5$ Pa to $p = 0.300$ Pa in a vessel of volume $V = 10.0$ l with the aid of a pump having a rate of evacuation of $C = 1.00$ l/s.

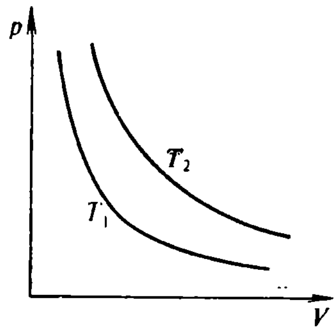

Fig. 2.3

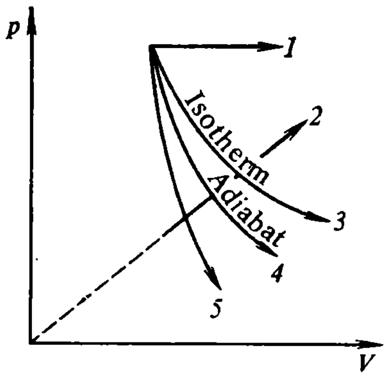

Fig. 2.4

2.27. Plot approximate graphs of an isochoric, isobaric, isothermal, and adiabatic processes for an ideal gas in diagrams: (a) p-V; (b) T-V; (c) T-p. Plot all the graphs so that they pass through a common point.

2.28. Figure 2.3 shows two isotherms for the same mass of an ideal gas. Which of the temperatures is higher?

2.29. Figure 2.4 depicts five processes occurring with an ideal gas. How does the internal energy of the gas behave in each of the processes?

2.30. Plot for an ideal gas approximate graphs of:

(a) an isochoric, isobaric, and adiabatic process in a U-T diagram;

(b) an isochoric, isobaric, isothermal, and adiabatic process in U-V and U-p diagrams. Lay off U along the axis of ordinates. Take a common point as the initial one for all the graphs.

2.31. The temperature of one mole of an ideal gas with γ known grows by ΔT in an isobaric, isochoric, and adiabat-

ic process. Determine the increment of the internal energy ΔU of the gas for all three cases.

2.32. What does the heat capacity C of an ideal gas equal in (a) an isothermal, and (b) an adiabatic process?

2.33. A certain amount of an ideal gas with triatomic rigid molecules passed adiabatically from a state with the temperature $T_1 = 280$ K to a state characterized by the values of the parameters $T_2 = 320$ K, $p_2 = 2.00 \times 10^5$ Pa, and $V_2 = 50.0$ l. What work A does the gas do in this process?

2.34. A certain amount of gas passed from a state with $U_1 = 600$ kJ to a state with $U_2 = 200$ kJ, doing the work $A = 300$ kJ. What amount of heat Q did the gas receive if the transition process is (a) reversible; (b) irreversible?

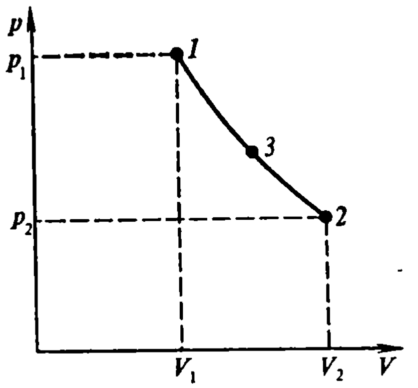

Fig. 2.5

2.35. A certain amount of a monatomic ideal gas was compressed adiabatically until the pressure exceeded its initial value p_1 10 times. Next the gas was expanded isothermally to its initial volume. How many times is the final pressure p_2 of the gas greater than its initial pressure p_1?

2.36. An ideal gas (with $\gamma = 1.40$) initially at a temperature of $t_1 = 0$ °C is compressed, as a result of which (a) the volume of the gas diminishes to one-tenth of its initial value; (b) the pressure of the gas increases 10 times. Considering the compression process to be adiabatic, find the temperature t_2 which the gas heats to because of compression.

2.37. Figure 2.5 shows a reversible transition of a biatomic ideal gas from state *1* to state *2*. The transition process consists of an isothermal section *1-3* and an adiabatic one *3-2*. In the initial state, $V_1 = 1.00 \times 10^{-3}$ m³, $p_1 = 3.00 \times 10^5$ Pa, and in the final state, $V_2 = 2.00 \times 10^{-3}$ m³, $p_2 = 1.33 \times 10^5$ Pa. Calculate the work A done by the gas in the course of the process *1-3-2*. No vibrational degrees of freedom of the gas molecules are excited.

2.38. The temperature in a room of volume V increased from the value T_1 to T_2. How did the internal energy of the

air in the room change? Assume that the atmospheric pressure remained unchanged.

2.39. The atmospheric pressure changed from $p_1 = 983$ hPa to $p_2 = 1003$ hPa. What increment ΔU did the internal energy of the air in a room of volume $V = 50.0$ m^3 receive? Assume that the temperature in the room remained unchanged.

2.40. A closed cylinder is divided into two parts by a piston of radius $r = 10.0$ cm and mass $m = 1.00$ kg that can move without friction. The piston was placed in its middle position, and then both parts of the cylinder were filled with a gas to an identical pressure of $p_0 = 1.00 \times 10^5$ Pa. The volume of the gas in each half is $V_0 = 5.00$ l. The gas may be considered ideal, its $\gamma = 1.40$. Disregarding heat exchange through the walls of the cylinder and through the piston, find the frequency ν of oscillations of the piston produced when it is displaced slightly from its middle position.

2.41. A certain amount of an ideal gas with monatomic molecules completed at $p = 1.00 \times 10^5$ Pa a reversible isobaric process during which the volume of the gas changed from $V_1 = 10.0$ l to $V_2 = 20.0$ l. Determine:

(a) the increment ΔU of the internal energy of the gas;
(b) the work A done by the gas;
(c) the amount of heat Q received by the gas.

2.42. An ideal gas with $\gamma = 1.40$ expands isothermally from the volume $V_1 = 0.100$ m^3 to $V_2 = 0.300$ m^3. The final pressure of the gas is $p_2 = 2.00 \times 10^5$ Pa. Determine:

(a) the increment ΔU of the internal energy of the gas;
(b) the work A done by the gas;
(c) the amount of heat Q received by the gas.

2.43. In isobaric heating from 0 to 100 °C, a mole of an ideal gas absorbs $Q = 3.35$ kJ of heat. Determine:

(a) the value of γ;
(b) the increment ΔU of the internal energy of the gas;
(c) the work A done by the gas.

2.44. A mole of an ideal gas having initially a temperature of $T_1 = 290$ K expands isobarically until its volume increases 2.00 times. Next the gas is cooled isochorically to its initial temperature T_1. Determine:

(a) the increment ΔU of the internal energy of the gas;
(b) the work A done by the gas;
(c) the amount of heat Q received by the gas.

2.45. Figure 2.6 shows the process of the transition of a certain amount of an ideal gas from state *1* to state *2*. Does the gas receive or give up heat in this process?

2.46. A certain amount of an ideal gas passes from state *1* to state *2* one time by means of process *I*, and another time by means of process *II* (Fig. 2.7). In which of the processes is the amount of heat received by the gas greater?

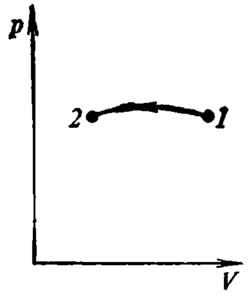

Fig. 2.6

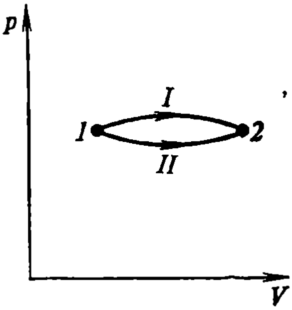

Fig. 2.7

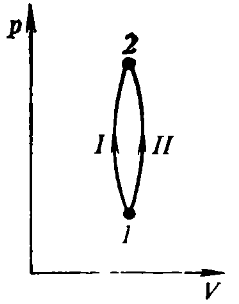

Fig. 2.8

2.47. A certain amount of an ideal gas passes from state *1* to state *2* one time by means of process *I*, and another time by means of process *II* (Fig. 2.8).

(a) What sign does the work A_{I} and A_{II} done by the gas in each of the processes have?

(b) In which of the processes does the gas receive more heat?

2.48. 1.00 kg of nitrogen (N_2) is initially confined in a volume of $V_1 = 0.300$ m^3 under a pressure of $p_1 = 5.00 \times \times 10^5$ Pa. Next the gas was expanded, as a result of which its volume becomes equal to $V_2 = 1.00$ m^3, and its pressure, to $p_2 = 1.00 \times 10^5$ Pa.

(a) Determine the increment ΔU of the internal energy of the gas.

(b) Can the work done by the gas in expansion be calculated?

2.49. As a result of the reversible isothermal (at $T = = 300$ K) expansion of 531 g of nitrogen (N_2), the pressure of the gas diminishes from $p_1 = 20.0 \times 10^5$ Pa to $p_2 = = 2.00 \times 10^5$ Pa. Determine:

(a) the work A done by the gas in expansion;

(b) the amount of heat Q received by the gas.

2.50. As a result of reversible adiabatic expansion, the temperature of 1.00 kg of nitrogen (N_2) lowers by 20.0 K. Determine the work A done by the gas in expansion. Have in view that the vibrational degrees of freedom of the nitrogen molecules are not excited at the temperatures being considered.

2.51. 321 g of helium (He) originally at a temperature of 20 °C and a pressure of $p_1 = 1.00 \times 10^5$ Pa were compressed adiabatically to a pressure of $p_2 = 1.00 \times 10^7$ Pa. Considering the process of compression to be reversible, determine:

(a) the temperature T_2 of the gas at the end of compression;

(b) the work A done by the gas;

(c) the number of times the volume of the gas diminished.

2.52. A monatomic ideal gas performs a process during which the molar heat capacity of the gas remains constant and equal to $\frac{5}{2}R$. What is the polytropic exponent n of this process?

2.53. The heat capacity of an ideal gas in a polytropic process is $C = C_V + 0.1R$. Find the value of the polytropic exponent n of this process.

2.54. The molar heat capacity of an ideal gas (with $\gamma = 1.40$) varies during a process according to the law $C = 20.0 + 500/T$ [J/(mol·K)].

(a) Is this process a polytropic one?

(b) Find the work A done by a mole of the gas when heated from $T_1 = 200$ K to $T_2 = 544$ K.

2.55. An ideal gas completes a process during which the pressure p grows in proportion to the volume V. Is this process polytropic?

2.56. For the process in the preceding problem, find:

(a) the polytropic exponent n;

(b) the molar heat capacity C.

2.57. Express the work A_{12} done by ν moles of an ideal gas in a polytropic process (with a polytropic exponent n) in terms of the temperatures T_1 and T_2 of the initial and final states.

2.58. An ideal gas expands according to the law $pV^2 = \text{const}$.

(a) Is it heated or cooled?

(b) What is the molar heat capacity C of the gas in this process?

2.59. Express the molar heat capacity C of an ideal gas in a polytropic process in terms of the polytropic exponent n and the ratio of the heat capacities γ.

2.60. Using the answer to the preceding problem, determine the values of the polytropic exponent n at which the heat capacity C of an ideal gas in the course of a polytropic process: (a) is positive; (b) is negative; (c) is zero; (d) is infinitely great.

2.61. In a polytropic process, an ideal gas (with $\gamma = 1.40$) was compressed from the volume $V_1 = 10.0$ l to $V_2 = 5.00$ l. The pressure increased from $p_1 = 1000$ hPa to $p_2 = 5000$ hPa. Determine:

(a) the polytropic exponent n;

(b) the molar heat capacity C of the gas for the process being considered.

2.62. Determine the molar heat capacity C of an ideal gas (with $\gamma = 1.40$) in a polytropic process with a polytropic exponent equal to (a) $n = 0.9$; (b) $n = 0.99$; (c) $n = 0.999$; (d) $n = 1.1$. Express C in terms of R.

2.63. For an ideal gas with $\gamma = 1.40$, draw an approximate graph showing how the molar heat capacity C in a polytropic process depends on the polytropic exponent n. Indicate the asymptotes and the characteristic points on the graph.

2.64. An ideal gas expands in the course of a polytropic process. At what values of the polytropic exponent n will the temperature of the gas (a) grow; (b) lower; (c) remain constant?

2.65. A certain amount of an ideal gas (with $\gamma = 1.40$) expands from $V_1 = 20.0$ l to $V_2 = 50.0$ l so that the process in a p-V diagram has the form of a straight line. The initial pressure is $p_1 = 1000$ hPa, and the final one is $p_2 = 2000$ hPa.

(a) Is the process a polytropic one?

(b) Find the amount of heat Q absorbed by the gas in the course of expansion.

2.3. The Kinetic Theory

2.66. How many molecules ν collide in 1 s with 1 m^2 of the wall of a vessel containing nitrogen (N_2) at a pressure of 1013 hPa (1 atm) and a temperature of 20 °C?

2.67. A spherical vessel with an internal radius of $r = 5.00$ cm contains hydrogen (H_2) at a temperature of $T = 300$ K and a pressure of $p = 1.00 \times 10^5$ Pa. How many molecules ν collide with the walls of the vessel in 1 s?

2.68. Determine the number and nature of the degrees of freedom of the molecules in a gas for which γ equals (a) 1.67; (b) 1.40; (c) 1.33; (d) 1.29; (e) 1.17.

2.69. Calculate the molar heat capacities C_V and C_p (express them in terms of R), and also the ratio of these heat capacities γ for an ideal gas with (a) monatomic molecules; (b) biatomic rigid molecules; (c) biatomic elastic molecules; (d) triatomic rigid molecules (whose atoms do not lie on one straight line).

2.70. How many atoms do the molecules of a gas consist of if γ increases 1.20 times when the vibrational degrees of freedom are "frozen"?

2.71. At $T = 1.00 \times 10^3$ K in tetratomic molecules of an ideal gas, all the degrees of freedom (including the vibrational ones) are excited. Determine the internal energy U_m of a mole of the gas.

2.72. A vessel is filled with argon (Ar). The temperature of the gas is 0 °C. The vessel initially travels at a speed of $v = 100$ m/s, then suddenly stops. Disregarding the exchange of heat between the gas and the walls of the vessel, determine the temperature of the gas after stopping of the vessel.

2.73. The mean energy of the molecules of a monatomic ideal gas $\langle\varepsilon\rangle = 6.00 \times 10^{-21}$ J. The pressure of the gas is $p = 2.00 \times 10^5$ Pa. Find the number of gas molecules n in a unit of volume.

2.4. Distributions

2.74. We are given $f(x)$—the probability distribution function of the quantity x. Write an expression for $P(a \leqslant x \leqslant b)$—the probability of the fact that the magnitude of the quantity x is within the interval from a to b.

2.75. We are given $f_1(x)$ and $f_2(y)$—the probability distribution functions for the statistically independent quantities x and y. Write an expression for $P(a_1 \leqslant x \leqslant a_2; b_1 \leqslant y \leqslant b_2)$—the probability of the fact that the value of the quantity x is within the interval from a_1 to a_2, and the

value of the quantity y is at the same time within the interval from b_1 to b_2.

2.76. Figure 2.9 contains plots of four different probability distribution functions of the values of a quantity x. For each of the plots, find the value of the constant A at which the function is normalized. Next calculate the mean values of x and x^2. For case (a) also calculate $\langle |x| \rangle$.

2.77. The probability distribution function of the quantity x has the form $f(x) = A \exp(-\alpha x^2)\, 4\pi x^2$, where A and α are constants. Write an approximate expression for

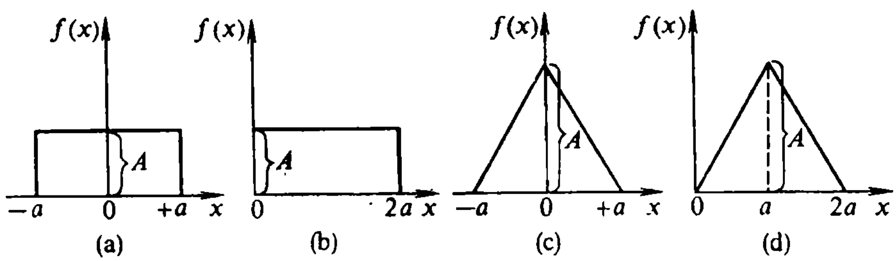

Fig. 2.9

the probability P of the fact that the value of x will be within the limits from 7.9999 to 8.0001.

2.78. A harmonic oscillator performs oscillations with the amplitude A. The mass of the oscillator is m, and its natural frequency is ω. Find:

(a) the function $f(x) = dP_x/dx$ showing the probability distribution of the values of the oscillator's coordinate x;

(b) the mean value of the coordinate $\langle x \rangle$;

(c) the mean value of the magnitude of the coordinate $\langle | x | \rangle$;

(d) the mean value of the square of the coordinate $\langle x^2 \rangle$;

(e) the mean value $\langle U \rangle$ of the potential energy of the oscillator.

2.79. Find the temperature T at which the root-mean-square speed of molecules of nitrogen (N_2) is greater than the mean speed by 50.0 m/s.

2.80. At what temperature T of the air will the mean speeds of molecules of nitrogen (N_2) and oxygen (O_2) differ by 20.0 m/s?

2.81. Transform the Maxwell distribution function, going over from the variable v to the variable $u = v/v_{\text{prob}}$,

where v_{prob} is the most probable speed of the molecules.

2.82. A sealed glass cylinder contains a mole of a monatomic ideal gas at a temperature of $T = 293$ K. What amount of heat Q must be imparted to the gas for the mean speed of its molecules to increase by 1%?

2.83. Calculate the most probable, mean, and root-mean-square speeds of molecules of oxygen (O_2) at 20 °C.

2.84. A mole of nitrogen (N_2) is in its equilibrium state at $T = 300$ K. Determine:

(a) the sum $\sum v_x$ of the x-components of the velocities of all the molecules;

(b) the sum $\sum \mathbf{v}$ of the velocities of all the molecules;

(c) the sum $\sum \mathbf{v}^2$ of the squares of the velocities of all the molecules;

(d) the sum $\sum v$ of the speeds of all the molecules.

2.85. Find the mean value of the magnitude of the x-component of the velocity for the molecules of a gas in an equilibrium state at the temperature of T. The mass of a molecule is m.

2.86. Find the sum of the magnitudes of the momenta of the molecules contained in a mole of nitrogen (N_2) at a temperature of 20 °C.

2.87. Determine, proceeding from classical notions, the root-mean-square angular speed $\sqrt{\langle\omega^2\rangle}$ of rotation of molecules of nitrogen (N_2) at $T = 300$ K. The distance between the nuclei of a molecule is $l = 3.7 \times 10^{-10}$ m.

2.88. A gas is in an equilibrium state. What per cent of the gas molecules have speeds differing from the most probable one by not over 1%?

2.89. Write an expression determining the relative fraction η of the molecules of a gas having speeds exceeding the most probable velocity.

2.90. The mean energy of molecules of helium (He) is $\langle\varepsilon\rangle = 3.92 \times 10^{-21}$ J. Determine the mean speed $\langle v\rangle$ of the helium molecules in the same conditions.

2.91. Nitrogen (N_2) is in an equilibrium state at $T = 421$ K.

1. Find the value of the most probable speed v_{prob} of the molecules.

2. Determine the relative number $\Delta N/N$ of the molecules whose speeds are within the limits: (a) from 499.9 to 500.1 m/s;

(b) from 249.9 to 250.1 m/s; (c) from 749.9 to 750.1 m/s; (d) from 999.9 to 1000.1 m/s.

2.92. Figure 2.10 shows how the number n of air molecules per unit volume depends on the altitude h. What is the meaning of the hatched area?

2.93. The molecules of an ideal gas are in equilibrium in a centrally symmetric force field so that the potential energy of an individual molecule has the form $\varepsilon_p = \varepsilon_p(r)$. Write an expression for dN_r—the number of molecules whose distances from the force centre are within the interval from r to $r + dr$. The number of molecules per unit volume at the distance r_1 is known to be n_1.

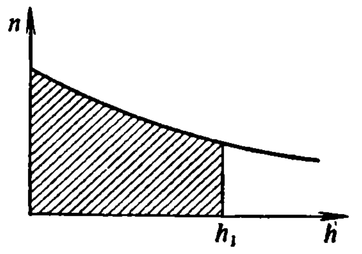

Fig. 2.10

2.94. In an experiment by means of which J. Perrin determined the Avogadro constant N_A, he used a suspension of globules of gamboge ($\rho = 1.254$ g/cm^3) in water. The temperature of the suspension was 20 °C. The radius of the globules was $r = 0.212$ μm. When the tube of the microscope was moved over a distance of $\Delta h = 30$ μm, the number of globules observed in the microscope changed 2.1 times. Proceeding from these data, find N_A.

2.95. Considering the atmosphere to be isothermal, and the acceleration of free fall independent of the altitude, evaluate the pressure (a) at an altitude of 5 km; (b) at an altitude of 10 km; (c) in a shaft at a depth of 2 km. Perform the calculations for $T = 293$ K. The pressure at sea level is p_0.

2.96. Near the Earth's surface, the ratio of the volume concentrations of oxygen (O_2) and nitrogen (N_2) in the air is $\eta_0 = 20.95/78.08 = 0.268$. Assuming the temperature of the atmosphere to be independent of the altitude and equal to 0 °C, find this ratio η for an altitude of $h = 10$ km.

2.97. A (vertical) chimney closed at both ends is filled with gaseous oxygen (O_2). The height of the chimney is $h = 200$ m, and its volume is $V = 200$ l. The chimney walls have the same temperature of $T = 293$ K everywhere. The pressure of the gas inside the chimney near its base is $p_0 = 1.00 \times 10^5$ Pa. Find:

(a) the pressure p in the chimney near its top end;

(b) the number N of oxygen molecules contained in the chimney.

2.98. 1. Assuming that the temperature of the air and the acceleration of free fall do not depend on the altitude, determine the altitude h at which the density of the air is smaller than its value at sea level (a) 2 times; (b) e times. Assume the temperature of the air to be 0 °C.

2. Having obtained the results, convince yourself that g may be assumed to be independent of h. For this purpose, use the formula obtained in Problem 1.226 to appraise in per cent the difference between the acceleration of free fall at the found altitudes and its value g at sea level.

2.99. A pipe with a length of $l = 1.00$ m closed at one end rotates about a vertical axis perpendicular to it and passing through the open end of the pipe at the angular speed $\omega = 62.8$ rad/s. The surrounding air has a pressure of $p_0 = 1.00 \times 10^5$ Pa and a temperature of $t = 20$ °C. Find the pressure p of the air in the pipe near its closed end.

2.100. The energy of each of N particles can be either E_1 or E_2. The particles are in an equilibrium state at the temperature T. What is the total energy E of all the particles in this state?

2.5. Entropy

2.101. A vessel contains five molecules.

(a) In how many ways can these molecules be distributed between the left and right halves of the vessel?

(b) What is the value of Ω (0, 5)—the number of ways in which a distribution can be achieved whereupon all five molecules will be in the right half of the vessel? What is the probability P (0, 5) of such a state?

(c) What is the value of Ω (1, 4)—the number of ways in which a distribution can be achieved whereupon one molecule will be in the left half of the vessel and four in the right half? What is the probability P (1, 4) of such a state?

(d) What is the value of Ω (2, 3)? What is the probability P (2, 3) of such a state?

2.102. How does the statistical weight Ω of the state of a thermodynamic system behave when a reversible adiabatic process occurs?

2.103. A thermodynamic system passed from state *1* to state *2*. The statistical weight of the second state is double that of the first one. What is the increment ΔS_{12} of the system's entropy?

2.104. The statistical weight of the state of a mass of a gas is Ω_1. Determine the statistical weight Ω_2 of the state in an η times greater mass of the same gas. The temperature and pressure of the gas in both cases are the same.

2.105. The statistical weight Ω of a state of a thermodynamic system is: (a) $1.00 \times 10^{10^{20}}$; (b) $5.00 \times 10^{10^{20}}$. What is the entropy S of the system in this state?

What does the relative difference of the entropies $\Delta S/S$ for cases (a) and (b) equal in its order of magnitude?

2.106. The logarithm of $N!$ can be calculated approximately by the Stirling formula

$$\ln N! = N \ln N - N + \frac{1}{2} \ln (2\pi N)$$

The relative error produced by this formula diminishes with increasing N.

Compare the exact values of $\ln N!$ with the ones computed by the Stirling formula for (a) $N = 5$; (b) $N = 10$.

2.107. In statistical physics, the third term in the Stirling formula (see the preceding problem) is disregarded, and it is assumed that

$$\ln N! \approx N \ln N - N$$

Determine the relative error obtained when using this formula for (a) $N = 5$; (b) $N = 10$; (c) $N = 20$; (d) $N = 30$; (e) $N = 100$. For $N \geqslant 20$, take the value computed by the three-term Stirling formula given in the preceding problem as the exact value of $\ln N!$

2.108. Appraise the linear dimensions l of the volume $\Delta V = l^3$ falling on an average to the number of molecules $\Delta N = 10^3$ for a gas at S.T.P. ($p = 1.013 \times 10^5$ Pa and $T = 273$ K).

2.109. The entropy of a mole of hydrogen (H_2) at a temperature of 25 °C and a pressure of 1.013×10^5 Pa (1 atm) is $S_m = 130$ J/(mol·K). Determine the statistical weight Ω: (a) of one mole; (b) of two moles of hydrogen in the indicated conditions.

2.110. Determine how many times the statistical weight Ω of a mole of water increases when it passes from the liquid to the gaseous state at a temperature of 100 °C.

2.111. How does the entropy of a thermodynamic system behave in an adiabatic process?

2.112. Can the entropy of a system grow during a process in which the system gives up heat to the surroundings?

2.113. A gas passes from state *1* to state *2* by means of a reversible adiabatic process. Can this gas pass from state *1* to state *2* by means of an irreversible adiabatic process?

2.114. Figure 2.11 shows two isentropes for the same mass of an ideal gas. Which of the entropies is greater?

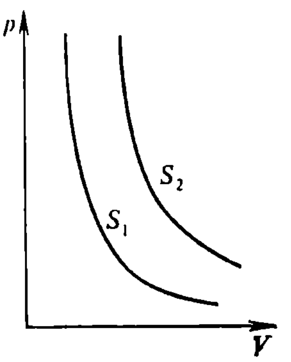

Fig. 2.11

2.115. Depict graphs of an isothermal and an adiabatic process in a U-S diagram for an ideal gas.

2.116. Depict for an ideal gas approximate graphs of an isothermal, isochoric, isobaric, and adiabatic process in the diagrams: (a) T-S; (b) V-S; (c) p-S. Lay off S along the axis of abscissas. Draw the graphs passing through a common point.

2.117. How does the entropy behave in the course of each of the processes shown in Fig. 2.4? (See Problem 2.29.)

2.118. Depict for an ideal gas approximate graphs of an isochoric, isobaric, isothermal, and adiabatic process in the diagrams: (a) S-T; (b) S-V; (c) S-p. Lay off S along the axis of ordinates. Adopt a common point as the initial one for all the graphs.

2.119. A certain amount of a gas passes from equilibrium state *1* to equilibrium state *2* by means of: (a) a reversible adiabatic process, (b) an irreversible process. The initial and final states of the gas are the same for both processes.

1. What is the increment ΔS of the entropy of the gas in both cases?

2. Can the second process also be adiabatic?

2.120. Figure 2.12 depicts a process performed by a certain amount of an ideal gas. The entropy increment ΔS_{12}

in the course of process *1-2* is known to differ from the increment ΔS_{23} in the course of process *2-3* only in the sign. What can we say about states *1* and *3*?

2.121. Figure 2.13 depicts process *1-2-3* transferring an ideal gas from state *1* to state *3*. Process *1-2* is reversible, and *2-3* is irreversible. States *1* and *3* are on the same adiabat. Process *1-2* is isothermal; it takes place at $T = 300$ K and is attended by the gas doing the work $A_{12} = 3.00$ J. What

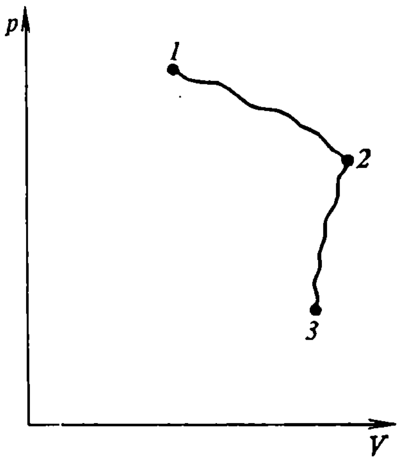

Fig. 2.12

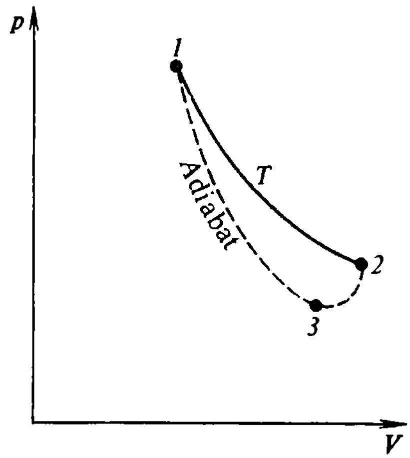

Fig. 2.13

is the increment ΔS_{23} of the entropy in the course of process *2-3*?

2.122. In a certain temperature region, the entropy of a thermodynamic system varies with the temperature according to the law $S = a + bT$, where $a = 100$ J/K and $b = 5.00$ J/K^2. What amount of heat Q does the system receive upon reversible heating in this region from $T_1 = 290$ K to $T_2 = 310$ K?

2.123. A mole of a monatomic ideal gas is heated reversibly from $T_1 = 300$ K to $T_2 = 400$ K. In the course of heating, the pressure of the gas varies with the temperature according to the law $p = p_0 \exp(\alpha T)$, where $\alpha = 1.00 \times 10^{-3}$ K^{-1}. Determine the amount of heat Q received by the gas when heated.

2.124. A cyclic process is depicted in a *T-S* diagram by an ellipse as shown in Fig. 2.14. Using the data contained in

the figure, determine the work A done by the working substance during a cycle.

2.125. The entropy of 1 g of nitrogen at a temperature of 25 °C and a pressure of 1.00×10^5 Pa is $S_1 = 6.84$ J/(g·K). Determine the entropy of 2 g of nitrogen at 100 °C and 2.00×10^5 Pa.

2.126. The entropy of a mole of oxygen at a temperature of 25 °C and a pressure of 1.00×10^5 Pa is $S_1 = 204.8$ J/(mol·K). As a result of isothermal expansion, the volume

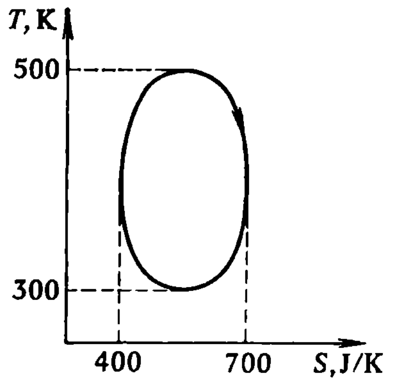

Fig. 2.14

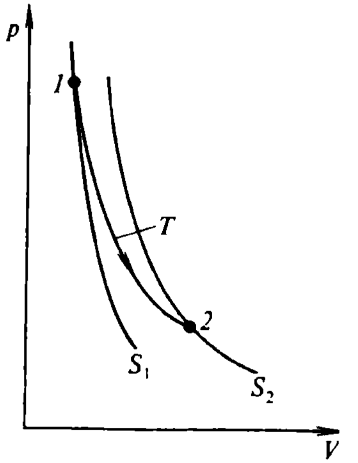

Fig. 2.15

occupied by the gas increased twofold. Determine the entropy S_2 of the oxygen in the final state.

2.127. Find the increment ΔS_m of the entropy of a mole of a monatomic ideal gas when heated from 0 to 273 °C if heating occurs: (a) at constant volume; (b) at constant pressure.

2.128. An ideal gas, expanding isothermally (at $T = 400$ K), does the work $A = 800$ J. What happens to the entropy of the gas?

2.129. In the course of a reversible isothermal process occurring at a temperature of $T = 350$ K, a body does the work $A = 80.0$ J, while the internal energy of the body receives an increment of $\Delta U = 7.50$ J. What happens to the entropy of the body?

2.130. An ideal gas, expanding isothermally at a temperature of T, passes from state *1* to state *2* (Fig. 2.15). State *1*

is on the adiabat corresponding to the value of the entropy of S_1, and state *2* is on the adiabat corresponding to the value of the entropy of S_2. What work A does the gas do in the course of the process?

2.131. Find the increment ΔS of the entropy when $m = 200$ g of ice at a temperature of -10.7 °C transfer into water at 0 °C. Consider that the heat capacity of the ice does not depend on the temperature. Assume the melting point to equal 273 K.

2.132. Find the increment ΔS of the entropy upon the condensation of $m = 1.00$ kg of steam at a temperature of 100 °C to water and the following cooling of the water to a temperature of 20 °C. Consider that the heat capacity of the water does not depend on the temperature. The condensation occurs at a pressure of 1 atm.

2.133. Within a limited temperature interval, the increment of the entropy of a substance is proportional to the increment of the temperature: $\Delta S = \alpha\, \Delta T$. How does the heat capacity C of the substance depend on the temperature within the same interval?

2.134. The heat capacity of bodies with simple crystal lattices varies near absolute zero according to the law $C = aT^3$, where a is a constant. Find the entropy of the body in these conditions.

2.135. Find how the entropy S_m of a mole of an ideal gas (with a known γ) depends on the volume V_m for a process in which the pressure of the gas is proportional to its volume.

2.136. A mole of an ideal gas (with $\gamma = 1.40$) completes a reversible process during which the entropy of the gas varies in proportion to the absolute temperature. As a result of the process, the internal energy of the gas changes from the value of $U_1 = 6.00$ kJ/mol to $U_2 = 7.00$ kJ/mol. The value of the entropy in the initial state is $S_1 = 200$ J/(mol $\times$ K). Find the work A done by the gas in the course of the process.

2.137. A kilogram of oxygen is initially contained in a volume of $V_1 = 0.200$ m^3 under a pressure of $p_1 = 5.00 \times 10^5$ Pa. Next the gas was allowed to expand, as a result of which its volume became equal to $V_2 = 0.500$ m^3, and its pressure to $p_2 = 2.00 \times 10^5$ Pa.

(a) Determine the increment ΔS of the gas's entropy.

(b) What is the increment ΔU of the internal energy of the gas?

2.138. A vessel is divided into two equal parts by a partition with an opening closed by a plug. One half of the vessel contains a mole of an ideal gas. There is a vacuum in its other half. The plug is removed, and the gas occupies the entire volume. Considering the process to be adiabatic, determine:

(a) the increment ΔU_m of the internal energy of the gas;
(b) the increment ΔS_m of the entropy of the gas.

2.139. Prove that the internal energy U, the entropy S, and the free energy F of a mixture of two ideal gases equal the sum of the relevant quantities for the components of the

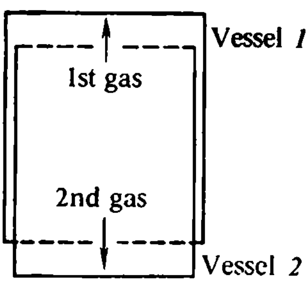

Fig. 2.16

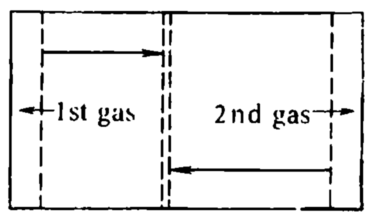

Fig. 2.17

mixture. For this purpose, consider the process of separation of a mixture containing ν_1 moles of component *1* and ν_2 moles of component *2*. Place the mixture into a vessel consisting of two vessels of the same volume V, one of which is inserted into the other one (Fig. 2.16). The bottom of vessel *1* freely passes molecules of *2* and is impermeable to molecules of *1*. The lid of vessel *2* freely passes molecules of *1* and is impermeable to molecules of *2*. Initially vessel *2* is completely moved into vessel *1*, and their common volume V contains the gas mixture. The walls of the vessels do not transmit heat. By moving vessel *2* very slowly out of vessel *1*, we can perform the reversible adiabatic separation of the components. It is necessary to prove that U, S, and F of the system do not change. This will prove the statement made initially.

2.140. A vessel is divided by a partition into two parts. One of them contains ν_1 moles of one gas, and the other,

ν_2 moles of a second gas. Both gases are ideal, and their temperature and pressure are the same. The partition is removed and the gases mix completely. Using the result of the preceding problem, find the increment ΔS of the entropy.

2.141. Solve Problem 2.140 by considering an isothermal process of separating the mixture by moving two partitions, one of which freely passes molecules of gas *1* and is impermeable to molecules of gas *2*, while the second one freely passes molecules of gas *2* and is impermeable to molecules of gas *1*. The first partition was initially at the left end of the vessel, and the second one was at the right end (Fig. 2.17). Next the partitions are moved in turn to a position at which the pressure in both parts of the vessel will be the same.

2.142. At $t = 25$ °C and $p = 1013$ hPa, the entropy of a mole of nitrogen is 192 J/(mol·K), and of a mole of oxygen is 205J/(mol·K). Assuming that atmospheric air contains four nitrogen molecules per oxygen molecule and disregarding the remaining components of the air, find:

(a) the entropy S_m of a mole of air at 25 °C;

(b) the dependence of S_m on T in the region of temperatures in which air obeys the ideal gas laws.

In both cases, $p = 1013$ hPa.

2.143. The temperature in a room with a volume of $V = 50.0$ m^3 increased from 15 to 20 °C. Using the result of the preceding problem, determine the increment ΔS of the entropy of the air contained in the room. Assume that the atmospheric pressure does not change and is $p = 1013$ hPa.

2.144. An ideal gas at a temperature of $T = 300$ K completes a reversible isothermal process during which the work $A' = -900$ J was done on the gas. Find the increment ΔS of the entropy and the increment ΔF of the free energy of the gas.

2.145. The transition of a thermodynamic system from equilibrium state *1* to equilibrium state *2* is attended by the reception of the heat Q and the increment ΔF of the free energy. The temperature and entropy change from the values T_1, S_1 to T_2, S_2. What work A is done by the system?

2.146. As a result of reversible adiabatic expansion, the temperature of a mole of a monatomic ideal gas lowers by 10.0 K. The entropy of the gas is $S_m = 20.0$ J/(mol·K). Find the increment ΔF_m of the free energy of the gas.

2.147. Find the increment ΔF of the free energy of a gas in the course of process *1-2* described in Problem 2.130.

2.6. Cycles

2.148. An ideal gas completes a cycle consisting of two isotherms and two isochors (Fig. 2.18).

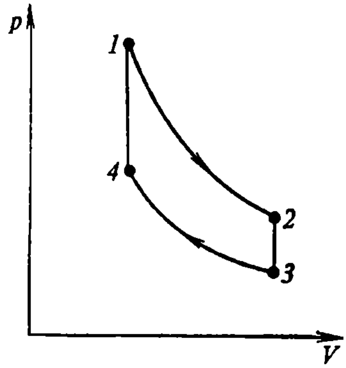

Fig. 2.18

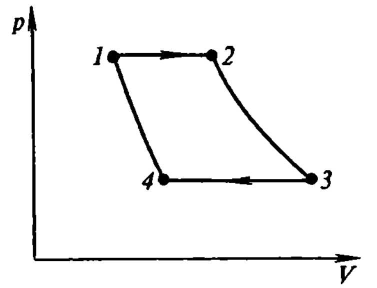

Fig. 2.19

1. How will (a) the internal energy; (b) the entropy behave on different sections of the cycle?
2. On what sections is (a) the work A done by the gas; (b) the heat Q received by the gas greater (smaller) than zero?

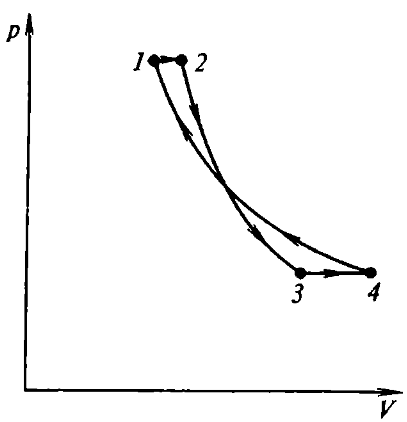

Fig. 2.20

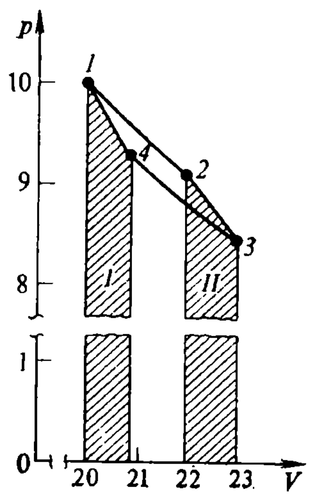

Fig. 2.21

2.149. An ideal gas completes a cycle consisting of two adiabats and two isobars (Fig. 2.19).

1. How does (a) the internal energy; (b) the entropy behave on different sections of the cycle?

2. On what sections is (a) the work A done by the gas; (b) the heat Q received by the gas greater (smaller) than zero?

2.150. Depict in a T-V diagram a cycle completed by an ideal gas and consisting of (a) two isotherms and two isobars; (b) two isobars and two isochors.

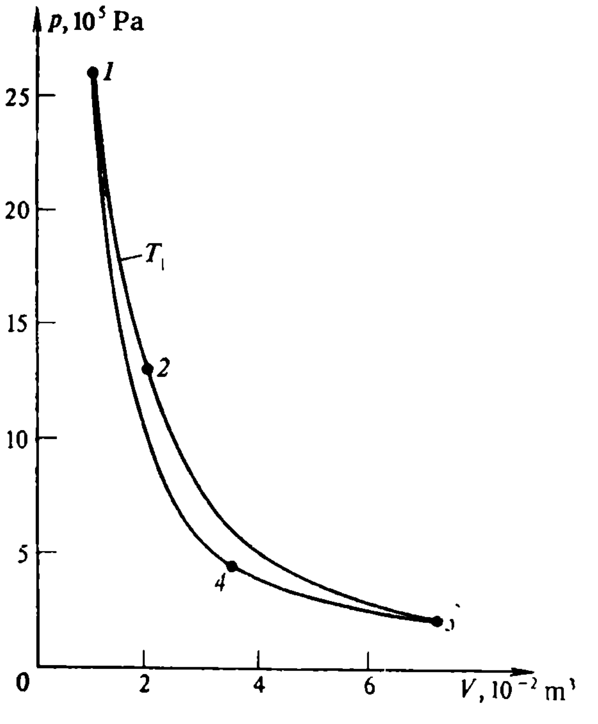

Fig. 2.22

2.151. Depict in a T-p diagram a cycle completed by an ideal gas and consisting of (a) two isotherms and two isochors; (b) two isochors and two isobars.

2.152. Depict in a T-S diagram a cycle completed by an ideal gas and consisting of two isobars and two isochors.

2.153. A cyclic process consists of an isotherm, an adiabat, and two isobars (Fig. 2.20). Depict this process in a T-S diagram.

2.154. In the course of a Carnot cycle, the working substance receives the heat $Q_1 = 300$ kJ from a high tempera-

ture reservoir. The temperatures of the high and low temperature reservoirs are $T_1 = 450$ K and $T_2 = 280$ K, respectively. Determine the work A done by the working substance during a cycle.

2.155. Figure 2.21 shows a Carnot cycle for an ideal gas in a p-V diagram. Which of the hatched areas—*I* or *II*—is larger?

2.156. Figure 2.22 depicts a Carnot cycle. Oxygen in an amount of $m = 200$ g is the working substance. The volume of the gas in states *1* and *2* is respectively $V_1 = 1.00 \times \times 10^{-2}$ m^3 and $V_2 = 2.00 \times 10^{-2}$ m^3. The temperature $T_1 = 500$ K. The work done during the cycle is $A = = 7.20$ kJ. Evaluate the work A_{41} done by the gas in the course of the process *4-1*. The vibrational degrees of freedom of the molecules are not excited.

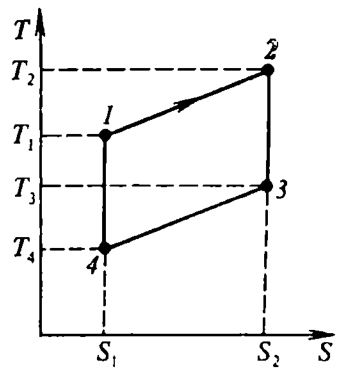

Fig. 2.23

2.157. An ideal gas (with a known γ) completes a cyclic process consisting of two isotherms and two isochors. The isothermal processes go on at the temperatures T_1 and T_2 ($T_1 > T_2$), and the isochoric ones at the volumes V_1 and V_2 (V_2 is e times larger than V_1). Find the efficiency η of the cycle.

2.158. An ideal gas (with a known γ) completes a cyclic process consisting of two isotherms and two isobars. The isothermal processes go on at the temperatures T_1 and T_2 ($T_1 > T_2$), and the isobaric ones at the pressures p_1 and p_2 (p_2 is e times larger than p_1). Find the efficiency η of the cycle.

2.159. Adopting an ideal gas with $\gamma = 1.40$ as the working substance and assuming that the values of the temperatures are $T_1 = 500$ K and $T_2 = 300$ K, evaluate the efficiency η:

(a) of a Carnot cycle;

(b) of the cycle considered in Problem 2.157;

(c) of the cycle considered in Problem 2.158.

2.160. Prove that the efficiency of any reversible cycle completed within the interval of temperatures from T_1 to

T_2 ($T_1 > T_2$) is less than the efficiency of the relevant Carnot cycle.

Indication. Compare the cycles in a T-S diagram.

2.161. A reversible cycle completed by a thermodynamic system has the form shown in Fig. 2.23 in a T-S diagram. Find the efficiency η of the cycle.

2.7. Van der Waals Equation

2.162. A mole of nitrogen was cooled to a temperature of —100 °C. Determine the pressure p exerted by the gas on the walls of the vessel if the volume V occupied by the gas is (a) 1.00 litre, and (b) 0.100 litre. Compare p with the pressure p_{id} which the nitrogen would have if it retained the properties of an ideal gas in the conditions being considered.

2.163. Solve the preceding problem for two moles of nitrogen and the same values of the temperature and volume. Compare the result obtained with the answer to the preceding problem.

2.164. To determine the van der Waals constants, a certain amount of gas occupying a volume of $V_1 = 6.79 \times 10^{-4}$ m^3 at $T_1 = 300$ K and $p_1 = 1.00 \times 10^7$ Pa was compressed isothermally to a volume of $V_2 = 4.00 \times 10^{-4}$ m^3, as a result of which the pressure increased to the value $p_2 = 1.65 \times 10^7$ Pa. Next the gas was cooled at a constant volume to the temperature $T_2 = 200$ K, with the pressure diminishing to $p_3 = 0.819 \times 10^7$ Pa. Using these data, calculate the values of the constants a and b for a mole of gas.

2.165. A mole of nitrogen expands adiabatically into an evacuated enclosure, as a result of which its volume increases from $V_1 = 1.00$ l to $V_2 = 10.0$ l. Determine the increment ΔT of the temperature of the gas.

2.166. Two moles of hydrogen expand into an evacuated enclosure, as a result of which the volume of the gas increases from $V_1 = 2.00$ l to $V_2 = 10.0$ l. What amount of heat Q must be imparted to the gas for its temperature to remain unchanged?

2.167. Obtain an expression for the work A done by a mole of a van der Waals gas upon isothermal expansion from the volume V_1 to the volume V_2. The temperature of the gas is T, and the van der Waals constants are a and b. Compare

the expression obtained with a similar expression for an ideal gas.

2.168. A mole of oxygen that initially occupied a volume of $V_1 = 1.000$ litre at a temperature of -100 °C expanded isothermally to the volume $V_2 = 9.712$ l. Find:

(a) the increment ΔU_m of the internal energy of the gas;

(b) the work A done by the gas (compare A with the work A_{id} calculated by the formula for an ideal gas);

(c) the amount of heat Q received by the gas.

2.169. Obtain for a van der Waals gas an equation of an adiabat in the variables V and T, and also in the variables V and p. Compare the equations obtained with similar equations for an ideal gas.

2.170. (a) Determine for a van der Waals gas the difference between the molar heat capacities $C_p - C_V$.

(b) Calculate the value of this difference for nitrogen at $V = 1.00$ l and a temperature equal to -100 °C (express it in terms of R).

2.171. Use the formula obtained in the preceding problem to calculate the value of $C_p - C_V$ for oxygen at $p = 5.00 \times 10^7$ Pa and a temperature of $T = 273$ K. In these conditions, a mole of oxygen occupies a volume of $V = 0.564 \times 10^{-4}$ m³.

2.172. When passing a stream of a gas through a porous partition installed in a thermally insulated tube, Joule and Thomson observed a change in the temperature of the gas due to its deviating from ideal properties (this phenomenon was named the "Joule-Thomson effect").

Assume that the state of the gas before the partition is characterized by the molar volume V_1 and temperature T_1. After the partition, the temperature of the gas is T_2. Considering the process to be adiabatic and applying the first law of thermodynamics to the portion of the gas passing through the partition, find the increment $\Delta T = T_2 - T_1$ of the temperature of the gas (express ΔT in terms of V_1 and T_1). Before expansion, assume that the gas is a van der Waals one, and after expansion, an ideal one.

2.173. What will happen to a gas because of the Joule-Thomson effect (see the preceding problem) if $T_1 > 2a/bR$?

2.174. Calculate the temperature increment ΔT of hydrogen due to the Joule-Thomson effect (see Problem 2.172) obtained if $p_1 = 10.0 \times 10^6$ Pa and T_1 is (a) 273 K;

(b) 210.5 K; (c) 173 K. The values of V_1 may be determined according to the equation of an ideal gas.

2.175. Calculate the temperature increment ΔT of nitrogen because of the Joule-Thomson effect (see Problem 2.172) obtained if $p_1 = 10.0 \times 10^5$ Pa and T_1 is (a) 0 °C; (b) 100 °C. The values of V_1 may be determined according to the equation of an ideal gas.

2.176. Find an expression for the entropy of a mole of a van der Waals gas (depending on T and V). Compare the for-

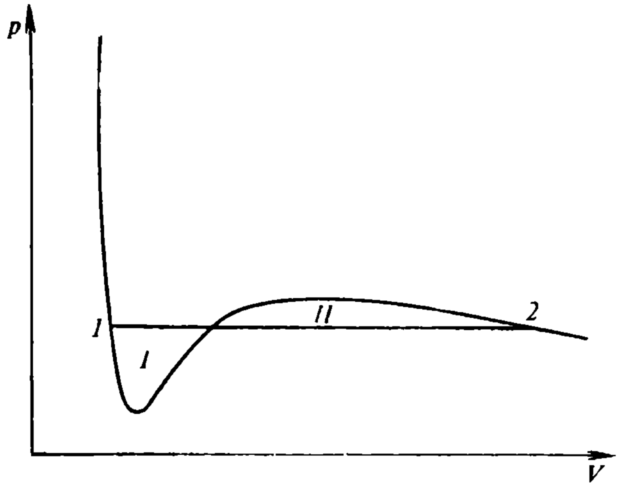

Fig. 2.24

mula obtained with a similar formula for an ideal gas.

2.177. A vessel of volume V is divided into two equal parts by a partition with an opening closed by a plug. One of the halves of the vessel contains a mole of a van der Waals gas (with known a, b, and C_V) having the temperature T. The plug is removed, and the gas occupies the entire volume. Considering the expansion process to be adiabatic, determine:

(a) the increment ΔU_m of the internal energy of the gas;
(b) the increment ΔT of the temperature of the gas;
(c) the work A_{mol} of the forces of intermolecular attraction;
(d) the increment ΔS_m of the entropy of the gas.

2.178. Prove that areas I and II confined between the sections of a van der Waals isotherm and the straight section of a real isotherm (Fig. 2.24) are identical in magnitude.

2.8. Liquids and Crystals

2.179. Can (a) a non-homogeneous body be isotropic; (b) a homogeneous body be anisotropic?

2.180. The salt NaCl has a cubic lattice. Its points accommodate alternating ions of sodium and chlorine. Determine the distance d between the crystal planes (i.e. the planes passing through the lattice points) parallel to the faces of an elementary crystal cell.

2.181. Using the Dulong and Petit law, determine the specific heat capacity c: (a) of copper; (b) of aluminium.

2.182. A spherical drop of mercury was divided into (a) 10; (b) 100; (c) 1000 identical drops. How did the capillary pressure inside the drops change?

2.183. A U-shaped vessel consists of a wide and narrow communicating tubes (Fig. 2.25). When water is poured into the vessel, a difference of $h = 80.0$ mm sets in between its levels in the narrow and wide tubes. The inner radius of the wide tube is $r_1 = 5.00$ mm. Considering wetting to be complete, find the radius r_2 of the narrow tube.

2.184. A glass rod with a diameter of $d_2 = 1.500$ mm is inserted coaxially into a glass tube with an internal diameter of $d_1 = 2.000$ mm. Considering wetting to be complete, determine the height h of capillary rise of water in the annular gap between the tube and the rod.

2.185. A drop of mercury with a volume of $V = 22.5\ \text{mm}^3$ is placed between two horizontal glass plates. With what force F must the plates be pressed together for a gap a equal to 3.00 μm to set in between them? Consider that the non-wetting of the plates by the mercury is complete.

2.186. There is an annular projection with a height of $h = 2.00$ μm along the edge of one of a pair of round glass plates. A water drop with a volume of $V = 15.0\ \text{mm}^3$ is placed between the plates, after which they are pressed together (Fig. 2.26). What force F must be applied to the plates to tear them apart? Consider wetting to be complete.

2.187. Two vertical and parallel glass plates are partly submerged in water. The gap between the plates is $a = 0.500$ mm, the horizontal dimension of the plates is $l = 10.0$ cm. Considering wetting to be complete, determine:
(a) the height h to which the water in the gap rises;
(b) the force F with which the plates attract each other.

2.188. Two glass plates are placed together so that the gap between them forms a vertical wedge with the very small angle φ at its apex. The plates are partly submerged into a liquid of density ρ. Considering wetting to be complete, determine the equation of the curve along which each of the plates intersects the surface of the liquid in the gap. Direct the y-axis upward along the edge of the wedge, and the x-axis horizontally along the inner surface of a plate, arranging this axis at the level of the liquid outside the wedge. The surface tension of the liquid is α.

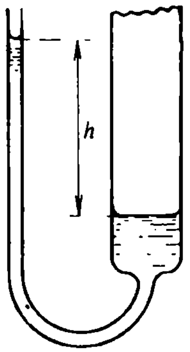

Fig. 2.25

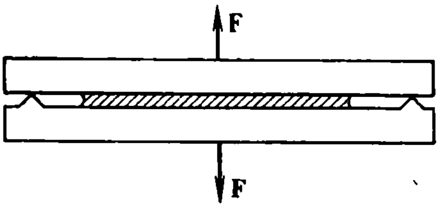

Fig. 2.26

2.189. After being coated with a layer of paraffin, the radius of the openings in a sieve becomes equal to $r = 1.50$ mm. Taking into account that water does not wet paraffin, determine the height h of the layer of water that can be carried in the sieve without the water flowing out through the openings.

2.190. Determine the depth h at which bubbles of gas form in water if upon rising of the bubbles to the surface their radius increases $\eta = 1.10$ times, reaching the value $r = 1.00$ μm at the surface. The atmospheric pressure is $p = 1.00 \times 10^5$ Pa. It is assumed that the temperature of the gas in a bubble does not change when the latter rises to the surface.

2.9. Phase Equilibria and Transitions

2.191. In what substances does the equilibrium transition from the solid phase to the gaseous one occur at atmospheric pressure without passing through the liquid phase?

2.192. Find the dependence of the saturated vapour pressure $p_{s.v}$ on the temperature T for the region of temperatures in which the specific volume of the liquid may be disregarded

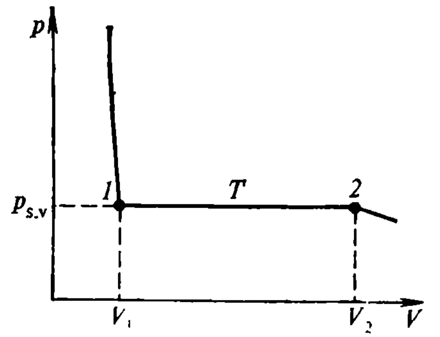

Fig. 2.27

in comparison with the specific volume of the saturated vapour. Consider that the heat of vaporization L does not depend on the temperature.

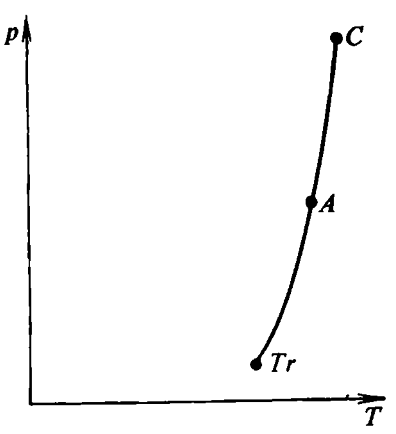

Fig. 2.28

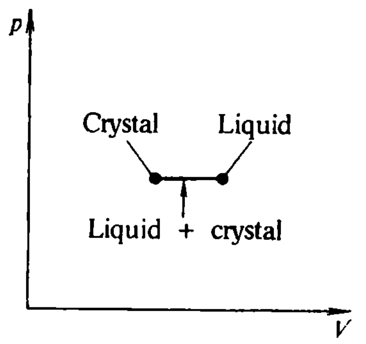

Fig. 2.29

2.193. Figure 2.27 depicts the isotherm of a substance. The horizontal section of the isotherm corresponds to the two-phase states "liquid + saturated vapour". The following quantities are known: the temperature T, the saturated vapour pressure $p_{s.v}$ at this temperature, the mass m of the substance, the heat of vaporization L_{12}, and the specific volumes of the liquid V_1' and of the saturated vapour V_2'. Find:

(a) the work A_{12} done by the substance when passing from state *1* to state *2*;
(b) the amount of heat Q_{12} obtained in this transition;
(c) the increment $U_2 - U_1$ of the internal energy;
(d) the increment $S_2 - S_1$ of the entropy;
(e) the increment $F_2 - F_1$ of the free energy.

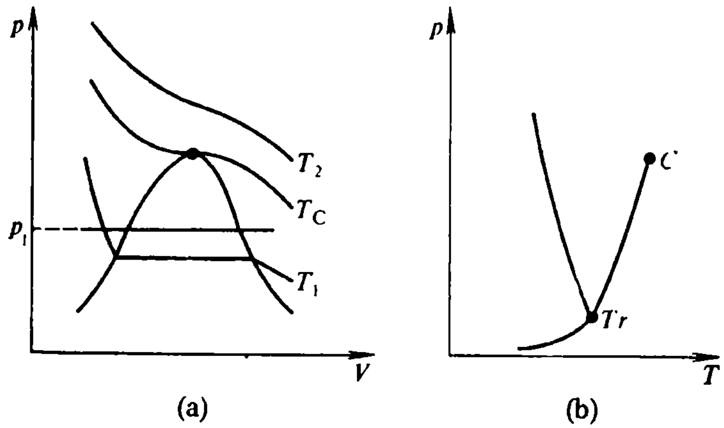

Fig. 2.30

2.194. Figure 2.28 shows point A on the saturated vapour pressure curve of a substance. What does this point correspond to in a p-V diagram?

2.195. Figure 2.29 shows a section of an isotherm corresponding to the transition of a substance from the crystalline to the liquid state. What corresponds to this section in a p-T diagram?

2.196. Figure 2.30*a* depicts three isotherms and an isobar.

(a) Depict these lines in the p-T diagram shown in Fig. 2.30*b*.

(b) What in the p-T diagram corresponds to the region under the bell-shaped curve in Fig. 2.30*a*?

2.10. Physical Kinetics

2.197. The gap between two very long coaxial cylindrical surfaces is filled with a homogeneous isotropic substance. The radii of the surfaces are $r_1 = 5.00$ cm and $r_2 = 7.00$ cm.

The inner surface is maintained at $T_1 = 290$ K, and the outer surface at $T_2 = 320$ K. Find for the middle part of the cylinders the dependence of the temperature T on the distance r to the axis.

2.198. The gap between two concentric spheres is filled with a homogeneous isotropic substance. The radii of the spheres are $r_1 = 10.0$ cm and $r_2 = 12.0$ cm. The surface of the inner sphere is kept at a temperature of $T_1 = 320$ K,

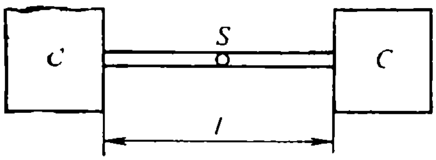

Fig. 2.31

and that of the outer sphere at a temperature of $T_2 = 300$ K. In these conditions, a steady heat flux of $dQ/dt = 2.00$ kW flows from the inner sphere to the outer one. Considering the thermal conductivity coefficient $\varkappa$ for the substance in the gap to be independent of the temperature, determine:

(a) the value of $\varkappa$;

(b) the temperature $T(r)$ in the gap as a function of the distance r from the centre of the spheres.

2.199. Two bodies each having a heat capacity of $C = 500$ J/K are joined together by a rod of length $l = 40.0$ cm having a cross-sectional area of $S = 3.00$ cm^2 (Fig. 2.31). The thermal conductivity coefficient of the rod is independent of the temperature and equals $\varkappa = 20.0$ W/(m·K). The bodies and the rod form a thermally insulated system. At the initial instant, the temperatures of the bodies differ from each other. Find the time τ after which the difference between the temperatures of the bodies diminishes $\eta = 2$ times. Disregard the heat capacity of the rod and the non-uniformity of the temperature within each of the bodies.

2.200. Oxygen is at a temperature of $T = 300$ K under a pressure of $p = 1.00 \times 10^5$ Pa. Determine:

(a) the mean free path l of the molecules;

(b) the mean free time τ of the molecules.

Compare l with the mean distance $\langle a \rangle$ between the molecules (see Problem 2.6).

2.201. Find the number ν of collisions per second be-

tween the molecules of nitrogen contained in 1 m^3 at $p = 1.00 \times 10^5$ Pa and $T = 300$ K.

2.202. The thermal conductivity coefficient of helium at $t = 0$ °C and $p = 1013$ hPa is $\varkappa = 0.143$ W/(m·K). Considering the numerical coefficient in the expressions for D, $\varkappa$, and η to be 1/3, appraise the coefficients of self-diffusion D and viscosity η of helium in the same conditions. Compare the values obtained with tabulated ones (obtained experimentally).

2.203. The coefficient of self-diffusion of oxygen at $t = 0$ °C and $p = 1013$ hPa is $D = 1.8 \times 10^{-5}$ m^2/s. Appraise the mean free path l of oxygen molecules in the same conditions. Compare l with the mean distance $\langle a \rangle$ between the molecules (see Problem 2.6).

2.204. Calculate the coefficient D_{12} of mutual diffusion of hydrogen and nitrogen at a temperature of $T = 300$ K and a pressure of $p = 1.00 \times 10^5$ Pa.

2.205. There is a gap of $a = 1.00$ cm between two very large parallel flat plates. A temperature difference of $\Delta T = 1.00$ K is maintained between the plates ($T_1 = 299.5$ K and $T_2 = 300.5$ K). The gap is filled with argon. Appraise the density of the heat flux dQ/dt if the pressure of the argon is: (a) 1.00×10^5 Pa; (b) 1.00×10^4 Pa; (c) 1.00×10^{-1} Pa; (d) 1.00×10^{-2} Pa.

Note. It must be had in view that the formula $\varkappa = \frac{1}{3} \rho \langle v \rangle l c_V$ gives only the order of magnitude of the thermal conductivity coefficient of gases. The numerical value of $\varkappa$ determined by this formula may differ several times from the experimental value.

2.206. Water with a mass of $m = 1.00$ kg is poured into a vacuum bottle. The area of the inner surface of the bottle's flask is $S = 700$ cm^2. The gap between the inner and outer vessels of the flask is $a = 5.00$ mm. The pressure of the gas in the gap is $p = 0.100$ Pa. Assuming that the withdrawal of heat from the contents of the bottle is accomplished only at the expense of the thermal conductivity of the gas in the gap, determine in approximately what time τ the temperature of the water in the bottle will diminish from 90 to 80 °C. The ambient temperature is 20 °C.

2.207. A horizontal disk of radius $R = 0.200$ m is suspended on a thin elastic filament over an identical disk secured

on a vertical axle. The torsion coefficient of the filament (the ratio of the applied torque to the angle of twist) is $\chi = 3.62 \times 10^{-4}$ N·m/rad. The gap between the disks is $a = 5.00$ mm. Through what angle α will the filament be twisted if the lower disk is brought into rotation at an angular speed of $\omega = 20.0$ rad/s?

2.208. One way of measuring the viscosity of gases consists in observing the rate of damping of the torsional oscillations of a horizontal disk suspended on a thin elastic filament over an identical stationary disk (Fig. 2.32). Obtain a formula relating the viscosity η of a gas between the disks to the mass m of a disk, the radius R of a disk, the size of the gap a, and the oscillation damping factor β. Consider that there is no friction in the suspension.

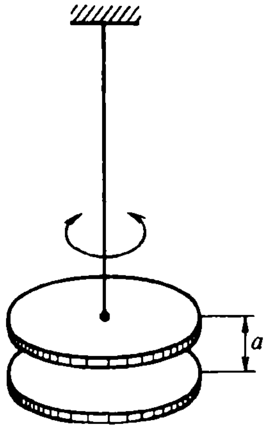

Fig. 2.32

2.209. A thin partition dividing a vessel into two parts has a round opening with a radius of $r = 1.00$ mm. The vessel contains helium under a pressure of $p = 1.00$ Pa. The walls of the vessel are kept at a temperature of $T = 300$ K. Determine the number ν of molecules flying through the opening in each direction in unit time.

2.210. A gas confined in a vessel of volume V flows out into a vacuum through an opening whose diameter is much smaller than the free path of the molecules. The area of the opening is S. The process is isothermal at a temperature of T. Find the time τ in which the pressure of the gas in the vessel diminishes η times. The molar mass of the gas is M.

2.211. Two vessels are separated by a thin spacer that does not conduct heat. The walls of vessel *1* are maintained at a temperature $T_1 = 300$ K, and those of vessel *2*—at $T_2 = 500$ K. The vessels communicate through an opening whose dimensions are 1/15 of the mean free path of the molecules of the gas filling the vessel. The pressure that sets in in vessel *1* is $p_1 = 0.100$ Pa. What is the pressure p_2 of the gas in vessel *2*?

PART 3

ELECTRICITY AND MAGNETISM

SYMBOLS

A	work
A_r	relative atomic mass
$\mathbf{a}$	acceleration
a	radius
$\mathbf{B}$	magnetic induction
b	distance; radius
C	capacitance
C_a	circulation of vector **a**
$\mathbf{D}$	electric displacement
d	diameter; distance; thickness
$\mathbf{E}$	electric field strength
$\mathscr{E}$	effective value of e.m.f.; electromotive force (e.m.f.)
e	elementary charge
$\mathbf{F}$	force
f	coefficient of friction
$\mathbf{H}$	magnetic field strength
I	current
$\mathbf{j}$	current density
k	torsion coefficient
$\mathbf{L}$	angular momentum
L	inductance
L_{12}	mutual inductance
l	distance; length
$\mathbf{M}$	magnetization
m	mass
n	number of revolutions per unit of time; number of turns per unit of length
$\mathbf{P}$	polarization
P	power
P_r	residual polarization
$\mathbf{p}$	electric dipole moment; momentum
$\mathbf{p}_m$	magnetic moment
Q	amount of heat; quality of circuit
q	electric charge
R	radius; resistance
$\mathbf{r}$	position vector
r	distance; radius
S	area; surface
s	distance
T	period of oscillations
t	time
U	potential difference; voltage
V	volume
W	energy
w	energy density
α	angle
δ	density (mass of unit volume); relative error
ε	relative permittivity
θ	polar angle
λ	linear charge density
μ	permeability
ν	frequency
ρ	resistivity; volume density of charge
σ	surface charge density
σ_u	ultimate tensile strength
τ	time
Φ_a	flux of vector **a**
φ	potential
ω	angular frequency; angular speed

3.1. Electric Field in a Vacuum

3.1. With what relative accuracy δ do charges of the order of 10^{-9} C have to be measured to detect the discrete nature of a charge?

3.2. A charge has the magnitude q in the reference frame K. What will the magnitude q' of this charge be in the frame K' moving at the speed v_0 relative to K?

3.3. What is the total charge q of a mole of electrons?

3.4. Find the total charge q of the atomic nuclei of copper contained in 1 cm^3.

3.5. Compare the force of Coulomb interaction F_e of two electrons with the force of their gravitational interaction F_g.

3.6. Calculate the acceleration a imparted by one electron to another one at a distance of $r = 1.00$ mm from the first one.

3.7. What mass m'_p should a proton have for the force of electrostatic repulsion of two protons to be balanced by the force of their gravitational attraction?

3.8. What charges q_S and q_E (proportional to the masses m_S and m_E) should be imparted to the Sun and the Earth for the force of Coulomb interaction between them to be equal to the force of gravitational interaction?

3.9. At what specific charge q/m identical for the Sun and the Earth will the force of Coulomb interaction between them be equal to the force of gravitational interaction? Compare the obtained value of q/m with the specific charge e/m_e of an electron.

3.10. Two systems of point charges $q_1, q_2, \ldots, q_i, \ldots, q_{N_1}$ and $q'_1, q'_2, \ldots, q'_k, \ldots, q'_{N_2}$ are fixed at points with position vectors $\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_i, \ldots, \mathbf{r}_{N_1}$ and $\mathbf{r}'_1, \mathbf{r}'_2, \ldots, \mathbf{r}'_k, \ldots, \mathbf{r}'_{N_2}$. Find the force $\mathbf{F}$ with which the system of charges q'_k acts on the system of charges q_i.

3.11. The charge q is distributed with a density of $\rho = \rho(\mathbf{r})$ over a body of volume V; another charge q' is distributed with a density of $\rho = \rho(\mathbf{r}')$ over a body of volume V'. Write an expression for the force $\mathbf{F}$ with which the charge q' acts on the charge q.

3.12. Point charges of the same magnitude q are placed at the vertices of a regular hexagon with a side of a. Find the potential φ and the field strength $\mathbf{E}$ at the centre of the

hexagon provided that: (a) the sign of all the charges is the same; (b) the signs of adjacent charges are opposite.

3.13. N point charges $q_1, q_2, \ldots, q_i, \ldots, q_N$ are in a vacuum at points with the position vectors $\mathbf{r}_1, \mathbf{r}_2, \ldots$ $\ldots, \mathbf{r}_i, \ldots, \mathbf{r}_N$. Write expressions for the potential φ and the field strength $\mathbf{E}$ at a point determined by the position vector $\mathbf{r}$.

3.14. A charge is distributed with the density $\rho = \rho(\mathbf{r})$ over the region V. Write expressions for the potential φ and the field strength $\mathbf{E}$ at a point determined by the position vector $\mathbf{r}'$.

3.15. Find the potential φ and the field strength $\mathbf{E}$ at the centre of a sphere of radius R charged uniformly with the surface density σ.

3.16. A charge of $q = 2.00$ μC is distributed uniformly over the volume of a sphere of radius $R = 40.0$ mm. Find the potential φ and the field strength $\mathbf{E}$ at the centre of the sphere.

3.17. Find the potential φ and the magnitude E of the field strength at the centre of a hemisphere of radius R charged uniformly with the surface density σ.

3.18. What are the equipotential surfaces of a uniform electric field?

3.19. The strength of a field has the form $\mathbf{E} = E\mathbf{e}_x$, where E is a constant. Write an expression for the field potential φ.

3.20. An electrostatic field has the form $\mathbf{E} = E_1\mathbf{e}_x + E_2\mathbf{e}_y + E_3\mathbf{e}_z$, where E_1, E_2, and E_3 are constants.

(a) Is this field uniform?

(b) Write an expression for φ.

3.21. The strength of an electrostatic field is determined by the expression $\mathbf{E} = (a/r^{3/2})\,\mathbf{e}_r$, where a is a constant.

(a) Is this field uniform?

(b) Find the potential $\varphi(r)$ of this field.

3.22. The potential of an electrostatic field has the form $\varphi = \varphi(x^2 + y^2 + z^2)$.

(a) What can be said about the nature of the field?

(b) Find the magnitude E of the field strength at the point x, y, z.

3.23. The potential of an electrostatic field has the form $\varphi = \varphi(r, \theta)$, where r is the distance from the origin of coordinates, and θ is the polar angle.

(a) What can be said about the nature of the field?
(b) Find the magnitude E of the field strength at the point r, θ.

3.24. The potential of the field produced by a system of charges has the form $\varphi = a(x^2 + y^2) + bz^2$, where a and b are positive constants.
(a) Find the field strength **E** and its magnitude E.
(b) What shape do the equipotential surfaces have?
(c) What shape do surfaces have for which $E = \text{const}$?

3.25. The potential of the field produced by a system of charges has the form $\varphi = a(x^2 + y^2) - bz^2$, where a and b are positive constants.
Answer the same questions as in the preceding problem.

3.26. Find the potential φ and the magnitude E of the field strength of a dipole as a function of r and θ (r is the distance from the centre of the dipole, and θ is the angle between the axis of the dipole and the direction from its centre to the given point). The dipole moment is p.

3.27. What property does the electric dipole moment **p** of a neutral system of charges have?

3.28. What work A must be done to turn a dipole with the moment **p** from a position along the field **E** to one against the field?

3.29. What does the electric dipole moment **p** equal: (a) for a quadrupole; (b) for an octupole?

3.30. Find the force F of interaction of two water molecules at a distance of $l = 1.00 \times 10^{-8}$ m (100 Å) apart. The electric dipole moment of a water molecule is $p = 0.62 \times 10^{-29}$ C·m. Consider the dipole moments of the molecules to be arranged along the straight line joining the molecules.

3.31. A very thin straight rod of length $2a$ in a vacuum is charged with the same linear density λ everywhere. For points on the straight line perpendicular to the axis of the rod and passing through its centre, find the magnitude E of the field strength as a function of the distance r from the centre of the rod.

3.32. For the rod of the preceding problem, find the potential φ and the magnitude E of the field strength at points on the axis of the rod outside it as a function of the distance r from the centre of the rod. Investigate the case $r \gg a$.

3.33. Using the answer to Problem 3.31, obtain an expression for the magnitude $E(r)$ of the field strength of an endless straight filament charged uniformly with a linear density of λ (r is the distance from the axis of the filament).

3.34. A charge of $q = 20.0$ nC is uniformly distributed over a thin wire ring of radius $r = 60.0$ mm.

(a) Adopting the axis of the ring as the x-axis, find the potential φ and the field strength $\mathbf{E}$ on the axis of the ring as a function of x (place the zero of the x-axis at the centre of the ring).

(b) Investigate the cases $x = 0$ and $|x| \gg r$.

(c) Determine the maximum value of the field strength magnitude E_m and the coordinates x_m of the points at which it is observed.

(d) Plot approximate graphs of the functions $\varphi(x)$ and $E_x(x)$. Establish what the points x_m are for the curve $\varphi(x)$.

Calculate the field strength in two ways: (1) proceeding from the expression for the field strength of a point charge and the principle of field superposition; (2) proceeding from the expression for the potential. Compare the two ways of calculations.

3.35. A charge of $q = 1.00$ μC is uniformly distributed over a very thin round plate of radius $r = 0.100$ m. Adopting the axis of the plate as the x-axis,

(a) find φ and E_x for points on the axis as a function of x; investigate the expressions obtained for the case when $|x| \gg r$;

(b) calculate φ and E_x for the point $x = 100$ mm.

3.36. A very thin plate has the shape of a ring with an inner radius a and an outer radius b. The charge q is uniformly distributed over the plate. Adopting the axis of the plate as the x-axis, find φ and E_x on the axis of the plate as a function of x. Investigate the case $|x| \gg b$.

3.37. Using the result of Problem 3.35, obtain an expression for E_x of the field of an infinite plane charged uniformly with the density σ (the x-axis is perpendicular to the plane).

3.38. In the preceding problem, we obtained the expression $E = \sigma/2\varepsilon_0$ for the strength of the field produced by an infinite uniformly charged plane. Let us take point P at a distance of b from the plane (Fig. 3.1). We draw a circle of radius a around the base of a perpendicular erected from the plane to point P. Find the value of a at which the field

strength produced at P by the charges inside the circle forms half of the total strength. Also determine r and θ corresponding to this value of a.

3.39. A parallel-plate capacitor has round plates of radius r with a separation distance of $2a$ ($a \ll r$). Opposite charges identical in magnitude are imparted to the plates. Let us denote the axis passing through the centres of the plates by the letter x and place the origin of coordinates at the centre of the capacitor. Assuming that the charges are distributed over the plates uniformly with a density of $+\sigma$ and $-\sigma$, investigate the field strength $\mathbf{E}$ at points on the x-axis. For this purpose find:

(a) E_x as a function of x;

(b) $E_x(0)$, i.e. E_x at the centre of the capacitor;

(c) $E_x(a-0)$, i.e. E_x at a point with the coordinate $x = a - \delta$ ($\delta \to 0$);

(d) $E_x(a+0)$, i.e. E_x at a point with the coordinate $x = a + \delta$ ($\delta \to 0$);

(e) E_x as a function of x at points for which $|x| \gg r$.

Disregard the thickness of the plates.

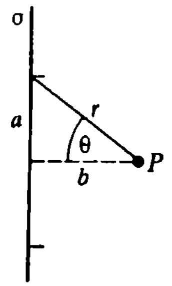

Fig. 3.1

3.40. Proceeding from the definition of the divergence of the vector $\mathbf{a}$ as the limit of the ratio of the flux Φ_a through a closed surface to the volume V confined by this surface: $\nabla\mathbf{a} = \lim\limits_{V\to 0} (\Phi_a/V)$, determine the divergence of the following vector fields:

(a) $\mathbf{a} = f(x)\,\mathbf{e}_x$, where $f(x)$ is a function of the Cartesian coordinate x;

(b) $\mathbf{a} = \mathbf{r}$, where $\mathbf{r}$ is the position vector of the point for which the divergence is being determined;

(c) $\mathbf{a} = \mathbf{e}_r$, where $\mathbf{e}_r$ is the unit vector of the position vector of the point for which the divergence is being found;

(d) $\mathbf{a} = f(r)\,\mathbf{e}_r$, where $f(r)$ is a function of the magnitude of the position vector.

3.41. We have a uniform field of a vector $\mathbf{a}$. Determine:

(a) the divergence of this field $\nabla\mathbf{a}$;

(b) the flux of the vector $\mathbf{a}$ through an arbitrary closed surface Φ_a.

3.42. Taking advantage of the fact that a uniform vector field has no sources, prove that for an arbitrary closed surface $\oint d\mathbf{S} = 0$.

3.43. Calculate the flux Φ_r of the position vector $\mathbf{r}$ through a sphere of radius R with its centre at the origin of coordinates.

3.44. We know the function $f(r)$ determining the divergence of the vector field of $\mathbf{a}$: $\nabla\mathbf{a} = f(r)$. Write an expression for the flux Φ_a of the vector $\mathbf{a}$ through a sphere of radius R with its centre at the origin of coordinates.

3.45. In the region of the vector field of $\mathbf{a}$, there is an imaginary closed surface S inside of which everywhere $\nabla\mathbf{a} = 0$. Let us arbitrarily divide S into two parts S_1 and S_2. What is the relation between the fluxes Φ_1 and Φ_2 of the vector $\mathbf{a}$ through S_1 and S_2?

3.46. What does $\nabla\mathbf{E}$ for a uniform field equal?

3.47. Find the dependence of the charge density ρ on the Cartesian coordinates x, y, z at which the field strength would be described by the function $\mathbf{E} = 1x\mathbf{e}_x + 2y^2\mathbf{e}_y + 3z^3\mathbf{e}_z$.

3.48. Find the dependence of the charge density ρ on the magnitude r of the position vector at which the field strength would be described by the function $\mathbf{E} = A\exp(-\alpha r)\,\mathbf{e}_r$, where A and α are constants.

3.49. Proceeding from the definition of the projection of the curl of the vector $\mathbf{a}$ onto the direction $\mathbf{n}$ as the limit of the ratio of the circulation C_a over the contour in a plane perpendicular to the direction $\mathbf{n}$ to the surface S confined by the contour: $[\nabla\mathbf{a}]_{\text{pr }\mathbf{n}} = \lim\limits_{S\to 0} (C_a/S)$, determine the curl of the following vector fields:

(a) $\mathbf{a} = f(x)\,\mathbf{e}_x$, where $f(x)$ is a function of the Cartesian coordinate x;

(b) $\mathbf{a} = \mathbf{r}$, where $\mathbf{r}$ is the position vector of the point where the curl is being determined;

(c) $\mathbf{a} = \mathbf{e}_r$, where $\mathbf{e}_r$ is the unit vector of the position vector of the point at which the curl is being determined;

(d) $\mathbf{a} = f(r)\,\mathbf{e}_r$, where $f(r)$ is a function of the magnitude of the position vector.

3.50. Taking advantage of the fact that $\oint d\mathbf{l}$ taken over

any closed contour is zero, prove that a uniform vector field is a non-vortex one.

3.51. Can an electrostatic field have the form $\mathbf{E} = a\,(y\mathbf{e}_x - x\mathbf{e}_y)$?

3.52. For the field $\mathbf{E} = -a\,(y\mathbf{e}_x - x\mathbf{e}_y)$, calculate:

(a) the curl at a point with the coordinates (x, y, z);

(b) the circulation C over a circle of radius b in the plane x, y (with its centre at an arbitrary point); the direction of circumvention forms a right-handed system with the z-axis.

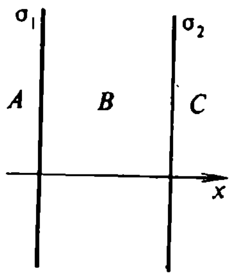

Fig. 3.2

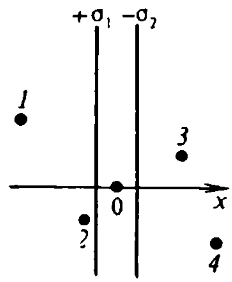

Fig. 3.3

3.53. An infinite plane is charged uniformly with the density σ. The x-axis is perpendicular to the plane; the zero point of the x-axis is at the point of intersection of the axis with the plane.

(a) Using Gauss's theorem, find an expression for E_x at a point with the coordinate x. Compare the result obtained with the answer to Problem 3.37.

(b) Find how φ depends on x.

(c) Is it possible to normalize the expression for φ so that it becomes equal to zero at infinity?

3.54. Can a field outside uniformly and oppositely charged parallel infinite planes be other than zero?

3.55. Two parallel infinite planes are charged: one with a density of $\sigma_1 = +4.42 \times 10^{-10}$ C/m^2, and the other with a density of $\sigma_2 = -8.84 \times 10^{-10}$ C/m^2 (Fig. 3.2). Find the field strength $\mathbf{E}$ for each of the regions A, B, and C.

3.56. Two parallel infinite planes are charged oppositely with the densities $+\sigma_1$ and $-\sigma_2$ differing in magnitude. The abscissas of the points indicated in Fig. 3.3 are: $x_1 = -3.00$ m, $x_2 = -1.00$ m, $x_3 = +2.00$ m, and

$x_4 = +3.00$ m. The potential difference between points *2* and *1* is $\varphi_2 - \varphi_1 = 400$ V.

(a) Which of the densities ($+\sigma_1$ or $-\sigma_2$) is larger in magnitude?

(b) What is the potential difference $\varphi_4 - \varphi_3$?

3.57. We have an infinite very thin straight filament charged uniformly with the linear density λ. Using Gauss's theorem, find the magnitude of the field strength E as a function of the distance r from the filament. Compare the result obtained with the answer to Problem 3.33.

3.58. An infinite thin straight filament is charged uniformly with the density $\lambda = 2.00$ μC/m.

(a) Find E and φ as a function of the distance r from the filament. Let the potential at the distance $r_0 = 1$ m be zero.

(b) Calculate E and φ for $r = 10.0$ m.

(c) Can the potential be normalized so that it vanishes at infinity?

3.59. The electrodes of a two-electrode valve (diode) have the form of a filament of radius $a = 0.100$ mm (the cathode) and a cylinder coaxial to it of radius $b = 2.72$ mm (the anode). A voltage of $U = 100$ V is fed to the electrodes. Find the magnitude of the force F_e that will be experienced by an electron and the force F_m that will be experienced by a water molecule at a point that is at the distance $r = 1.00$ mm from the axis of the cathode. The dipole moment of a water molecule is $p = 0.62 \times 10^{-29}$ C·m.

3.60. Two infinitely long parallel filaments charged with a like linear density of $\lambda = 3.00$ μC/m are at a distance of $b = 20.0$ mm apart. With what force F (per unit length) will the filaments repel each other? What work A (per unit length) has to be done to make the filaments approach each other to a distance of $a = 10.0$ mm?

3.61. We have a sphere of radius R charged uniformly with the surface density σ.

(a) Find the field strength **E** at a point at a distance of r from the centre of the sphere ($r < R$).

(b) What conclusion follows from the answer to (a)?

(c) What is the potential φ inside the sphere?

3.62. What force **F** acts on an electron in the space formed by a charged spherical layer if the volume density ρ of the charge in the layer depends only on the distance r to its centre?

3.63. A sphere of radius R is charged uniformly with the volume density ρ. Find the field strength $\mathbf{E}$ and the potential φ for points inside the sphere.

3.64. Inside a sphere charged uniformly with the volume density ρ there is a spherical space in which charges are absent. The displacement of the centre of the space relative to the centre of the sphere is determined by the vector $\mathbf{a}$. Find the field strength $\mathbf{E}$ inside the space. Consider the case when $\mathbf{a} = 0$.

3.65. A space is filled with a charge whose density varies according to the law $\rho = \rho_0/r$, where ρ_0 is a constant, and r is the distance from the origin of coordinates. Find the field strength $\mathbf{E}$ as a function of the position vector $\mathbf{r}$. Investigate the nature of the field strength lines. Do not consider the region near the origin of coordinates.

3.66. A space is filled with a charge of density $\rho = \rho_0 \exp(-\alpha r^3)$, where ρ_0 and α are constants. Find $\mathbf{E}$ as a function of $\mathbf{r}$. Investigate the nature of the field for large and small r's (consider that the values of r are large when $\alpha r^3 \gg 1$, and small when $\alpha r^3 \ll 1$).

3.2. Electric Field in Dielectrics

3.67. A dielectric body is charged uniformly with a volume density of $\rho_0 = 1.00\ \mu\text{C/m}^3$. What will the volume density ρ of the charge be if the body is brought into motion at a speed of $v = 0.500c$?

3.68. A cubic dielectric body is charged uniformly with a surface density of $\sigma_0 = 1.00\ \mu\text{C/m}^2$. What will the surface density σ of the charge be if the body is brought into motion in the direction of one of its edges at a speed of $v = 0.500c$?

3.69. A thin dielectric rod is charged uniformly with a linear density of $\lambda_0 = 1.00\ \mu\text{C/m}$. What will the linear density λ of the charge be if the rod is brought into motion at a speed of $v = 0.500c$ in a direction making the angle $\alpha = 30°$ with the initial direction of the rod axis?

3.70. At a certain point of an isotropic dielectric with the permittivity ε, the displacement is $\mathbf{D}$. What is the polarization $\mathbf{P}$ at this point?

3.71. We have two infinite parallel planes charged with the densities $+\sigma$ and $-\sigma$. They were initially in a vacuum.

Next the gap between the planes is filled with a homogeneous isotropic dielectric with the permittivity ε. What will happen: (a) to the strength **E** of the field in the gap; (b) to the displacement **D**; (c) to the potential difference U between the planes?

3.72. An infinite plane-parallel plate of a homogeneous and isotropic dielectric with the permittivity $\varepsilon = 2.00$ is placed in a uniform electric field with the strength $E_0 = 100.0$ V/m. The plate is arranged at right angles to $\mathbf{E}_0$. Determine:

(a) the field strength E and the electric displacement D inside the plate;

(b) the polarization P of the dielectric;

(c) the surface density σ' of the bound charges.

3.73. An infinite plate of thickness a consisting of an isotropic dielectric is polarized so that the polarization near one boundary of the plate is $\mathbf{P}_1 = P_1\mathbf{n}$, and near the other boundary is $\mathbf{P}_2 = P_2\mathbf{n}$, where $\mathbf{n}$ is a unit vector perpendicular to the plate and directed from the first boundary to the second one. Find the mean volume density $\langle \rho' \rangle$ of the bound charges over the volume of the plate.

3.74. An infinite isotropic dielectric plate is placed in a uniform external electric field with the strength $\mathbf{E}_0$ perpendicular to it (Fig. 3.4). The thickness of the plate is a. The permittivity of the plate varies linearly from the value ε_1 at the left-hand boundary to ε_2 at the right-hand one. Outside the plate, $\varepsilon = 1$. Find:

(a) $\nabla \mathbf{E}$ inside the plate as a function of x;

(b) the flux Φ_E of the vector **E** through an imaginary cylindrical surface with generatrices parallel to the x-axis; the bases of the cylinder are at points with $x_1 = -a/2$ and $x_2 = +a/2$; the area of each base is S;

(c) the volume density ρ' of the bound charges as a function of x.

3.75. Find ρ' at the middle of the plate in the preceding problem if $\varepsilon_1 = 2.00$, $\varepsilon_2 = 4.00$, $a = 1.00$ cm, and $E_0 = 3.00$ kV/m.

3.76. An infinite dielectric plate of thickness a (Fig. 3.5) is placed in an external uniform electric field of strength $\mathbf{E}_0$ perpendicular to the plate. The permittivity of the plate changes according to a certain law $\varepsilon(x)$ $[\varepsilon(0) = \varepsilon_1]$. What form must the function $\varepsilon(x)$ have for the density of the

bound charges to vary according to the law $\rho' = \rho_1'/(1 + \alpha x)$, where ρ_1' and α are constants? Outside the plate, $\varepsilon = 1$.

3.77. A glass plate with the permittivity $\varepsilon_2 = 6.00$ is introduced into a uniform electric field with the strength $E_1 = 10.0$ V/m and is arranged so that the angle α_1 between a normal to the plate and the direction of the external field is 30°. Find the strength E_2 of the field in the plate, the angle α_2 made by this field with a normal to the plate, and also the density σ' of the bound charges that appeared on

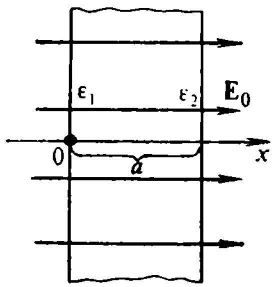

Fig. 3.4

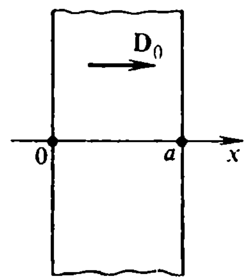

Fig. 3.5

the surfaces of the plate. Assume the permittivity ε_1 o the medium outside the plate to be unity.

3.78. A dielectric plate is introduced into the gap between two planes charged oppositely (Fig. 3.6). The plate carries no extraneous charges. The dashed line in the figure shows an imaginary closed surface partly inside the dielectric and partly outside it. What is the flux of the vector **D** through this surface?

3.79. An imaginary closed surface S is partly outside a plate made from an isotropic dielectric, and partly inside it (Fig. 3.7). The flux of the vector **D** through this surface is zero, and the flux of the vector **E** is greater than zero. What conclusions can be made from this?

3.80. An infinite plate made from a dielectric with the permittivity ε is charged uniformly with the volume density ρ. The thickness of the plate is $2a$. Outside the plate, $\varepsilon = 1$. Let us direct the x-axis at right angles to the plate and place the origin of coordinates at the middle of the plate. Find φ and E_x inside and outside the plate as a function of x (assume that the potential at the middle of the plate is zero). Plot graphs of φ and E_x.

3.81. For the plate from the preceding problem, find:
(a) the polarization $\mathbf{P}$ of the dielectric as a function of x;
(b) the surface density σ' of the bound charges on the left-hand ($x = -a$) and right-hand ($x = +a$) boundaries of the plate;
(c) the volume density ρ' of the bound charges.

3.82. The plate of Problem 3.80 is charged with a density of $\rho = \rho_0 \exp(-\alpha |x|)$, where ρ_0 and α are constants. Find:
(a) the projection of the field strength onto the x-axis;

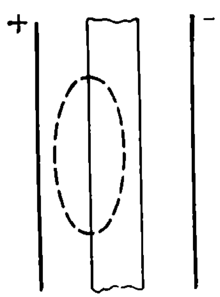

Fig. 3.6

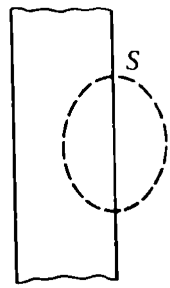

Fig. 3.7

(b) the volume density of the bound charges as a function of x.

3.83. The polarization $\mathbf{P}$ of a medium is proportional to the expression $\mathbf{e}_r/r^2$, where $\mathbf{e}_r$ is the unit vector, and r is the magnitude of the position vector $\mathbf{r}$. What is the volume density ρ' of the bound charges?

3.84. A uniform electric field having the strength $E = 100$ V/m has been produced inside a sphere made from a homogeneous isotropic dielectric with $\varepsilon = 5.00$. Find the maximum surface density $\sigma'_{\max}$ of the bound charges and the mean value of σ' of one sign.

3.85. A ferroelectric rod having a residual polarization P_r directed along the rod's axis is suspended by its middle in a horizontal position on a thin inelastic thread. Determine the frequency ω of the small-amplitude oscillations the rod will perform in a uniform horizontally directed field E which is so weak that it has no appreciable influence on the polarization of the rod. The length of the rod is l, its density is δ.

3.3. Conductors in an Electric Field

3.86. A point charge of $q = 20.0$ nC is in a vacuum at a distance of $a = 50.0$ mm from an earthed flat metal wall. Find the force F with which the wall attracts the charge.

3.87. A point charge q is near an earthed flat metal wall at a distance of a from it. Determine the surface density σ of the charges induced on the wall as a function of the distance x from the base of a perpendicular from the charge

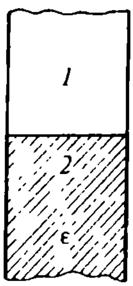

Fig. 3.8

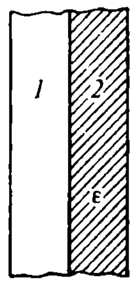

Fig. 3.9

to the wall. Calculate the total induced charge q_{ind}, assuming the wall to have infinitely great dimensions.

3.88. A metal sphere with a radius of $r = 1$ cm is charged to a potential of $\varphi = 1$ V. In what digit will the charge q of the sphere change if 100 electrons fly out of the sphere?

3.89. There is initially a vacuum in the space between the plates of a parallel-plate capacitor. In this case, the field strength in the gap is E, and the electric displacement is D. Next half of the gap is filled as shown in Fig. 3.8 with a homogeneous isotropic dielectric having the permittivity ε. Find the resulting values of E_1 and D_1 in part *1* of the gap, and also the values of E_2 and D_2 in part *2* of the gap. Consider two cases:

(a) the voltage between the plates remains constant;

(b) the charges on the plates remain constant.

Depict the approximate course of the lines of **E** and **D** in the gap.

3.90. Solve a problem similar to the preceding one, with the only difference that the dielectric fills half of the gap as shown in Fig. 3.9.

3.91. The area of each plate of a parallel-plate capacitor is $S = 1.00\ m^2$, and the separation distance is $d = 5.00$ mm. The gap between the plates is filled with a two-layer dielectric. The permittivity and thickness of the first layer are $\varepsilon_1 = 2.00$, $d_1 = 3.00$ mm, and of the second layer are $\varepsilon_2 = 3.00$, $d_2 = 2.00$ mm. Find the capacitance C of the capacitor.

3.92. The area of each plate of a parallel-plate capacitor is $S = 1.00\ m^2$, the separation distance is $d = 5.00$ mm. The gap between the plates is filled with a dielectric whose permittivity changes in a direction perpendicular to the plates according to a linear law from the value of $\varepsilon_1 = 2.00$ near one plate to $\varepsilon_2 = 5.44$ near the other one. Determine the capacitance C of the capacitor.

3.93. Disregarding the dissipation of the field near the edges of the plates, obtain an expression for the capacitance C of a cylindrical capacitor. The radii of the plates are r_1 and r_2 $(r_1 < r_2)$, and their length is l. The gap between the plates is filled with a dielectric having the permittivity ε.

3.94. A gas-discharge counter of elementary particles consists of a tube of radius $r_2 = 10.0$ mm and a filament of radius $r_1 = 50.0$ μm stretched taut along the axis of the tube. The length of the counter is $l = 150$ mm. Assuming that $\varepsilon = 1$, appraise the interelectrode capacitance C.

3.95. Obtain an expression for the capacitance C of a spherical capacitor. The radii of the plates are r_1 and r_2 $(r_1 < r_2)$. The gap between the plates is filled with a dielectric having the permittivity ε.

3.96. The radii of the plates of a spherical capacitor are $r_1 = 9.00$ cm and $r_2 = 11.00$ cm. The gap between the plates is filled with a dielectric whose permittivity varies with the distance r from the centre of the capacitor according to the law $\varepsilon = 2.00\ (r_1/r)$. Find the capacitance C of the capacitor.

3.97. We have N capacitors whose capacitances are $C_1, C_2, \ldots, C_N$. Obtain an expression for the capacitance C of a system of capacitors with (a) parallel, and (b) series connection to one another.

3.98. Ten identical capacitors each with a capacitance of 100 pF are connected in series. What is the capacitance C of this system?

3.99. How must we connect the capacitors $C_1 = 2$ pF,

$C_2 = 4$ pF, and $C_3 = 6$ pF to obtain a system with a capacitance of $C = 3$ pF?

3.100. A constant voltage of $U = 300$ V is applied across two capacitors $C_1 = 100$ pF and $C_2 = 200$ pF connected in series. Determine the voltages U_1 and U_2 across the capacitors and the charge q on their plates. What is the capacitance C of the system?

3.101. In the diagram shown in Fig. 3.10, $\mathscr{E} = 100$ V, $C_1 = 1.00$ μF, $C_2 = 2.00$ μF, $C_3 = 3.00$ μF. First switch Sw_1 is closed. Next it is opened, and switch Sw_2 is closed.

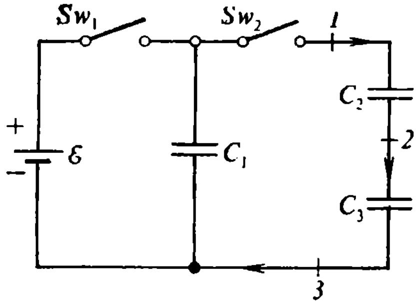

Fig. 3.10

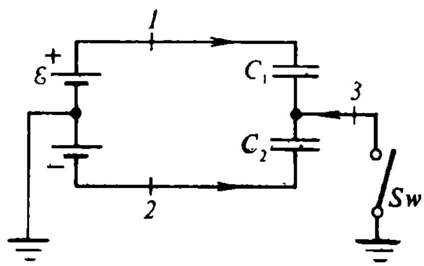

Fig. 3.11

What charges q_1, q_2, and q_3 will flow through sections *1*, *2*, and *3* in the directions indicated by the arrows?

3.102. The capacitors $C_1 = 2.00$ μF and $C_2 = 3.00$ μF are connected in series to a battery with $\mathscr{E} = 120$ V whose middle point is earthed (Fig. 3.11). The wire connecting the capacitors can be earthed with the aid of switch Sw. Determine the charges q_1, q_2, and q_3 that will pass after the switch is closed through sections *1*, *2*, and *3* in the directions indicated in the figure.

3.103. Two long wires with a radius of $a = 0.50$ mm are arranged in the air parallel to each other. The distance between their axes is $b = 10.0$ mm. Find the mutual capacitance C_1 of the wires per unit of their length.

3.104. Find the capacitance C of the capacitor formed by two identical spheres of radius a in a medium with the permittivity ε. The distance between the centres of the spheres is b $(b \gg a)$. Calculate C for $a = 10.0$ mm and $\varepsilon = 1.00$.

3.4. Energy of an Electric Field

3.105. Calculate the energy W of Coulomb interaction of two electrons at a distance of $r = 1.00$ mm from each other.

3.106. The mean distance of the electron from the nucleus in a hydrogen atom is $\langle r \rangle = 0.79 \times 10^{-10}$ m. Appraise:

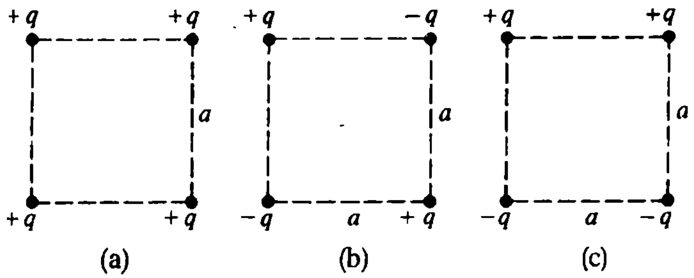

Fig. 3.12

(a) the energy W of Coulomb interaction of the electron and the nucleus;

(b) the sum of the energies W for a mole of atomic hydrogen.

3.107. Find the mutual potential energy W for each of the systems of point charges shown in Fig. 3.12. All the charges are identical in magnitude and are at the corners of a square with a side of a.

3.108. Find the mutual potential energy W of a system of N point charges $q_1, q_2, \ldots, q_i, \ldots, q_N$ arranged in a vacuum at points with the position vectors $\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_i, \ldots, \mathbf{r}_N$.

3.109. A charge is distributed with a volume density of $\rho = \rho(\mathbf{r})$ over a body with a volume of V. Find an expression for the energy W of this body, assuming ε inside and outside the body to be unity.

3.110. A charge of $q = 1.00 \times 10^{-10}$ C is uniformly distributed over the surface of a sphere of radius $r = 1.00$ cm. The permittivity of the medium surrounding the sphere is unity.

(a) Evaluate the energy W of the field associated with the sphere.

(b) What part η of this energy is confined within an imaginary sphere of radius $R = 1.00$ m concentric to the charged sphere?

(c) What is the radius R of a sphere within which half of the energy is confined?

3.111. A charge of $q = 1.00 \times 10^{-10}$ C is uniformly distributed over the volume of a sphere of radius $r = 1.00$ cm. Determine:

(a) the energy W of the field associated with the sphere;

(b) the energy W_1 confined within the sphere;

(c) the energy W_2 contained in the space surrounding the sphere.

Assume that the permittivity inside and outside the sphere is unity.

3.112. A charge of $q = 1.00 \times 10^{-10}$ C is first distributed uniformly over the volume of a sphere of radius $r = 1.00$ cm. Next owing to mutual repulsion, the charges pass to the surface of the sphere. What work A do the electric forces do on the charges ($\varepsilon = 1$)?

3.113. Find what we call the classical radius r_{cl} of an electron on the basis of the following reasoning. In classical physics, an electron is considered as a charged sphere whose rest energy is identified with the energy of the electrostatic field associated with it. To make no assumptions on the nature of charge distribution over the volume of the sphere, instead of the numerical factor 1/2 (corresponding to surface distribution of the charge; see the answer to Problem 3.110) or 3/5 (corresponding to uniform distribution of the charge over the volume; see the answer to Problem 3.111), a factor of one is taken in the expression for the energy of the field.

3.114. A point charge $q = 3.00$ μC is placed at the centre of a spherical layer of a homogeneous and isotropic dielectric with $\varepsilon = 3.00$. The inner radius of the layer is $a = 250$ mm and the outer one is $b = 500$ mm. Find the energy W confined within the limits of the dielectric.

3.115. The outer plate of a spherical capacitor can contract, retaining a strictly spherical shape and remaining concentric to the inner rigid plate.

(a) After identical charges $q = 2.00$ μC having opposite signs were imparted to the plates, the outer plate is compressed under the action of electrical forces, as a result of which its radius diminishes from $a = 100.0$ mm to $b = 95.0$ mm.

Find the work A done by the electrical forces. Consider that the permittivity of the medium between the plates is unity.

(b) Why is an incorrect result obtained when the work is calculated by the formula $\int_a^b \frac{-q^2}{4\pi\varepsilon_0}\frac{dr}{r^2}$?

3.116. Determine the work A that must be done to increase by $\Delta x = 0.200$ mm the separation distance x of a parallel-plate capacitor charged with opposite charges having the magnitude $q = 0.200$ μC. The area of each plate is $S = 400$ cm^2. The gap between the plates contains air.

3.117. A charged parallel-plate capacitor was originally in a vacuum. The gap between its plates was then filled with a dielectric having the permittivity ε. What happens to the energy density w of the field in the gap if the capacitor is (a) connected to a voltage source; (b) disconnected from a voltage source?

3.5. Electric Current

3.118. We have N resistors $R_1, R_2, \ldots, R_N$. Obtain an expression for R of the system of resistors when they are connected (a) in parallel; (b) in series.

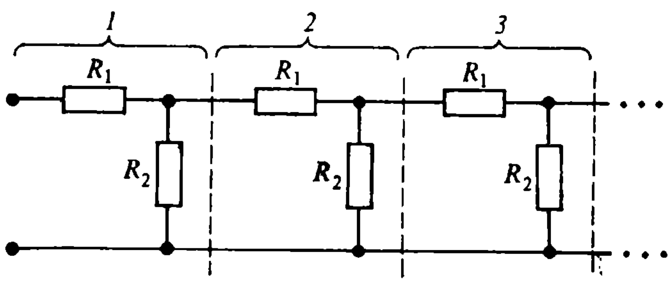

Fig. 3.13

3.119. How do resistors with resistances of $R_1 = 2\,\Omega$, $R_2 = 3\,\Omega$, and $R_3 = 6\,\Omega$ have to be connected to obtain a system with $R = 4\,\Omega$?

3.120. Figure 3.13 shows an infinite circuit formed by repetition of the same unit consisting of resistors with resistances of $R_1 = 2\,\Omega$ and $R_2 = 4\,\Omega$. Find the resistance R of this circuit.

3.121. A section of a circuit is a body of revolution made from a homogeneous material with the resistivity ρ (Fig. 3.14). The cross-sectional area of the body depends on x according to the law $S(x)$. Write an expression for the resistance R of this section of the circuit.

3.122. It is necessary to make a heating coil for an electric stove with a power of 0.50 kW intended for connection to mains at 220 V. How much nichrome wire 0.40 mm in

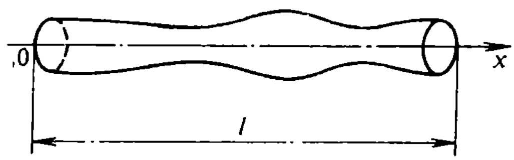

Fig. 3.14

diameter must be taken for this purpose? The resistivity of nichrome in the heated state is 1.05 $\mu\Omega\cdot$m.

3.123. A material with a resistivity of ρ is used to make a flat ring of thickness d. The radii of the ring are a and b $(b > a)$. A potential difference is maintained between the outer and inner cylindrical surfaces of the ring. Find the resistance R of the ring in these conditions.

3.124. A metal sphere of radius a is surrounded by a concentric metal shell of radius b. The space between these electrodes is filled with a homogeneous and isotropic conducting medium of resistivity ρ. Find the resistance R of the interelectrode space. Consider the case when $b \to \infty$.

3.125. A steady current I flows through an imaginary closed surface. What does it equal and how is it directed if during the time Δt the electric displacement flux through the surface increases from the value Φ_1 to Φ_2 $(\Phi_1 < \Phi_2)$?

3.126. Two square plates with a side of $a = 300$ mm secured at a distance of $d = 2.00$ mm from each other form a parallel-plate capacitor that is connected to a source of steady voltage at $U = 250$ V. The plates, arranged vertically, are immersed in a vessel with kerosene $(\varepsilon = 2.00)$ at a speed of $v = 5.00$ mm/s. Find the current I flowing in the leads.

3.127. A capacitor of capacitance $C = 300$ pF is connected through a resistor with a resistance of $R = 500\ \Omega$

to a source of steady voltage U_0. Determine the time t at which the voltage U across the capacitor is $0.990U_0$.

3.128. Opposite charges with a magnitude of $q_0 = 1.00$ mC are imparted to the plates of a capacitor with a capacitance of $C = 2.00$ μF. Next the plates are connected to each other through a resistance of $R = 5000\ \Omega$. Find:

(a) the law of the change in the current flowing through the resistor;

(b) the charge q that passed through the resistor during 2.00 ms;

(c) the amount of heat Q evolved in the resistor during the same time.

3.129. A capacitor of capacitance C is charged to a voltage of U after which it is connected to a resistor R. What

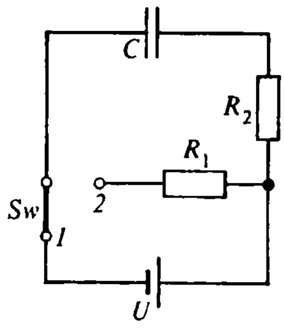

Fig. 3.15

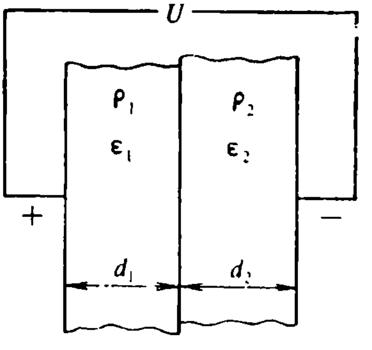

Fig. 3.16

amount of heat Q is evolved in the resistor when the capacitor discharges?

3.130. A capacitor of capacitance $C = 5.00$ μF is connected to a steady current source with a voltage of $U = 200$ V (Fig. 3.15). Next switch Sw is transferred from contact *1* to contact *2*. Find the amount of heat evolved in the resistor with a resistance of $R_1 = 500\ \Omega$. The resistance of R_2 is $300\ \Omega$. Disregard the resistance of the connecting wires.

3.131. $N = 24$ identical current sources have an e.m.f. of $\mathcal{E} = 1.00$ V and an internal resistance of $R_0 = 0.200\ \Omega$. The sources are connected so as to form a battery of n series-connected sections each of which consists of N/n sources connected in parallel. An appliance with a resistance of $R = 0.30\ \Omega$ is connected to the battery. At what n will the

power P used by the appliance be maximum? What is the maximum value of P?

3.132. A copper plate is placed between the plates of a parallel-plate capacitor parallel to them. Its thickness equals one-third of the gap between the plates. The capacitance of the capacitor in the absence of this plate is $C = 0.0250$ μF. The capacitor is connected to a current source, as a result of which it is charged to a voltage of $U = 100.0$ V. Determine:

(a) the work A_1 that must be done to extract the plate from the capacitor;

(b) the work A_2 done during this operation by the current source.

Disregard heating of the plate.

3.133. Solve a problem similar to the preceding one with the difference that the plate is made from a dielectric with a permittivity of $\varepsilon = 3.00$ instead of from copper.

3.134. A paper capacitor (i.e. a capacitor in which paper impregnated with vaseline is the dielectric with $\varepsilon = 2.10$) loses half the charge imparted to it during a time of $\tau =$ $= 5.00$ min. Assuming that leakage of the charge occurs only through the dielectric spacer, calculate its resistivity ρ.

3.135. The gap between the plates of a parallel-plate capacitor is filled with a substance whose conductivity σ changes in a direction perpendicular to the plates according to a linear law from the value $\sigma_1 = 1.00 \times 10^{-12}$ S/m to $\sigma_2 = 1.00 \times 10^{-11}$ S/m. Find the leakage current I through the capacitor provided that the voltage across the plates is $U = 300$ V. The area of the plates is $S = 100$ cm^2, and their separation distance is $d = 2.00$ mm.

3.136. The dielectric of a parallel-plate capacitor consists of two layers (Fig. 3.16) characterized by permittivities of $\varepsilon_1 = 2.00$, $\varepsilon_2 = 3.00$, and resistivities of $\rho_1 = 10.0$ GΩ·m and $\rho_2 = 20.0$ GΩ·m. The thicknesses of the layers are $d_1 = 2.00$ mm and $d_2 = 1.00$ mm. A voltage of $U = 100.0$ V is applied to the capacitor (the plus to the left-hand plate, the minus to the right-hand one).

1. Determine:

(a) the values of the field strength E_1 and E_2, and also the values of the electric displacement D_1 and D_2 in both layers;

(b) the density of the extraneous charges on the left-hand plate σ_1, on the right-hand plate σ_2, and at the interface of the layers σ;

(c) the density of the bound charges near the left-hand plate σ'_1, near the right-hand plate σ'_2, and at the interface of the layers σ';

(d) the density j of the current flowing through the capacitor.

2. Determine the quantities listed in clause 1 for the case when $\rho_1 = \infty$.

3.137. The gap between the plates of a parallel-plate capacitor is filled with a substance having a permittivity of $\varepsilon = 7.00$ and a resistivity of $\rho = 100\ \text{G}\Omega\cdot\text{m}$. The capacitance of the capacitor is $C = 3000$ pF. Find the leakage current I through the capacitor when a voltage of $U = 2000$ V is applied across it.

3.138. The plates of a capacitor having an arbitrary shape are separated by a poorly conducting medium with the permittivity ε and the resistivity ρ. The capacitance of the capacitor is C. Find the leakage current I through the capacitor when a voltage of U is applied across it.

3.139. Two electrodes in the form of metal spheres of radius a are placed in a medium with a resistivity of ρ. The distance between the sphere centres is b ($b \gg a$). Find the resistance R between the electrodes.

3.140. The radii of the plates of a spherical capacitor are a and b ($a < b$). The space between the plates is filled with a substance having the permittivity ε and the conductivity σ. Initially, the capacitor is not charged. Next the charge q_0 is imparted to the inner plate. Find:

(a) the law of the change in the charge q on the inner plate;

(b) the amount of heat Q evolved when the charge spreads; compare Q with the change in the electrical energy of the capacitor.

3.141. A steady current I flows in a section of a circuit with the resistance R. Can the potential difference across the ends of the section be zero?

3.142. Figure 3.17 shows a steady current circuit consisting of three current sources and three resistors connected in series. Determine the potential difference $\varphi_1 - \varphi_2$ between points *1* and *2*. Disregard the resistances of the current sources and of the connecting wires.

3.143. N identical current sources with an e.m.f. of $\mathscr{E}$ and an internal resistance of R_0 and the same number of identical resistors with the resistance R form a closed cir-

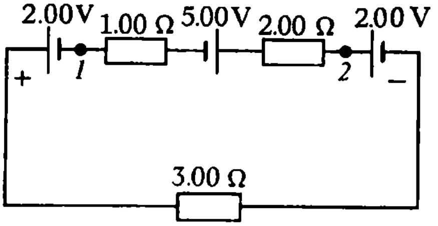

Fig. 3.17

cuit of N units as shown in Fig. 3.18. Find the potential difference between points A and B dividing the circuit into n and $N - n$ units. Disregard the resistance of the connecting wires.

3.144. In the diagram shown in Fig. 3.19, $\mathscr{E} = 5.0$ V, $R_1 = 1.00\ \Omega$, $R_2 = 2.00\ \Omega$, $R_3 = 3.00\ \Omega$. The resistance of the current source is $R_0 = 0.10\ \Omega$. Find the currents I_1 and I_2.

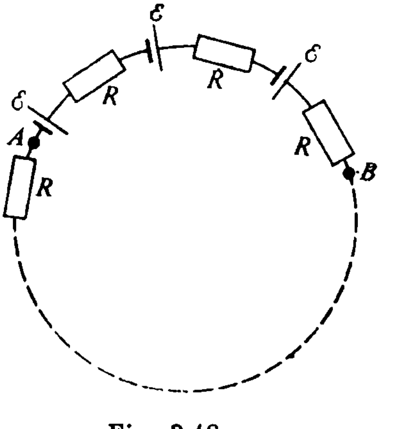

Fig. 3.18

3.145. In the diagram shown in Fig. 3.20, $\mathscr{E}_1 = 10.0$ V, $\mathscr{E}_2 = 20.0$ V, $\mathscr{E}_3 = 30.0$ V, $R_1 = 1.00\ \Omega$, $R_2 = 2.00\ \Omega$, $R_3 = 3.00\ \Omega$, $R_4 = 4.00\ \Omega$, $R_5 = 5.00\ \Omega$, $R_6 = 6.00\ \Omega$, $R_7 = 7.00\ \Omega$. The internal resistance of the current sources is negligibly small. Find the currents I_1, I_2, and I_3.

3.146. Figure 3.21 shows two branched steady current circuits. Determine the currents flowing through the resistors in both cases. Disregard the resistances of the current sources and of the connecting wires.

How will the currents change in case (a) if the circuit is broken at points A and B?

3.147. The elements of the diagram depicted in Fig. 3.22 have the following values: $\mathscr{E}_1 = 1.00$ V, $\mathscr{E}_2 = 2.00$ V,

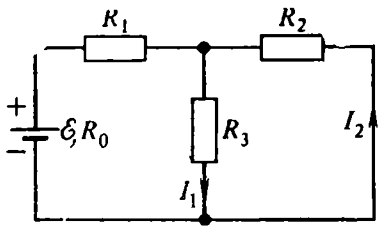

Fig. 3.19

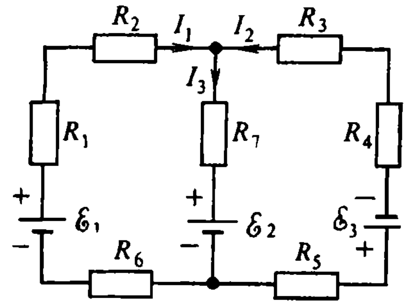

Fig. 3.20

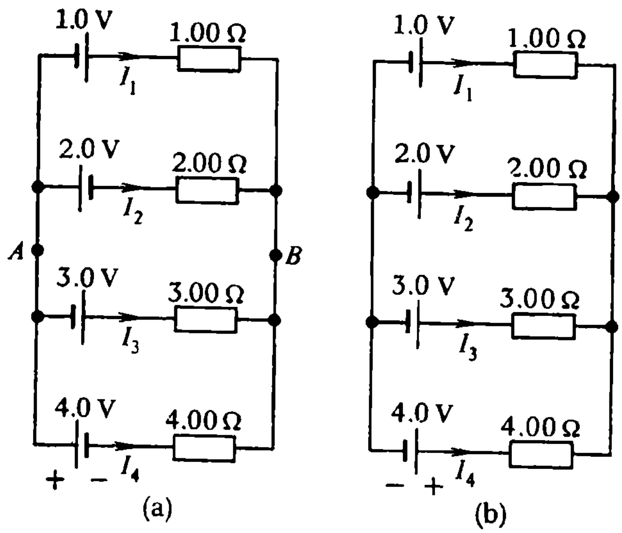

Fig. 3.21

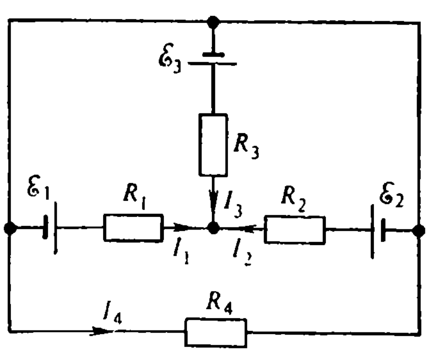

Fig. 3.22

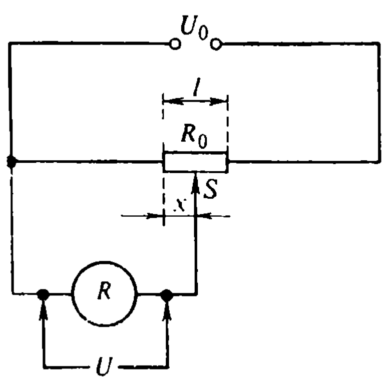

Fig. 3.23

$\mathscr{E}_3 = 3.00$ V, $R_1 = 100\ \Omega$, $R_2 = 200\ \Omega$, $R_3 = 300\ \Omega$, $R_4 = 400\ \Omega$. Determine the currents flowing through the resistors. Disregard the resistances of the current sources and of the connecting wires.

3.148. Figure 3.23 contains a diagram of a potentiometer. This device can be used to regulate the voltage U supplied to appliance R within the limits from 0 to U_0, where U_0 is the voltage of the steady current source. In the simplest potentiometers, the resistor R_0 is made in the form of a homogeneous wire along which slide S travels. Find the voltage U fed from the potentiometer to appliance R as a function of the distance x to the potentiometer slide from the end of wire R_0. Investigate the case $R \gg R_0$.

3.6. Magnetic Field in a Vacuum

3.149. An electron moves rectilinearly and uniformly at a speed of $v = 3.00 \times 10^5$ m/s. Find the magnetic induction B of the field produced by the electron at a point at a distance of $r = 1.00 \times 10^{-9}$ m (10 Å) from it and located on a perpendicular to **v** passing through the instantaneous position of the electron.

3.150. Find the infinite steady current I at which the magnetic induction B of the field at a distance from the conductor of $b = 1.00$ m is 4.8×10^{-3} T (see the answer to the preceding problem).

3.151. Two electrons are moving in a vacuum "side by side" along parallel straight lines at the same speed $v = 3.00 \times 10^5$ m/s. The distance between the electrons is $a = 1.00$ mm. Find the force F_m of magnetic interaction between the electrons. Compare F_m with the force F_e of Coulomb interaction between the electrons.

3.152. A current of $I = 1.00$ A is circulating in a round loop of radius $r = 100$ mm. Find the magnetic induction B:

(a) at the centre of the loop;

(b) on the axis of the loop at a distance of $b = 100$ mm from its centre.

3.153. In a closed circuit with a steady current I, there is a section in the form of two straight wires forming a right angle (Fig. 3.24). The length of these wires is so large that the influence of the remaining sections of the circuit on the

field in the vicinity of the vertex of the angle may be disregarded. Find the magnetic induction B at point A indicated in the figure.

3.154. The current $I = 1.00$ A flows in the plane circuit depicted in Fig. 3.25. The angle between the straight sections of the circuit is a right one. The radii are $r_1 = 10.0$ cm and $r_2 = 20.0$ cm. Find the magnetic induction B at point C.

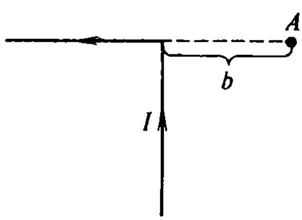

Fig. 3.24

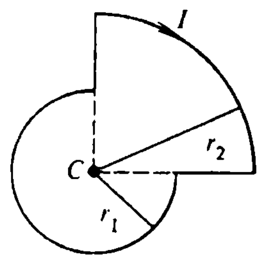

Fig. 3.25

3.155. A circuit with a steady current includes a homogeneous ring and two very long radial conductors connected to it (Fig. 3.26). The part of the circuit closing these conductors (including the current source) is so remote that it

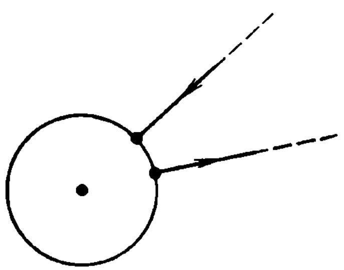

Fig. 3.26

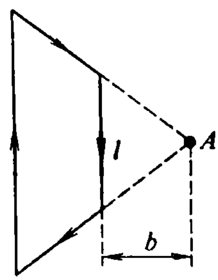

Fig. 3.27

has no appreciable influence on the field in the region of the ring. What is the magnetic induction B at the centre of the ring?

3.156. A closed circuit with a current I includes a straight section of length $2a$. Point A is at a distance b from this section on a perpendicular passing through its middle. Find the part of the magnetic induction B at point A that is pro-

duced by the given section. Investigate the case when $a \to \infty$.

3.157. A current I flows through a wire bent in the form of a regular polygon with n sides inscribed in a circle of radius r. Find the magnetic induction B at the centre of the polygon. Investigate the expression obtained for the case $n \to \infty$.

3.158. A current of $I = 6.28$ A flows in a circuit having the shape of an isosceles trapezium (Fig. 3.27). The ratio of the bases of the trapezium is 2.00. Find the magnetic induction B at point A in the plane of the trapezium. The length of the smaller base of the trapezium is $l = 100$ mm, the distance $b = 50.0$ mm.

3.159. A solenoid of radius r and length l has n turns per unit length. A current I flows in the solenoid. Determine the field strength H at the axis of the solenoid as a function of the distance x from its centre. Investigate the cases:
(a) x is finite, $l \to \infty$; (b) $x = l/2$, $l \to \infty$.

3.160. What influence of the field of a solenoid is rendered by the fact that passing from turn to turn is attended by motion along the solenoid axis?

3.161. A current with the identical density $\mathbf{j}$ over the entire cross section flows in a round straight wire of radius R. Find an expression for the field strength $\mathbf{H}$ at a point whose position relative to the axis of the wire is determined by the position vector $\mathbf{r}$ perpendicular to this axis. Consider the cases when the point is inside and outside the wire.

3.162. Inside a straight wire with a round cross section there is a round cylindrical space whose axis is parallel to that of the wire. The displacement of the space's axis relative to that of the wire is determined by the vector $\mathbf{a}$. A current with the identical density $\mathbf{j}$ over the entire cross section flows through the wire. Find the magnetic field strength $\mathbf{H}$ inside the space. Consider the case when $\mathbf{a} = 0$.

3.163. An ebonite sphere of radius $R = 50.0$ mm is charged uniformly with a surface density of $\sigma = 10.0\ \mu C/m^2$. The ball is made to rotate about its axis at the angular speed $\omega = 62.8$ rad/s. Find the magnetic induction B at the centre of the ball.

3.164. The sphere of Problem 3.84 is made to rotate about its axis parallel to the vector $\mathbf{E}$ inside the sphere. What is the magnetic induction $\mathbf{B}$ at the centre of the sphere?

3.165. The charge q is uniformly distributed over the volume of a homogeneous ball of mass m and radius R. The ball is made to rotate about its axis at the angular speed ω. Find the angular momentum L and the magnetic moment p_m produced as a result of the rotation, and also the ratio of p_m to L.

3.166. An insulated wire is wound so that it forms a flat coil with $N = 100$ turns. The radius of the innermost turn (over the axis of the wire) is $R_1 = 10.0$ mm, and of the outermost turn is $R_2 = 40.0$ mm. What magnetic moment p_m does this coil have when a current of $I = 10.0$ mA flows in it? What does the magnetic field strength H at the centre of the coil equal in this case?

3.167. A small magnetic needle performs small-amplitude oscillations in the Earth's magnetic field with the period $T_1 = 1.33$ s. When it is placed into a solenoid in which a current flows, the needle oscillates with the period $T_2 = 0.16$ s. Determine the magnetic induction B_2 of the field in the solenoid. The horizontal component of the induction of the Earth's magnetic field is $B_1 = 18.0$ μT. Disregard damping of the needle's oscillations.

3.168. Two small identical coils are arranged with their axes on one straight line. The distance between the coils $l = 200$ mm considerably exceeds their linear dimensions. The number of turns in each coil is $N = 100$, the radius of a turn is $r = 10$ mm. With what force F do the coils interact when an identical current of $I = 0.10$ A flows in them?

3.169. A square loop with a current of $I_2 = 2.00$ A is placed next to a long straight wire with a current of $I_1 = 30.0$ A. The loop and the wire are in one plane. The axis of the loop passing through the middles of opposite sides is parallel to the wire and is at a distance of $b = 30.0$ mm from it. The loop side is $a = 20.0$ mm. Find the force F acting on the loop and the work A that has to be done to turn the loop about its axis through 180°.

3.170. The coil of a mirror galvanometer is suspended on a thread whose torsion coefficient (the ratio of the applied torque to the angle of torsion) is $k = 10.0$ μN·m/rad. The coil consists of $N = 100$ rectangular turns of thin wire. The dimensions of a turn are 50×30 mm. The coil can rotate in the gap between the poles of a magnet having cy-

lindrical depressions. An iron cylinder is placed into the coil along the axis of the gap, as a result of which the field in the gap between the poles and the cylinder has axial symmetry. In the part of the gap containing one side of the coil, the field is directed toward the axis, and in the part containing the other side, away from the axis of the coil. The strength of the field in the gap can be considered constant in magnitude and equal to $H = 100$ kA/m. A scale inscribed on a rule with a length of $l_2 = 800$ mm is at a distance of $l_1 = 1200$ mm from the galvanometer mirror. In the absence of a current, the light spot reflected by the mirror is at the middle of the scale.

What maximum current I_m can be measured by this instrument?

3.171. The centre of a long solenoid with $n = 5000$ turns per metre accommodates a small coil with $N = 200$ turns

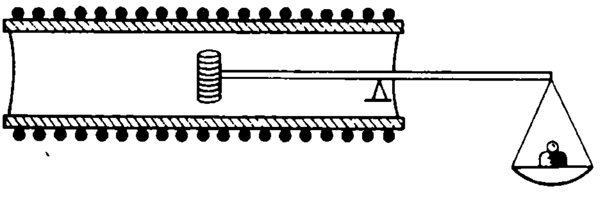

Fig. 3.28

secured on the end of the balance beam of a scale (Fig. 3.28). The axis of the coil is perpendicular to that of the solenoid. The diameter of the coil turns is $d = 10.0$ mm. The arm of the balance beam has a length of $l = 300$ mm. The coil is balanced by weights placed on the pan of the scale. When a current flows in the solenoid and the coil, the equilibrium of the balance beam is upset. By what amount ΔP must the weight of the weights placed on the pan of the scale be changed to restore equilibrium if an identical current of $I = 20.0$ mA flows in the solenoid and the coil?

3.172. A coil in which a current of $I = 1.00$ A flows is placed in a uniform magnetic field so that its axis coincides with the direction of the field. The coil winding is made from copper wire with a diameter of $d = 1.00$ mm; the radius of the turns is $r = 100$ mm. At what value of the magnetic induction B of the external field would the coil winding be

torn apart? The ultimate tensile strength of copper is $\sigma_t = 230$ MPa.

3.173. It is known that: (1) the density $\mathbf{j}$ of a steady current is parallel to the z-axis and depends only on the distance r from this axis; (2) the circulation C of the vector $\mathbf{H}$ along a flat contour of radius r perpendicular to the z-axis and with its centre on this axis is proportional to the third power of r: $C = \alpha r^3$. Find the form of the function $\mathbf{j}(r)$.

3.174. The gap between two parallel round plates is filled with a homogeneous weakly conducting substance having a conductivity of σ and a permittivity of ε (the permeability μ of the substance is unity). The magnitude of the gap d is much smaller than the radius R of the plates. A voltage is applied across the plates that varies according to the law $U = U_m \cos \omega t$ (ω is sufficiently small for the condition of a quasistationary nature to be observed). Find an expression for the magnetic field strength H in the gap at a distance r from the axis of the plates that is much smaller than R.

3.175. Show that notwithstanding the presence of conduction currents flowing in radial directions, the magnetic field strength H in the gap of the spherical capacitor of Problem 3.140 is zero.

3.7. Magnetic Field in a Substance

3.176. What will happen to the field of an infinite solenoid when the latter is filled with a homogeneous isotropic magnetic with the permeability μ?

3.177. What is the mean value of the magnitude of the tangential component of the magnetic field strength $\langle H_l \rangle$ for an arbitrary closed contour of length l surrounding a conductor in which the current I flows?

3.178. An infinite plane-parallel plate made from a homogeneous and isotropic magnetic with the permeability μ is placed in a uniform magnetic field with the induction $\mathbf{B}_0$. The plate is perpendicular to the lines of $\mathbf{B}_0$. Determine the magnetic induction $\mathbf{B}$ and the magnetic field strength $\mathbf{H}$ in the magnetic.

3.179. Two plates made from magnetics with the permeabilities μ_1 and μ_2 are placed together and put in a uniform field with the induction $\mathbf{B}_0$ perpendicular to them (Fig. 3.29).

The dashed lines show an imaginary cylindrical surface with generatrices parallel to $\mathbf{B}_0$ and with bases of area S perpendicular to $\mathbf{B}_0$. What do the flux Φ_B of the vector $\mathbf{B}$ and the flux Φ_H of the vector $\mathbf{H}$ through this surface equal?

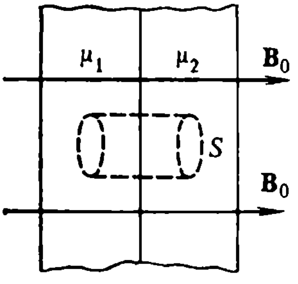

Fig. 3.29

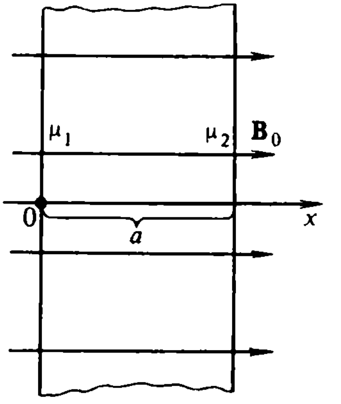

Fig. 3.30

3.180. An infinite plate made from an isotropic magnetic is placed in a uniform external field with the induction $\mathbf{B}_0$ perpendicular to it (Fig. 3.30). The permeability of the plate varies linearly from the value of μ_1 at the left-hand boundary to μ_2 at the right-hand one. Find:

(a) $\nabla\mathbf{H}$ inside the plate as a function of x;

(b) the flux Φ_H of the vector $\mathbf{H}$ through an imaginary cylindrical surface with generatrices parallel to the x-axis. The bases of the cylinder are at points with the coordinates $x_1 = a/2$ and $x_2 = 3a/2$. The area of each base is S.

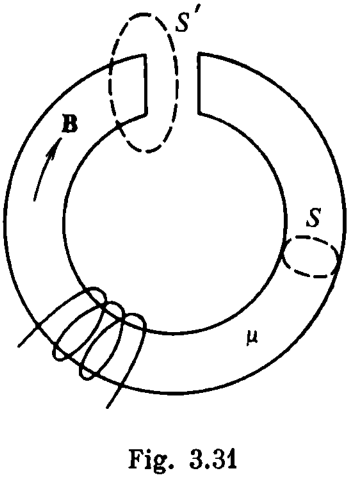

Fig. 3.31

3.181. A ball made from a homogeneous and isotropic magnetic with the permeability μ is put in a uniform magnetic field with the induction $\mathbf{B}_0$.

(a) Determine the strength **H** and induction **B** of the field in the magnetic. Consider the demagnetization factor to be known.

(b) Write an approximate expression for **B** if $\mu \gg 1$.

3.182. The iron core depicted in Fig. 3.31 carries a winding in which a steady current flows. As a result, a field having

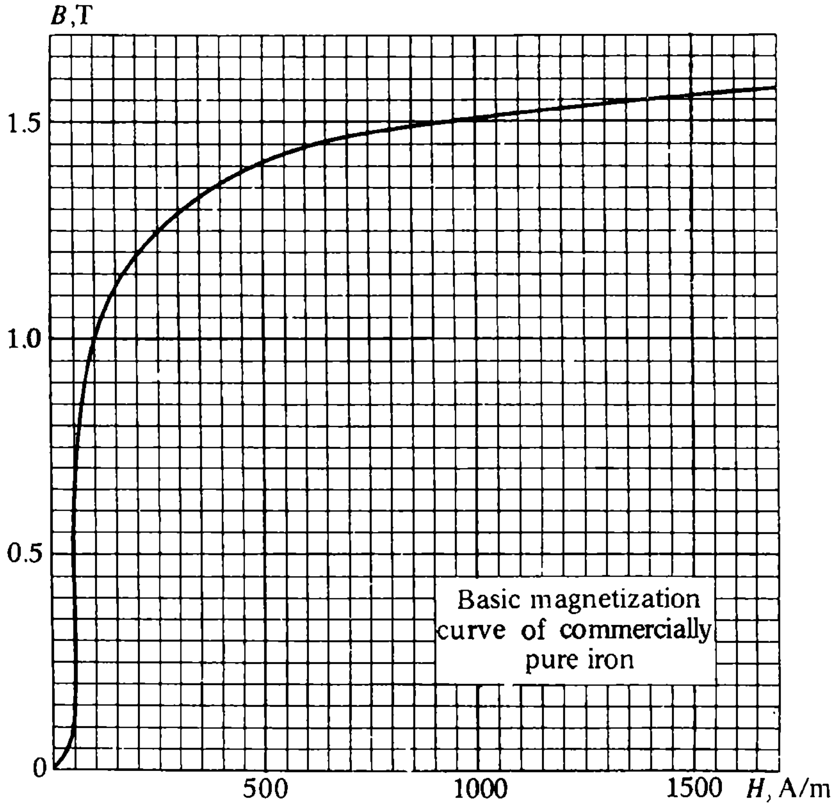

Fig. 3.32

the induction B appears in the core. The permeability of iron in these conditions is μ. The cross-sectional area of the core is S. One end of the core enters the imaginary closed surface S'. Find for this surface the flux Φ_B of the vector **B** and the flux Φ_H of the vector **H**.

3.183. An iron core in the form of a torus with a diameter of $d = 500$ mm carries a winding with $N = 1000$ turns. A lateral notch has been made in the core as a result of which

an air gap having a width of $b = 1.00$ mm has been formed. At a current in the winding of $I = 0.85$ A, the field strength in the gap is $H = 600$ kA/m. Determine the permeability μ of the iron in these conditions. Disregard scattering of the field at the edges of the gap.

3.184. Figure 3.32 shows a basic magnetization curve of commercially pure iron obtained experimentally. Using this graph, plot a curve showing how the permeability μ depends on the magnetic field strength H. Find the maximum value of the permeability μ_{max} and the strength H at which it is reached.

3.185. An iron ring has a square cross section. The middle diameter of the ring is $d = 300$ mm, its cross-sectional area is $S = 500$ mm^2. The ring carries a winding of $N = 800$ turns. A current of $I = 3.00$ A flows in the winding. The ring has a transverse notch with a width of $b = 2.00$ mm. Disregarding the scattering of the field at the edges of the notch, find:

(a) the permeability μ of the iron in these conditions;

(b) the flux Φ of the magnetic induction through the cross section of the ring;

(c) the energy W_1 in the iron, the energy W_2 in the air gap, and the total energy W of the field.

3.8. Electromagnetic Induction

3.186. Figure 3.33 depicts a plane loop placed in a uniform magnetic field directed toward us. Indicate the direction of the current appearing in the loop if (a) B grows; (b) B diminishes; (c) the loop is stretched; (d) the loop contracts.

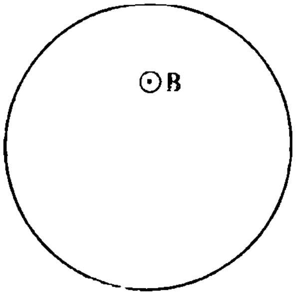

Fig. 3.33

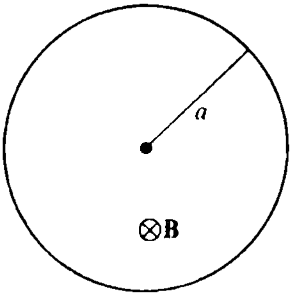

Fig. 3.34

3.187. A round conducting loop of radius a has a resistance of R (Fig. 3.34). Initially, there is no current in it. Next a uniform magnetic field with the induction **B** directed beyond the drawing and perpendicular to the plane of the loop is switched on.

(a) In what direction will the induced current flow?

(b) What charge q will flow in the loop?

3.188. A rod closing the circuit moves along a U-shaped wire at a constant speed v under the action of the force F (Fig. 3.35). The circuit is in a uniform magnetic field perpendicular to its plane. What does the force F equal if the amount of heat Q is evolved in the circuit every second?

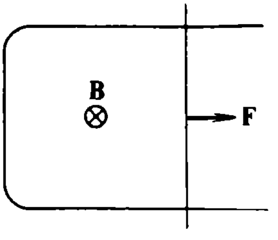

Fig. 3.35

3.189. A thin metal rod of length $l = 1.200$ m rotates in a uniform magnetic field about an axis perpendicular to the rod and at a distance of $l_1 = 0.250$ m from one of its ends. The rod's speed is 120 rpm. The vector **B** is parallel to the axis of rotation and has a magnitude of 1.00 mT. Find the potential difference U induced between the ends of the rod.

3.190. An insulated metal disk of radius $a = 0.250$ m rotates at $n = 1000$ rpm. Find the potential difference U between the centre and the edge of the disk induced:

(a) in the absence of magnetic fields;

(b) when there is a uniform field with the induction $B = 10.0$ mT perpendicular to the disk.

3.191. A small coil is placed between the poles of an electromagnet. It is arranged so that the axes of the coil and the pole shoes of the magnet coincide. The cross-sectional area of the coil is $S = 3.00$ mm^2, its number of turns is $N = 60$. When the coil is turned through 180°, a charge of $q = 4.50$ μC flows through the ballistic galvanometer connected to it. Determine the strength H of the field between the poles. The resistance of the coil, the galvanometer, and the connecting wires is $R = 40.0\ \Omega$.

3.192. One layer containing $N = 100$ turns of a wire is wound onto a cylindrical frame with a diameter of $d =$ 120 mm. The entire winding is accommodated over a

length of $l = 60$ mm. Determine the inductance L of the coil. Assume that the permeability of the core is unity.

Hint. The inductance of single-layer coils is evaluated by the formula $L = \alpha L_\infty$, where L_∞ is the inductance of an ideal solenoid within whose entire volume the field is the same as in an infinite solenoid with the same value of N/l, α is a coefficient determined approximately by the expression $\alpha = 1/[1 + 0.45(d/l)]$.

3.193. A wire with a radius of $a = 1.00$ mm has been used to make a rectangular frame whose length $l = 10.0$ m is considerably larger than its width $b = 0.100$ m (measured between the axes of the frame sides). Find the inductance L of the frame. Assume that the permeability of the medium is unity. Disregard the field inside the wire.

3.194. Find the inductance L_1 of the wires from Problem 3.103 per unit of their length. Assume that the permeability of the material of the wires and the surrounding medium is unity.

3.195. What is called a coaxial cable consists of two coaxial conductors separated by a dielectric layer. Determine the capacitance C_1 and the inductance L_1 per unit length of a cable in which the radius of the inner conductor is $a = 1.50$ mm and that of the outer conductor (having the shape of a thin-walled tube) is $b = 5.4$ mm. The dielectric is polyethylene ($\varepsilon = 2.3$). Take into account that at high frequencies (which coaxial cables are intended for), an alternating current flows along the surface of the conductor.

3.196. Determine the inductance L of the winding of Problem 3.185. It is recommended to calculate L in two ways—with the aid of the expression for the flux of the vector **B** and with the aid of the one for the energy of a field—and compare the results obtained.

3.197. Two turns of a wire are adjacent to each other. A current of $I = 10.0$ A flows in the first one. A ballistic galvanometer is connected to the circuit of the second one. The total resistance of the second circuit is $R = 5.00\ \Omega$. What is the mutual inductance L_{12} of the turns if a charge of $q = 1.00 \times 10^{-8}$ C passes through the galvanometer when the current I is switched off?

3.198. A coil with N turns is wound onto an infinite solenoid with n turns per unit length and with a cross-sectional area of S. Find the mutual inductance L_{12} of the coil and

the solenoid. The permeability of the medium filling the solenoid is μ.

3.199. Determine the mutual inductance L_{12} of a toroid and of an infinite straight wire located along its axis. The toroid has a rectangular cross section of width a. The inner radius of the toroid is r_1, and the outer one is r_2. The number of turns of the toroid is N. The toroid and the wire are immersed in a medium whose permeability is μ.

3.200. A straight wire with a resistance of R_1 per unit length is bent to form an angle of 2α (Fig. 3.36). A rod of the same wire perpendicular to the bisector of the angle 2α forms a closed triangular loop with the bent wire. This loop is placed in a uniform magnetic field with the induction **B** perpendicular to its plane. Find the direction and the intensity I of the current flowing in the loop when the rod moves at the constant speed v. Disregard the resistance at the contacts *1* and *2*.

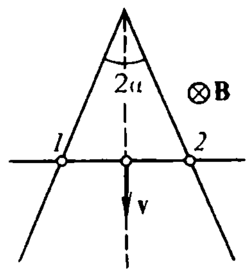

Fig. 3.36

3.201. We have an infinite straight wire in which a current I_0 flows. Two bare wires are parallel to it at the distances a and b and are short-circuited at one end by resistor R

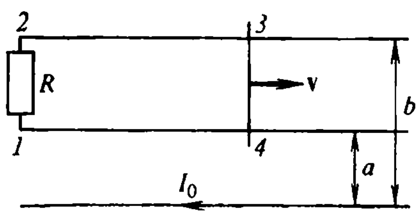

Fig. 3.37

(Fig. 3.37). All three wires are in one plane. Rod *3-4* closing the circuit in the wires short-circuited by the resistor slides along them at a speed of v. Determine:

(a) the current I in circuit *1-2-3-4* and its direction;

(b) the force F needed to maintain a constant speed of rod *3-4* and the distance x from the wire with the current

I_0 to the point at which this force must be applied for the rod to have forward motion;

(c) the power P used to move the rod.

Disregard the resistance of the wires, the rod, and the contacts at points *3* and *4*.

3.202. A copper bar of mass m slides under the action of the force of gravity along two copper buses installed at the angle α to the horizontal (Fig. 3.38). A uniform magnetic

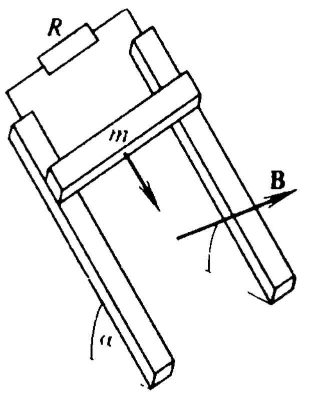

Fig. 3.38

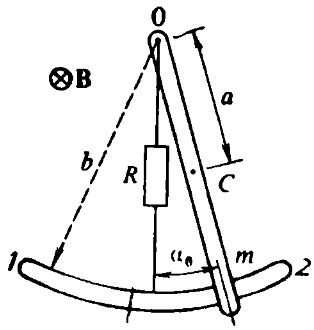

Fig. 3.39

field with the induction **B** perpendicular to the plane in which the bar moves has been set up in the space surrounding the buses. The top ends of the buses are connected through resistor R. The coefficient of friction between the surfaces of the buses and the bar is f ($f < \tan\alpha$). The distance between the buses is l. Disregarding the resistance of the buses, the bar, and the places of contact between them, find the steady value of the bar's speed v.

3.203. An arrangement differs from that considered in the preceding problem (see Fig. 3.38) only in that a capacitor of capacitance C is connected to the ends of the buses instead of the resistor R. The bar is placed on the buses and released. Determine the nature of motion of the bar, assuming that the electrical resistance of the circuit is zero.

3.204. A metal rod of mass m can oscillate like a pendulum about axis O (Fig. 3.39). The lower end of the rod contacts wire *1-2* bent along an arc of radius b. The middle of this wire is connected to point of suspension O through resistor R. The entire device is placed in a uniform magnetic

field with the induction **B** perpendicular to the plane of oscillations. The distance from the point of suspension to the centre of mass C of the rod is a; the moment of inertia of the rod relative to an axis passing through C is I_0. Disregarding friction, and also the electrical resistance of the rod, wire *1-2*, and the place of their contact, determine the nature of the motion performed after the rod is deflected through the small angle α_0 and is released with a zero initial speed.

3.205. An arrangement differs from that considered in the preceding problem (see Fig. 3.39) only in that a capacitor of capacitance C is connected instead of the resistor R. Considering the resistance of the circuit to be zero, determine the nature of the motion performed after the rod is deflected through the small angle α_0 and is released with a zero initial speed.

3.206. A rod of mass m can rotate without friction about axis O, sliding (also without friction) along an annular conductor of radius b arranged in a vertical plane (Fig. 3.40).

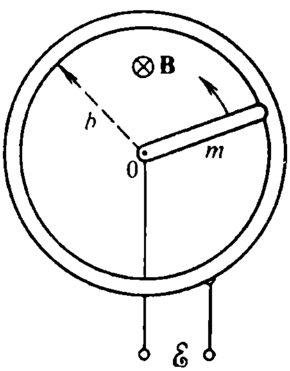

Fig. 3.40

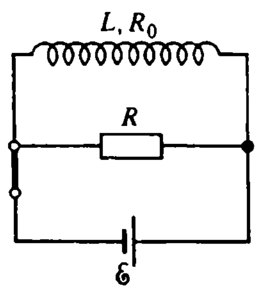

Fig. 3.41

The entire arrangement is placed in a uniform magnetic field with the induction **B** perpendicular to the plane of the annular conductor. The axis and the conductor are connected to the terminals of a current source. Determine:

(a) according to what law the current I flowing in the rod must vary for the rod to rotate at a constant angular speed ω. Begin to measure the time from the instant when the rod is in its right-hand horizontal position; consider the current

to be positive when it flows from the axis of rotation to the annular conductor;

(b) what the e.m.f. $\mathscr{E}$ of the source must be to maintain the required current.

Consider the total resistance of the circuit to be constant and equal to R. Disregard the inductance of the circuit.

3.207. A coil with an inductance of $L = 250$ mH and a resistance of $R = 0.300\ \Omega$ is connected to a source with a constant voltage. In what time τ will the current in the coil reach (a) 50%; (b) 75% of the steady value? Compare both values of τ.

3.208. A coil with an inductance of $L = 2.00\ \mu$H and a resistance of $R_0 = 1.00\ \Omega$ is connected to a source of steady current with an e.m.f. of $\mathscr{E} = 3.00$ V (Fig. 3.41). A resistor with a resistance of $R = 2.00\ \Omega$ is connected parallel to the coil. After the current in the coil reaches its steady value, the current source is switched off. Find the amount of heat Q evolved in the resistor R after breaking of the circuit. Disregard the resistance of the current source and the connecting wires.

3.209. An iron core having the shape of a ring with a square cross section carries a winding consisting of $N = 1000$ turns. The inner radius of the ring is $a = 0.200$ m, and the outer one $b = 0.250$ m. Determine the energy W accumulated in the core when a current of $I = 1.26$ A flows in the winding. Do this approximately, assuming the strength of the field over the entire section of the core to be the same and equal to the value of H at the centre of the cross section.

3.210. An additional winding consisting of $N_1 = 20$ turns is wound onto the winding of the preceding problem and is connected to a ballistic galvanometer. The total resistance of the additional winding, the galvanometer, and the connecting wires is $R = 31.0\ \Omega$. What charge q will pass through the galvanometer if the current flowing in the main winding is switched off? Disregard the residual magnetization of the core.

3.9. Motion of Charged Particles in Electric and Magnetic Fields

3.211. Calculate the speed v acquired by an electron passing through a potential difference U equal to (a) 100 V; (b) 100 kV.

3.212. For case (b) of the preceding problem, compare the values of v_{cl} and v_{rel} obtained according to the classical and relativistic formulas.

3.213. An electron first flies freely at the velocity $\mathbf{v}_0$. At the instant $t = 0$, a uniform electric field $\mathbf{E}$ is switched on that makes the angle α with the direction of $\mathbf{v}_0$.

(a) Along what trajectory will the electron travel after the field is switched on?

(b) What is the radius of curvature R of the trajectory at the point where the velocity of the electron is minimum?

(c) What is the increment of the electron's momentum $\Delta\mathbf{p}$ during the time τ?

(d) How will the magnitude of the electron's angular momentum L relative to the point where the electron was at the instant when the field was switched on change with time?

3.214. A charged droplet with a mass of $m = 6.40 \times 10^{-16}$ kg is in a horizontally arranged parallel-plate capacitor with a separation distance of $d = 10.0$ mm. In the absence of a voltage between the plates, the droplet falls at the constant speed of $v_1 = 0.078$ mm/s. After a voltage of $U = 95.0$ V is applied across the capacitor, the droplet moves uniformly upward at a speed of $v_2 = 0.016$ mm/s. Determine the charge e' of the droplet.

3.215. Determine the force F acting on an electron at the instant when it intersects the axis of a long solenoid at right angles in direct proximity to its end. The current in the solenoid is $I = 2.00$ A, the number of turns per unit of length is $n = 3000\ \text{m}^{-1}$. The speed of the electron is $v = 3.0 \times 10^7$ m/s. Assume the permeability of the medium to be unity.

3.216. An alpha particle initially moves freely at a speed of $v = 0.350 \times 10^7$ m/s. At a certain instant, a uniform magnetic field with the induction $B = 1.000$ T is created in the vicinity of the particle at right angles to its velocity. Find:

(a) the radius r of the particle's trajectory;

(b) the magnitude and the direction of its magnetic moment p_m;

(c) the ratio of the magnetic moment p_m of the particle to its angular momentum L.

The charge of the alpha particle is $e' = 2e$, and its mass is $m = 6.65 \times 10^{-27}$ kg.

3.217. The helical line along which an electron travels in a uniform magnetic field has a diameter of $d = 80$ mm and a pitch of $l = 200$ mm. The induction of the field is $B = 5.00$ mT. Determine the speed v of the electron.

3.218. A slightly diverging beam of monoenergetic electrons having a speed of $v = 6.0 \times 10^6$ m/s emerges from a point O in the direction of the x-axis in a uniform magnetic field with the induction $B = 10.0$ mT directed along this axis. Determine the distance l from point O to the nearest point at which the trajectories of all the electrons intersect (the point at which the beam is focussed).

3.219. We have uniform crossing fields of **E** and **B** ($E \ll cB$). We choose the coordinate axes so that the y-axis is directed along the vector **E**, and the z-axis along the vector **B**. We place a particle of mass m and charge e' at the origin of coordinates and release it with a zero initial speed.

(a) How will the particle travel?

(b) According to what law will the particle's speed v vary with time?

3.220. In a device similar to the one with whose aid J. J. Thomson determined the specific charge of an electron (Fig. 3.42), an electron beam can be deflected in a vertical

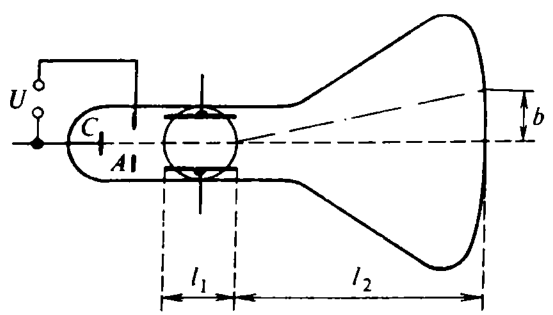

Fig. 3.42

direction either with the aid of a vertically directed electric field or with the aid of a horizontally directed magnetic field. Both fields act over a length of $l_1 = 50$ mm. The distance from the deflecting system to the fluorescing screen is $l_2 = 175$ mm. The electrons in the beam are accelerated by a voltage of $U = 500$ V applied between cathode C and anode A. With a certain electric field, the trace of the

beam deviates over the screen by a distance of $b = 50$ mm. The switching on of a magnetic field $B = 370$ μT returns the trace of the beam to its initial point. Determine the specific charge of an electron from the above data.

3.221. In a Bainbridge mass spectrometer (Fig. 3.43), the distance between the outlet slit of the velocity selector and the inlet slit of the instrument registering the ions is $l = 400$ mm. The induction of the magnetic field is $B' = B = 50.0$ mT. When the strength of the selector's electric field is smoothly changed, peaks of an ionic current are observed in the receiver at the values of $E_1 = 120$ V/cm and $E_2 = 160$ V/cm. Determine the atomic masses A_{r1} and A_{r2} of the relevant ions, assuming them to have a single charge. Identify these ions (i.e. indicate the chemical element they correspond to).

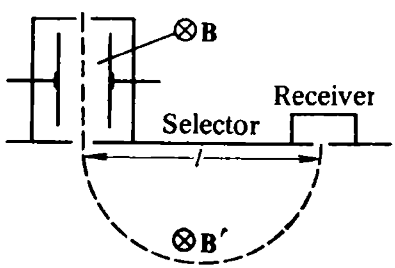

Fig. 3.43

3.222. The internal diameter of the dees of a cyclotron is $d = 1.000$ m. The induction of the magnetic field is $B = 1.20$ T. The accelerating voltage is $U = 100$ kV. Find:

(a) the maximum energy W to which protons can be accelerated in this cyclotron, and the speed v acquired by the protons by the end of their acceleration;

(b) the time τ during which the acceleration process lasts;

(c) the approximate value of the distance s travelled by the protons during this time.

3.223. The mean value $\langle B \rangle$ of the magnetic induction of the field produced by the magnet of a betatron, varying approximately according to a linear law, increases during the time $\tau = 1.00$ ms from zero to the value $B_1 = 200$ mT. The radius of the electrons' orbit is $r = 300$ mm. Find:

(a) the distance s travelled by the electrons during the time of their acceleration to an energy of $W = 50$ MeV;

(b) the speed v of the electrons accelerated to such an energy.

3.10. Electrical Oscillations

3.224. A capacitor of capacitance C is charged to a voltage of U_0 and is connected to a coil of inductance L. What is the amplitude I_0 of the current in the formed oscillatory circuit? Disregard the resistance of the circuit.

3.225. A closed circuit in the form of a frame with an area of $S = 60.0\ \text{cm}^2$ rotates in a uniform magnetic field with the induction $B = 20.0$ mT at a constant speed of $n = 20.0$ rps. The axis of rotation and the direction of the field are mutually perpendicular. Determine the amplitude $\mathscr{E}_m$ and effective $\mathscr{E}$ values of the e.m.f. in the circuit.

3.226. An alternating current circuit is formed by series-connected resistance $R = 800\ \Omega$, inductance $L = 1.27$ H, and capacitance $C = 1.59\ \mu$F. A 50-period effective voltage of $U = 127$ V is applied across the terminals of the circuit. Find:

(a) the effective value of the current I in the circuit;

(b) the phase shift φ between the current and the voltage;

(c) the effective values of the voltages U_R, U_L, and U_C across the terminals of each of the circuit elements;

(d) the power P evolved in the circuit.

3.227. An alternating voltage whose effective value is $U = 220$ V and whose frequency is $\nu = 50$ Hz is fed across a coreless coil with an inductance of $L = 31.8$ mH and a resistance of $R = 10.0\ \Omega$.

(a) Find the amount of heat Q evolved in the coil a second.

(b) How will Q change if a capacitor with the capacitance $C = 319\ \mu$F is connected in series with the coil?

3.228. An alternating voltage with an effective value of $U = 220$ V and a frequency of $\nu = 50$ Hz is supplied across the terminals of the circuit shown in Fig. 3.44. The resistance of the circuit is $R = 22\ \Omega$, its inductance is $L = 318$ mH. The capacitance of the circuit is selected so that the reading of a voltmeter connected parallel to the inductance is maximum. Find the readings U_1 of the voltmeter and I of the ammeter in these conditions. The impedance of the ammeter and the branching of the current into the voltmeter circuit may be disregarded.

3.229. An alternating voltage with an effective value of $U = 220$ V is fed to points A and B of the diagram depicted in Fig. 3.45. The capacitance of the circuit is $C = 1.00\ \mu$F,

its inductance is $L = 1.00$ mH, and its resistance is $R = 100$ mΩ.

(a) At what value of the frequency ω will the current through section *1* be minimum?

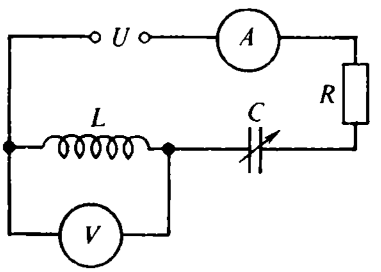

Fig. 3.44

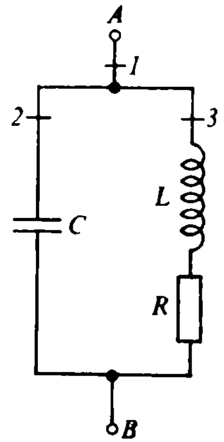

Fig. 3.45

(b) What are the effective values I_1, I_2, and I_3 of the currents flowing through sections *1*, *2*, and *3* at this frequency?

3.230. The oscillatory circuit of a radio set consists of a coil with an inductance of $L = 1.00$ mH and a variable capacitor whose capacity can vary from 9.7 to 92 pF. Within what range of wavelengths can the set receive radio station broadcasts?

3.231. The resistance of an oscillatory circuit is $R = 0.33\ \Omega$. What power P does the circuit use when undamping oscillations are maintained in it with a current amplitude of $I_m = 30$ mA?

3.232. The parameters of an oscillatory circuit have the values: $C = 1.00$ nF, $L = 6.00\ \mu$H, and $R = 0.50\ \Omega$. What power P must be supplied to the circuit to maintain undamping oscillations in it with a voltage amplitude across the capacitor of $U_m = 10.0$ V?

3.233. The parameters of an oscillatory circuit have the values: $C = 4.00\ \mu$F, $L = 0.100$ mH, and $R = 1.00\ \Omega$.

(a) What is the quality Q of the circuit?

(b) What relative error do we make in calculating the quality by the approximate formula $Q = (1/R)\sqrt{L/C}$?

3.234. The quality of an oscillatory circuit is $Q = 10.0$. Determine by what per cent the frequency of the free oscillations ω of the circuit differs from the natural frequency ω_0 of the circuit. [Find $(\omega_0 - \omega)/\omega_0$.]

3.235. The natural frequency of oscillations of a circuit is $\omega_0 = 50 \times 10^3\ s^{-1}$, the quality of the circuit is $Q = 72$. Damped oscillations are produced in the circuit.

(a) Find the law of diminishing of the energy W accumulated in the circuit with the time t.

(b) What part of the initial energy W_0 is retained in the circuit when the time $\tau = 1.00$ ms elapses?

3.236. What must the quality Q of a circuit be for the frequency at which resonance of the currents sets in to differ from the frequency at which resonance of the voltages sets in by not more than 1%?

3.237. The capacitance of the circuit shown in Fig. 3.46 is $C = 1000$ pF, and its inductance is $L = 1.00$ mH. Two

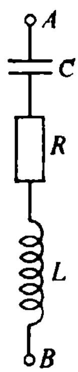

Fig. 3.46

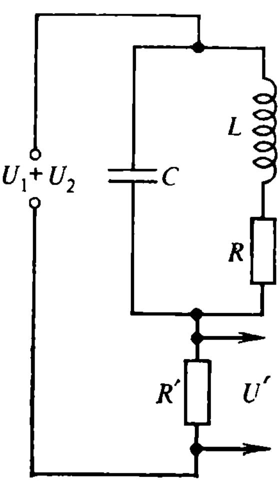

Fig. 3.47

alternating voltages of the same amplitude, but of different frequency, are simultaneously fed to points A and B: the frequency of the first voltage coincides with the natural frequency of the circuit ($\omega_1 = \omega_0$), and the frequency of

the second voltage exceeds the natural one by 10% ($\omega_2 = 1.10\omega_0$).

Find the ratio of the amplitudes of the currents I_1/I_2 produced in the circuit by both voltages for cases when the quality Q of the circuit is: (a) 100; (b) 10.

3.238. The oscillatory circuit shown in Fig. 3.47 has a capacitance of $C = 1.00 \times 10^{-9}$ F and an inductance of $L = 1.00 \times 10^{-5}$ H. Two voltages U_1 and U_2 identical in amplitude, but differing in frequency, are applied across the circuit and the resistance $R' = 10.0\ \Omega$ connected in series with it. The amplitude of each of the voltages is 10.0 V. The frequency of the first voltage coincides with the resonance frequency of the circuit ($\omega_1 = \omega_{res}$), and the frequency of the second voltage exceeds the resonance value by 10% ($\omega_2 = 1.10\omega_{res}$).

Find the amplitudes of the voltages U_1' and U_2' across the resistor R' when the quality Q of the circuit is: (a) 200; (b) 20.

PART 4

WAVES

SYMBOLS

A	amplitude
$\hat{A}$	complex amplitude
$\mathbf{E}$	electric field strength
E	Young's modulus
E_k	kinetic energy
$\mathbf{F}$	force
$\mathbf{H}$	magnetic field strength
I	current; wave intensity
i	imaginary unity; $i = \sqrt{-1}$
$\mathbf{j}$	density of energy flux
$\mathbf{K}$	momentum
$\mathbf{k}$	wave vector
L	sound loudness level
l	distance; length
m	mass
P	power
p	pressure
r	distance; radius
$\mathbf{S}$	Poynting vector
S	area; surface
$\mathbf{u}$, $\mathbf{v}$	velocity
V	volume
v	phase speed
W	energy
w	energy density
α	initial phase
γ	ratio of heat capacities at constant pressure and constant volume; wave attenuation coefficient
ε	permittivity
$\varkappa$	wave absorption coefficient
λ	wavelength
μ	permeability
ν	frequency
ξ	displacement of particle from equilibrium position
ρ	density
τ	time
φ	angle; phase
ω	angular frequency

4.1. Elastic Waves

4.1. Which wave—a longitudinal or a transverse one—is described by the equation $\xi = A \cos(\omega t - kx)$?

4.2. An elastic wave passes from a medium in which the phase speed is v into a medium in which the phase speed is twice this value. What happens to the wave frequency ω and the wavelength λ?

4.3. A plane wave of length λ propagates along the x-axis. What is the smallest distance Δx between points of the medium at which the oscillations are performed in counterphase?

4.4. Figure 4.1 shows an "instantaneous photograph" of the displacements ξ of the particles of a medium in which an elastic wave propagates along the x-axis. Indicate the directions of the particles' velocities at points A, B, and

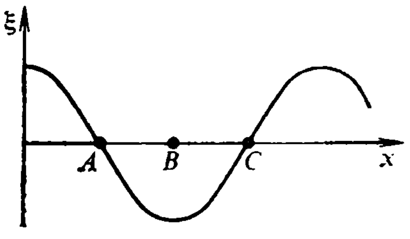

Fig. 4.1

C for: (a) a longitudinal wave; (b) a transverse wave in which the oscillations occur in the plane of the drawing.

4.5. A plane elastic wave described by the equation $\xi = A \exp(-\gamma x) \cos(\omega t - kx)$ propagates in a homogeneous medium. Assuming that $\lambda = 1.00$ m and $\gamma = 0.100\ \mathrm{m}^{-1}$, find the phase difference $\delta\varphi$ at points for which the ratio of the amplitudes of the displacement of the medium's particles is $\eta = 1.0100$.

4.6. What data does the complex amplitude $\hat{A}$ contain?

4.7. Two coherent oscillations of the same direction are characterized by the complex amplitudes $\hat{A}_1 = 5 \exp(i\pi/6)$ and $\hat{A}_2 = 6 \exp(i\pi/3)$. Find the complex amplitude $\hat{A}$ of the resultant oscillation.

4.8. Write the equation of a cylindrical harmonic wave emitted by a source in the form of an endless straight filament.

4.9. The investigation of a physical quantity showed that it satisfies the equation

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{\alpha} \frac{\partial^2 f}{\partial t^2}$$

where α is a constant quantity whose numerical value in the SI is 1.44×10^8.

(a) Determine the dimension of the quantity α from the form of the equation.

(b) What can be said about the quantity f?

4.10. What is described by an equation of the form $\xi = f(\omega t - kx)$, where f is a function, while ω and k are constants? What is the meaning of the quantity ω/k?

4.11. Determine the speed v of longitudinal elastic waves in a copper rod. Assume that Young's modulus $E = 1.12 \times 10^{11}$ Pa.

4.12. Figure 4.2 shows an "instantaneous photograph" of the displacements of particles in a running wave.

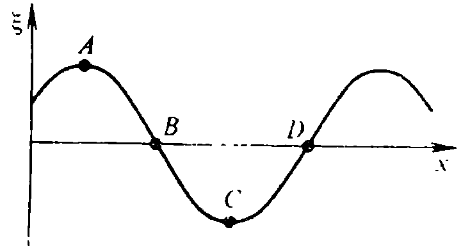

Fig. 4.2

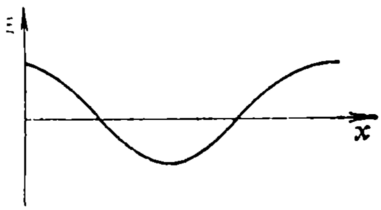

Fig. 4.3

1. Indicate the places at which the deformation of the medium: (a) is absent; (b) is maximum.

2. What does the density of the kinetic, potential, and total energy equal (zero, non-zero, maximum) at the points: (a) A and C; (b) B and D?

4.13. The longitudinal plane wave $\xi = A \cos(\omega t - kx)$ propagates in an elastic medium. Depict for $t = 0$, one under the other, approximate plots of how the displacement ξ depends on x and the density ρ of the medium depends on x.

4.14. Figure 4.3 contains a graph of the displacements ξ in a running wave for a certain instant t. Draw under this graph an approximate plot of the energy density w for the same instant t.

4.15. The wave $\xi = A \cos(\omega t - kx + \alpha)$ runs along the x-axis in an elastic medium of density ρ. Write an expression for Umov's vector $\mathbf{j}$ (the vector of the energy flux density).

4.16. What is the flux of Umov's vector through a surface S?

4.17. A plane damping wave [its amplitude diminishes according to the law $\exp(-\gamma x)$] travels in a tube of cross section S. In the section with the coordinate x_1, the mean (in

time) value of the magnitude of Umov's vector is j_1. What amount of energy W is absorbed during the time t much greater than the period of the wave in the volume confined between the sections with the coordinates x_1 and x_2?

4.18. According to what law does the intensity of a damping (a) spherical; (b) cylindrical wave diminish with an increasing distance r from the source?

4.19. Two waves propagate over the surface of water from two point coherent sources. What is the shape of the lines for which the amplitude of the oscillations is maximum?

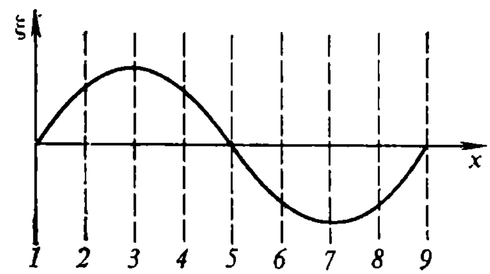

Fig. 4.4

4.20. Figure 4.4 shows a picture of the displacements in a standing wave for the instant t when the displacements reach their maximum.

1. What does the instantaneous value of the energy flux through each of surfaces *1*, *2*, *3*, . . ., *9* equal (zero or non-zero): (a) at the instant t; (b) at the instants following t?

2. What is the mean (in time) energy flux through the same surfaces?

3. How is Umov's vector **j** directed during the quarter of the period following the instant t for surfaces *2*, *4*, *6*, *8*?

4. The same question as in 3 for the following quarter of the period.

4.21. Find the nature of motion of the particles in an elastic medium in which two plane transverse waves propagate—one along the x-axis and the other along the y-axis. The oscillations in both waves occur along the z-axis. The lengths and amplitudes of both waves are the same and equal to λ and A. The difference in the initial phases of the waves is α.

4.22. Solve a problem similar to the preceding one for longitudinal waves instead of transverse ones.

4.2. Sound

4.23. At a certain tension of a string having a length of $l = 1.000$ m, the frequency of the string's fundamental tone is $\nu = 1000$ Hz. What is the speed v of propagation of a wave along the string in these conditions?

4.24. How will the frequency of the fundamental tone of a string change if:

(a) the middle of the string is pressed by a finger to the fingerboard;

(b) by changing the tension of the string, the speed of propagation of a wave along the string is increased three times?

4.25. In a string of mass m, the angular frequency of its fundamental tone is ω_1. The n-th harmonic is generated in the string ($n = 1$ corresponds to the fundamental tone). What is the amplitude A_a at the antinodes of the string if the mean kinetic energy of the string during a period of oscillations is $\langle E_k \rangle$?

4.26. The speed of propagation of a wave along a string is determined by the formula $v = \sqrt{F/\rho_{lin}}$, where F is the tension of the string, and ρ_{lin} is the linear density (mass per unit length) of the string. Determine the tension at which the fundamental tone of a steel string with a diameter of $d = 0.500$ mm and a length of $l = 0.500$ m will be A of the first octave ($\nu = 440$ Hz). Assume the density of steel to be $\rho = 7.80$ g/cm^3.

4.27. The maximum height of the sound reached by female singers is 2.35 kHz. With what force F would the string from the preceding problem have to be tightened for its fundamental tone to have such a frequency?

4.28. How will the frequency of the fundamental tone of a string change if the linear density of the string is doubled? Assume the tension of the string to remain constant.

4.29. A steel rod having a length of 1.00 m is fastened at its middle. Assuming Young's modulus to be 2.00×10^{11} Pa, find the frequency ν_n of the natural longitudinal vibrations of the rod. The density of steel is $\rho = 7.8$ g/cm^3.

4.30. A tube of length $l = 1.00$ m is closed at one end. Assuming the speed v of sound to be 340 m/s, determine the natural frequencies ν_n of the air vibrations in the tube.

4.31. The opening in the end of a lock key has a depth of $l = 17$ mm. If we blow near the end in a direction at right angles to the axis of the opening, sound vibrations will be produced in the column of air in the opening. What is the frequency ν of the fundamental tone of these vibrations?

4.32. What will a person hear if two sound waves with an approximately identical amplitude and with frequencies equal to: (a) 500 and 550 Hz; (b) 50 and 51 Hz; (c) 10 and 11 Hz act simultaneously on his ear?

4.33. Two strings, one 51.0 cm long and the other one 49.0 cm long, are made from wire one metre of which has a mass of 1.00 g. The strings are tensioned with the same force of $F = 20.0$ N. What will be the frequency $\Delta\nu$ of the beats that will appear if both strings are made to vibrate simultaneously?

4.34. In what gas at the same temperature is the speed v of sound greater—in nitrogen (N_2) or in carbon dioxide (CO_2)? How many times? The vibrational degrees of freedom of the gas molecules are not excited.

4.35. 1. Determine the speed of sound in air at a temperature of: (a) -40 °C; (b) 0 °C; (c) 40 °C.

2. Find the ratio of the calculated values, taking as unity the speed at 0 °C.

4.36. Let us assume that the temperature of the air varies with the height z according to a linear law from the value $T_1 = 300$ K at $z_1 = 0$ to $T_2 = 250$ K at $z_2 = 10.0$ km. Find the time t needed for a sound wave produced at the height z_2 to reach the Earth's surface?

4.37. How many times will the intensity of a wave diminish over the length l if the damping on this path is 30 dB?

4.38. A plane sound wave propagates along the x-axis in air. The sound absorption coefficient is $\varkappa = 2.07 \times 10^{-3}$ m^{-1}. In the plane $x = 0$, the sound loudness level is $L_0 = 100$ dB. Find the loudness L for x equal to: (a) 2.00 km; (b) 4.00 km; (c) 6.00 km; (d) 8.00 km; (e) 10.00 km.

4.39. A spherical sound wave propagates in air from an isotropic source. At a distance of $r_0 = 100$ m from the source, the sound loudness level is $L_0 = 100$ dB. Assuming that sound is not absorbed by the air, find the loudness level L at a distance of r equal to: (a) 2.00 km; (b) 4.00 km; (c) 6.00 km; (d) 8.00 km; (e) 10.00 km. Compare the result with the answer to the preceding problem.

4.40. An isotropic source produces a spherical sound wave with a frequency of 3 kHz in air. At a distance of $r_1 = 100$ m from the source, the sound loudness level is $L_1 = 60$ dB. Determine the loudness level L_2 at a distance of $r_2 = 200$ m and the distance r_0 at which the sound stops being heard: (a) assuming the sound absorption coefficient in air to be $\varkappa = 2.42 \times 10^{-3}$ m^{-1}; (b) disregarding absorption. Compare the results obtained.

4.41. Two sounds in a gas differ in their loudness level by $L_{12} = 20.0$ dB. Find the ratio of the amplitudes of the pressure oscillations $(\Delta p)_m$ for these sounds.

4.42. Find for sound of the frequency 3 kHz the amplitude of oscillations of the air pressure $(\Delta p)_m$ corresponding: (a) to the threshold of hearing; (b) to a loudness level of $L = 100$ dB. Assume that $T = 293$ K and $p = 1000$ hPa.

4.43. For a sound wave described by the equation

$$\xi = 1.00 \times 10^{-4} \cos (6280t - 18.5x)$$

where the factor of the cosine is expressed in m, the factor of t, in s^{-1}, and the factor of x, in m^{-1}, find:

(a) the amplitude of the velocity v_m of the medium's particles;

(b) the ratio of the amplitude A of displacement of the medium's particles to the wavelength λ;

(c) the ratio of the amplitude of velocity v_m of the particles to the speed v of wave propagation.

4.44. A source at rest emits a sound wave with a length of λ_0 in all directions. How will the wavelength change if the source is brought into motion at a speed equal to half the speed of sound?

4.45. An automobile travels at a speed of $v_1 = 60$ km/h along a straight highway. It is overtaken by a special vehicle travelling at a speed of $v_2 = 90$ km/h with its sound signal having a frequency of $\nu_0 = 1.00$ kHz switched on. A signal of what frequency ν will be heard by the automobile's passengers? Assume the speed v of sound to be 340 m/s.

4.46. Two electric trains run along a straight track section toward each other at the same speed of $v = 50$ km/h. When they meet, the drivers greet each other with prolonged whistles. The frequency of both signals is the same and equals $\nu_0 = 200$ Hz. What will a railwayman hear who is on the tracks at a certain distance from the meeting place of the trains? The temperature of the air is -10 °C.

4.47. Two electric trains run at the same speed of $v = 90$ km/h along a straight track one after the other with an interval of $l = 2.00$ km between them. At the instant when they are located symmetrically relative to point A at a distance of $b = 1.00$ km from the track (Fig. 4.5), both trains give a brief sound signal of the same frequency of $\nu = 500$ Hz. What will the nature of the sound be at point

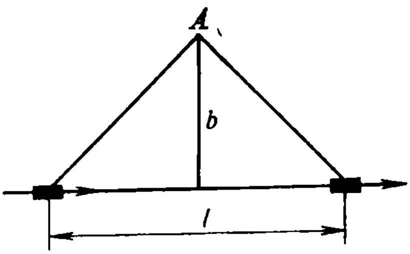

Fig. 4.5

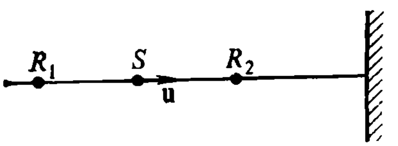

Fig. 4.6

A when the vibrations produced by the signals arrive at it? The speed of sound is $v = 350$ m/s.

4.48. Two automobiles travel one after the other along a straight section of a road at the same speed of 90 km/h. When a third vehicle travelling at a speed of 72 km/h in the opposite direction appeared at a distance, the driver of the leading automobile gave a long blast of his horn at a frequency of 700 Hz. A sound of what frequency will be heard by the occupants of the second and third vehicles? The temperature of the air is 30 °C.

4.49. Receivers R_1 and R_2, and also source S producing sound at a frequency of $\nu_0 = 1000$ Hz are arranged near a stationary wall in the order indicated in Fig. 4.6. The receivers are stationary, while the source moves toward the wall at a speed of $u = 8.5$ m/s. The speed of sound is $v = 340$ m/s.

(a) Which of the receivers will register beats?

(b) What is the frequency of these beats?

4.3. Electromagnetic Waves

4.50. A plane electromagnetic wave propagates along one of the coordinate axes in a vacuum. Write the possible expressions (in terms of the wave parameters and the unit vector of one of the axes) for the wave vector **k** if: (a) the vectors **E**

and $\mathbf{e}_y$ are collinear, the frequency of the wave is ω; (b) the vectors $\mathbf{H}$ and $\mathbf{e}_z$ are collinear, the wavelength is λ.

4.51. A plane electromagnetic wave propagates in a homogeneous and isotropic medium with $\varepsilon = 3.00$ and $\mu = 1.00$. The amplitude of the wave's electric field strength is $E_m = 10.0$ V/m. Find:

(a) the amplitude of the wave's magnetic field strength H_m;

(b) the phase speed v of the wave.

4.52. A plane electromagnetic wave propagating in a vacuum and described by the equations

$$\mathbf{E} = \mathbf{E}_m \cos(\omega t - kx), \quad \mathbf{H} = \mathbf{H}_m \cos(\omega t - kx)$$

is reflected without a loss in intensity from a plane perpendicular to the x-axis. Write equations describing the reflected wave.

4.53. Consider the superposition of two plane electromagnetic waves propagating along the x-axis in opposite directions. Determine:

1. The coordinates of the antinodes x_a and the nodes x_n for: (a) the electric vector $\mathbf{E}$ and (b) the magnetic vector $\mathbf{H}$ of the standing wave produced as a result of the superposition. To simplify the formulas, consider that the initial phase α in the equations of the forward and backward waves is zero. Compare the results obtained for $\mathbf{E}$ and $\mathbf{H}$.

2. What is the relation between the phases of oscillations of the vectors $\mathbf{E}$ and $\mathbf{H}$?

4.54. An electromagnetic wave of frequency ω propagates in a medium. The permittivity of the medium at the frequency ω is $\varepsilon = 2.00$, and the permeability is virtually equal to unity. Find the Poynting vector $\mathbf{S}$ for the point at which the electric vector varies according to the law $\mathbf{E} = 10.0 \cos(\omega t + \alpha)\, \mathbf{e}_z$ (V/m). The amplitude of the vector $\mathbf{H}$ has the form $H_m \mathbf{e}_x$.

4.55. A plane electromagnetic wave with ω of the order of 10^{10} s^{-1} propagates in a vacuum. The amplitude of the wave's electric vector is $E_m = 0.775$ V/m. A wave-absorbing surface having the shape of a hemisphere of radius $r = 0.632$ m is placed in the wave's path with its vertex facing the direction of propagation of the wave. What energy W will this surface absorb during the time $\tau = 1.00$ s?

4.56. A parallel-plate capacitor with round plates is charged with a steady current during the time τ to a voltage of U. The separation distance is d. Imagining that there is a cylindrical surface between the plates coaxial to them whose radius r is much smaller than the radius of the plates, determine:

(a) the magnitude and direction of the Poynting vector at points of the surface;

(b) the amount of energy W flowing through the surface during the time τ. Compare W with the energy of the electric field contained in the volume V confined by the surface after completion of the charging process.

4.57. The current in a very long solenoid increases uniformly from zero to I during the time τ. The number of solenoid turns per unit length is n. Imagining inside the solenoid in its middle part a closed surface coaxial to it of length l and radius r, determine:

(a) the magnitude and direction of the Poynting vector at points of the surface;

(b) the amount of energy W flowing through the surface during the time τ. Compare W with the energy of the magnetic field contained in the volume V confined by the surface after the current I has set in.

4.58. A plane electromagnetic wave propagates along the x-axis in a vacuum. The amplitude of the wave's magnetic field strength is $H_m = 0.0500$ A/m. Determine:

(a) the amplitude of the wave's electric field strength E_m;

(b) the time-averaged density $\langle w \rangle$ of the wave's energy;

(c) the intensity I of the wave;

(d) the time-averaged density of the wave's momentum $\langle \mathbf{K}_{u.v} \rangle$.

4.59. The wave described in the preceding problem is incident along a normal on the surface of a body completely absorbing the wave. What pressure p does the wave exert on the body?

4.60. A ferroelectric rod has a polarization of $\mathcal{P} = 0.050$ C/m^2 directed along its axis. The diameter of the rod is $d = 5.00$ mm, and its length is $l = 200$ mm. The rod is made to rotate about an axis perpendicular to it and passing through its centre at an angular velocity of $\dot{\varphi} =$

$= 314$ rad/s (3000 rpm). Find the wavelength λ and the power P radiated by the rod.

4.61. An electromagnetic wave emitted by an elementary dipole propagates in a vacuum. The amplitude of the electric field strength in the wave zone on a ray drawn from the dipole at right angles to its axis at a point located at a distance of $r = 1.00$ m from the dipole is $E_m = 1.00$ mV/m. Calculate the power P emitted by the dipole (i.e. the energy emitted by the dipole in unit time in all directions).

4.62. What part η of the entire power emitted by a dipole falls to the interval of angles θ from 70 to 110° (θ is the angle with the axis of the dipole)?

4.63. The radius of an electron's circular orbit in a betatron is $r = 15.0$ cm. At the end of an acceleration cycle, the speed of the electron reaches a value of $v = 0.999\,95c$. Find the power P emitted by the electron at this speed.

4.64. An electron travels in a uniform magnetic field in a plane perpendicular to the vector $\mathbf{B}$. The induction of the field is $B = 1.00$ T, the speed of the electron is $v = 1.00 \times 10^7$ m/s. Determine:

(a) the fraction η of its kinetic energy which the electron spends on radiation during one revolution;

(b) during what time τ the kinetic energy of the electron diminishes by 1%;

(c) the number of revolutions N completed by the electron during the time τ.

4.65. Solve the preceding problem, substituting a proton for the electron.

PART 5

OPTICS

SYMBOLS

A	amplitude of light wave; amplitude of oscillations; mechanical equivalent of light
a	amplitude; distance
b	distance; thickness; width
c	speed of light in a vacuum
D	dispersion of a diffraction grating
d	diameter; distance; period of a diffraction grating
$\mathbf{E}$	light vector
E	electric field strength; illuminance
F	first (front) focus of an optical system; force
F'	second (back) focus of an optical system
f	first focal length
f'	second focal length
H	first principal plane of an optical system; magnetic field strength
H'	second principal plane of an optical system
I	intensity of light; luminous intensity
j	flux density
$\mathbf{k}$	wave vector
L	luminance
l	distance; length
M	luminous emittance
m	integer; mass
N	nodal point of an optical system
n	concentration of particles; refractive index
P	degree of polarization
R	radius; resolving power of an optical instrument
r	distance; radius
u	group speed; speed
V	relative spectral sensitivity of eye; volume
v	parameter; phase speed; speed
λ_0	length of light wave in a vacuum
λ	length of light wave in a substance
ρ	coherence radius
ω	angular frequency

A thin lens is conditionally depicted as follows:

converging

diverging

5.1. Geometrical Optics. Photometry

5.1. How long will it take a light wave to travel a distance equal to: (a) the mean distance from the Sun to the Earth; (b) the mean distance from the Moon to the Earth; (c) the Sun's diameter; (d) the Earth's diameter?

5.2. Light having a wavelength of 665 nm in air has a wavelength of 500 nm in water. Does this mean that the colour perception by the human eye of this light in air and in water will differ?

5.3. A metre is defined as the length equal to 1 650 763.73 wavelengths in a vacuum of the radiation corresponding to the transition between the levels $2p_{10}$ and $5d_5$ of the krypton-86 atom. What is the wavelength λ of this radiation? In what colour region is this radiation?

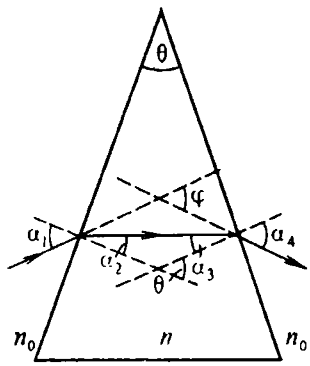

Fig. 5.1

5.4. A second is defined as the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom. What is the wavelength λ of this radiation?

5.5. Write the law of reflection of a light ray in the form of a relation expressing the unit vector $\mathbf{e}'$ of the reflected ray in terms of the unit vector $\mathbf{e}$ of the incident ray and the unit vector $\mathbf{n}$ of an outward normal to the reflecting surface.

5.6. Write the law of refraction of a light ray in the form of a relation expressing the unit vector $\mathbf{e}''$ of the refracted ray in terms of the unit vector $\mathbf{e}$ of the incident ray, the unit vector $\mathbf{n}$ of a normal to the refracting surface directed into the first medium, and the refractive index ν.

5.7. (a) Find an expression for the angle φ of deflection of a ray by a prism (Fig. 5.1), limiting yourself to the case when the prism angle θ is much smaller than 1 rad, while the angles α_1, α_2, α_3, α_4 are such that the sines of these angles may be replaced with sufficient accuracy by the angles themselves (if $\alpha = 5°$, then $\sin\alpha$ differs from α by

0.13%, and if $\alpha = 10^\circ$, by 0.5%). The refractive index of the prism is n, and that of the surrounding medium is n_0. It is assumed that the incident ray (and, consequently, the ray emerging from the prism) is in the principal section of the prism, i.e. in a plane at right angles to the refracting edge.

(b) What is noteworthy in the result obtained?

5.8. The second focal length f' of a lens is: (a) 200 mm; (b) —400 mm. What is the lens power Φ?

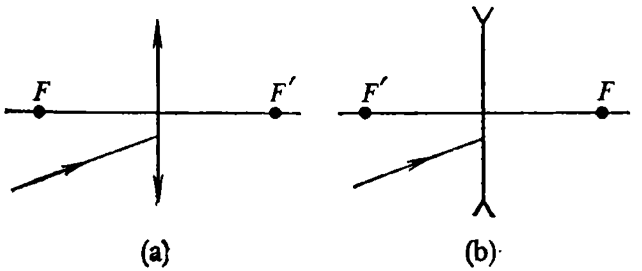

Fig. 5.2

5.9. In what case does a light ray pass through the centre of a thin lens without changing its direction?

5.10. Where are the nodal points N and N' of a thin lens if the medium at both sides of the lens is the same?

5.11. Construct the path of the ray after the thin lens (Fig. 5.2). The refractive index of the medium at both sides of the lens is the same.

5.12. Figure 5.3 shows a thin lens, its focal points F and F', and the coinciding nodal points N and N'. Construct the path of ray *1* after the lens.

5.13. Figure 5.4 shows a thin lens, its optical axis OO', conjugate rays *1* and *1'*, and also ray *2*. Construct ray *2'* that is the conjugate of ray *2* ($n = n'$).

5.14. A mirror is placed in a second focal plane of a thin converging lens (Fig. 5.5). By constructing the path of ray *1* in the space between the lens and the mirror, and then upon emergence from the lens, determine the relation between the direction of the ray, emerging from the lens to the left, and the direction of ray *1*.

5.15. Converging lens *1* and diverging lens *2* are arranged so that their second focal points F_1' and F_2' coincide (Fig. 5.6).

The medium between the lenses and at both sides of them is the same. An object is placed in the first focal plane of lens *1*. After constructing the image of the object, answer the following questions:

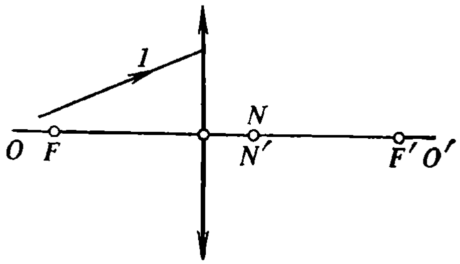

Fig. 5.3

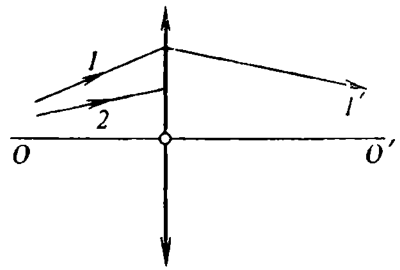

Fig. 5.4

1. Where will the image be?
2. Will the image be: (a) real or virtual; (b) erect or inverted?
3. In what case will the size of the image coincide with that of the object?

5.16. Figure 5.7 shows the axis, principal and focal planes of a centered optical system, and also an object. Planes *H*

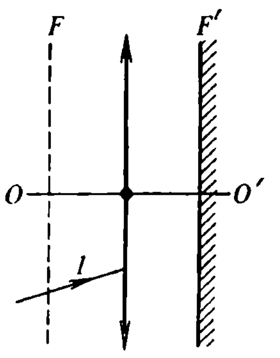

Fig. 5.5

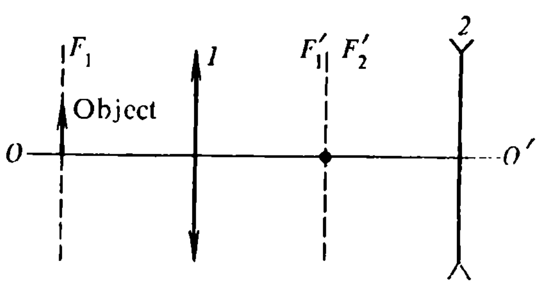

Fig. 5.6

and *H′* are outside the system.

(a) Construct the image of the object.

(b) On the basis of the properties of principal planes, answer the following question: what will happen to the image when the object is moved toward plane *H*?

5.17. Figure 5.8 depicts: optical axis OO' of a centered optical system, principal planes H and H', first focal point F, and ray *1*. The medium at both sides of the system is the same. Construct the conjugate ray *1'* of ray *1*.

5.18. A monochromatic light wave with $\lambda = 510$ nm is incident on a flat surface along a normal to it. The intensity of the wave is $I = 0.32$ W/m^2. Using the graph of the eye's

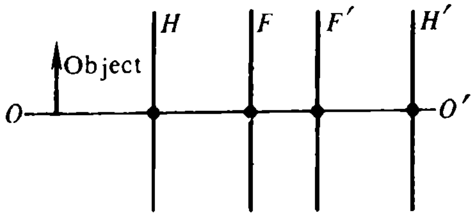

Fig. 5.7

relative spectral sensitivity shown in Fig. 5.9, determine the illuminance E of the surface. At $\lambda = 555$ nm, an energy flux equal to 0.001 60 W corresponds to a luminous flux of 1 lm. The quantity $A = 0.001\ 60$ W/lm is sometimes called the mechanical equivalent of light.

5.19. An energy flux equal to 0.001 60 W corresponds to a luminous flux of 1 lm formed by radiation with $\lambda = 555$ nm. What energy flux corresponds to a luminous flux of 100 lm formed by radiation for which the relative spectral sensitivity of the eye is $V = 0.762$?

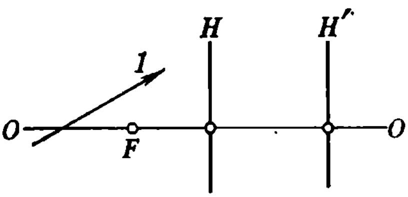

Fig. 5.8

5.20. What luminous flux corresponds to an energy flux of 1.00 W formed by radiation for which the relative spectral sensitivity of the eye is $V = 0.342$?

5.21. Assume that the energy flux associated with a light wave is distributed uniformly by wavelengths, i.e. $d\Phi_e/d\lambda =$ $=$ const. What would be the shape of the curve of luminous flux distribution by wavelengths in this case?

5.22. Assume that a luminous flux is distributed uniformly by wavelengths within the interval from 400 to 760 nm (Fig. 5.10).

(a) How would the graph of the distribution function of the luminous energy by wavelengths appear in this case?

(b) Is such a distribution possible?

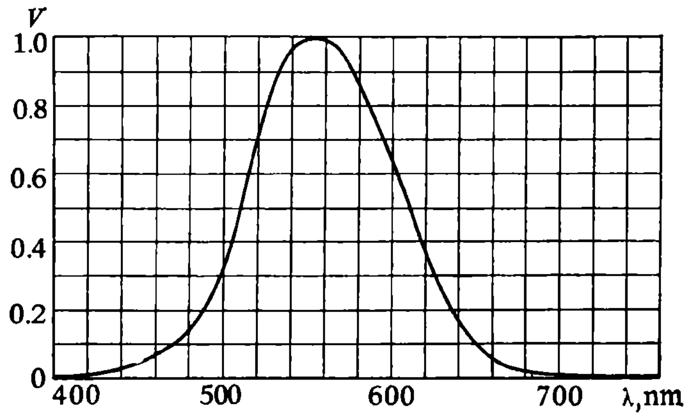

Fig. 5.9

5.23. A monochromatic light wave with $\lambda = 510$ nm upon normal incidence on a certain surface produces an illuminance of $E = 100$ lx. Determine the pressure p exerted by the light on the surface if half of the incident light is reflected.

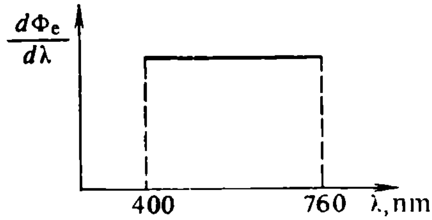

Fig. 5.10

5.24. The intensity (mean density of the luminous flux) of a monochromatic light wave is $I = 100$ lm/m^2. The frequency of the wave is $\omega = 3.69 \times \times 10^{15}$ s^{-1}. The refractive index of the medium in which the wave propagates is $n = = 1.50$, and the permeability is $\mu = 1$. Find the values of the amplitudes E_m and H_m of the electric and magnetic field strengths of this wave.

5.25. A point isotropic light source emits a flux of $\Phi = = 1257$ lm in all directions. What is the intensity I of the light of this source?

5.26. A parallel beam of rays carrying a uniform luminous flux of density $j = 200$ lm/m^2 is incident on a plane surface, an outward normal to which makes an angle of $\alpha = 120°$

with the direction of the rays. What is the illuminance E of this surface?

5.27. A point axisymmetric source hangs at a height of $h = 3.00$ m from the floor. Its light intensity is described by the function $I(\theta) = I_0 \cos^2\theta$ within the limits of $0 \leqslant \theta \leqslant \pi/2$ and is zero at $\theta > \pi/2$ (I_0 is a constant, and θ is the angle made by the light ray with the vertical). The illuminance of the floor under the source is $E = 100$ lx. Determine the luminous flux Φ emitted by the source.

5.28. A point isotropic light source is placed over the centre of a round table. The light intensity of the source is $I = 50.0$ cd, the radius of the table is $R = 0.500$ m, and the height of the source above the table is $h = 1.00$ m. Determine:

1. How the illuminance E of the table depends on the distance r from its centre.
2. The value of the illuminance: (a) at the centre; (b) at the edge of the table.
3. The luminous flux Φ incident on the table.
4. What fraction η of the total flux emitted by the source falls on the table.

5.29. How must the light intensity $I(\theta)$ of the source from the previous problem depend on the angle θ between the direction of the ray and a vertical for the flux of $\Phi = 33$ lm incident on the table (see the answer to item 3 of the preceding problem) to be distributed uniformly over the table's surface? What will the illuminance E of the table be? Compare the value of E with the answer to items 2(a) and 2(b) of the preceding problem.

5.30. The luminance of a uniformly luminescent flat surface is described by the function $L(\theta, \varphi)$ (θ is the angle made with a normal to the surface, and φ is the azimuth angle). Write an expression for the luminous emittance M of this surface.

5.31. A uniformly luminescent disk has a radius of $R = 10.0$ cm. The luminance of the disk is $L = L_0 \cos\theta$ (L_0 is a constant equal to 1.00×10^3 cd/m^2, and θ is the angle made with a normal to the surface). Find the luminous flux Φ emitted by the disk.

5.2. Interference of Light

5.32. What is the amplitude A of an oscillation that is the superposition of N incoherent oscillations of the same direction and the same amplitude a?

5.33. Two light waves produce oscillations of the same direction described by the functions

$$A \cos \omega t \quad \text{and} \quad A \cos [(\omega + \Delta\omega) t]$$

where $\Delta\omega = 0.628 \text{ s}^{-1}$, at a point of space. How does the intensity of the light at this point behave?

5.34. Find the intensity I of the wave formed by the superposition of two coherent waves polarized in mutually perpendicular directions. The values of the intensity of these waves are I_1 and I_2.

5.35. Light oscillations N in number and parallel to one another of the form

$$E_m = a \cos [\omega t + (m - 1)\delta] \quad (m = 1, 2, \ldots, N)$$

arrive at a certain point. Determine the amplitude A of the resultant oscillation by the method of graphical addition of the oscillations.

5.36. A source of light with a diameter of $d = 30.0$ cm is at a distance of $l = 200$ m from the place of observation. The radiation of the source contains wavelengths ranging from 490 to 510 nm. Appraise for this radiation:

(a) the coherence time t_{coh};
(b) the coherence length l_{coh};
(c) the coherence radius ρ_{coh};
(d) the coherence volume V_{coh}.

5.37. Appraise the coherence radius ρ_J of the light arriving at Jupiter from the Sun. Compare it with the coherence radius ρ_E of the light arriving at the Earth from the Sun. Assume the length of a light wave to be 500 nm.

5.38. The angular diameter of the star Betelgeuse (alpha Orion) is 0.047 angular second. What is the coherence radius ρ_{coh} of the light arriving at the Earth from this star?

5.39. The wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ of two plane coherent waves of the same intensity form the angle φ much smaller than unity. The waves are incident on a screen arranged so that the vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ are symmetrical relative to a normal to the screen. Determine the width Δx of the interference fringes observed on the screen.

5.40. What wavelength is understood in the expression for the phase difference δ of interfering light waves whose optical path difference is Δ ($\delta = 2\pi\Delta/\lambda$)—the wavelength in a vacuum or that in the medium in which the waves are propagating?

5.41. Figure 5.11 depicts an interference diagram with two identical luminescent slits. The oscillations from corresponding points of the slits (for example, from points adjoining the top edges of the slits, or from points at the

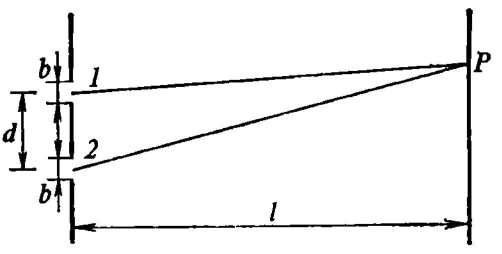

Fig. 5.11

middle of the slits, and so on) are coherent, whereas the oscillations from points at different distances from the top edges of the slits are incoherent.

(a) Assuming that the refractive index of the medium is unity, determine $\delta\Delta = \Delta_t - \Delta_b$, where Δ_t is the optical path difference to point P on the screen from the top edges of the 1st and 2nd slits, and Δ_b is the optical path difference to the same point P from the bottom edges of the 1st and 2nd slits.

(b) Appraise the maximum width b_{max} of the slits at which the interference fringes will still be distinguished sufficiently clearly.

5.42. First a red light filter and then a green one were placed in the path of white light in an interference arrangement. The transmission band $\Delta\lambda$ of both filters was the same. In what light—red or green—is the number of distinguishable interference fringes larger?

5.43. By what amount a will the optical path difference of interfering rays change when passing from the middle of one interference fringe to the middle of another one?

5.44. A laser beam with $\lambda = 632.8$ nm is incident along a normal on an obstacle with two narrow parallel slits.

A system of interference fringes is observed on a screen installed after the obstacle. In what direction and over what number of fringes will the interference pattern be shifted if one of the slits is covered with a transparent plate with a thickness of $a = 10.0$ μm made from a material with a refractive index of $n = 1.633$?

5.45. In an experiment similar to the one by means of which Young first determined the wavelength of light, a beam of sunrays passing through a light filter and a narrow slit in an opaque obstacle impinged on a second obstacle with two narrow slits spaced at $d = 1.00$ mm. A screen on which the interference fringes were observed was arranged after the obstacle at a distance of $l = 1.00$ m from it. The width of a fringe Δx was found to be: (a) 0.65 mm for red light and (b) 0.45 mm for blue light. What is the length λ_0 of a light wave?

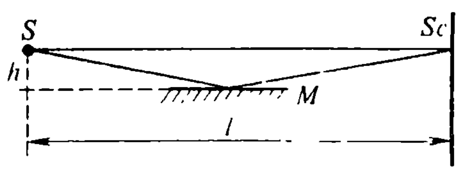

Fig. 5.12

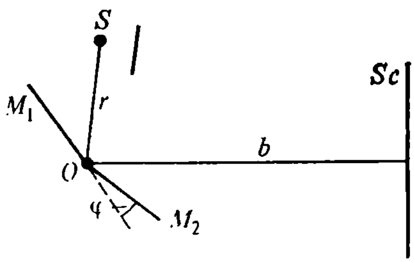

Fig. 5.13

5.46. In the arrangement proposed by Lloyd, the light wave falling on screen *Sc* directly from luminescent slit *S* interferes with the wave reflected from mirror *M* (Fig. 5.12). Let the distance from the slit to the plane of the mirror be $h = 1.00$ mm, the distance from the slit to the screen be $l = 1.00$ m, and the length of the light wave be $\lambda_0 = 500$ nm. Determine the width Δx of the interference fringes.

5.47. In the arrangement shown in Fig. 5.13 with Fresnel mirrors M_1 and M_2, S is a light source in the form of a slit perpendicular to the plane of the drawing, and *Sc* is a screen. The distance $r = 0.100$ m and $b = 1.00$ m. Determine:

(a) the value of the angle φ at which for $\lambda_0 = 500$ nm the width Δx of the interference fringes on the screen will be 1.00 mm;

(b) the maximum number N of fringes that can be observed in this case.

5.48. In the interference arrangement with a Fresnel biprism shown in Fig. 5.14, the distance from luminescent slit S to the biprism is $a = 0.300$ m, and the distance from the biprism to the screen is $b = 0.700$ m. The refractive

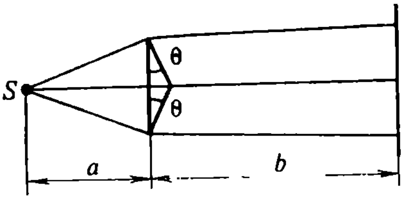

Fig. 5.14

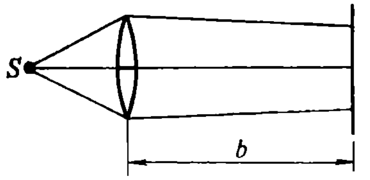

Fig. 5.15

index of the biprism is $n = 1.50$. Assuming $\lambda_0 = 500$ nm, determine:

(a) at what value of the prism angle θ the width Δx of the interference fringes observed on the screen will be 0.400 mm;

(b) the maximum number N of fringes that can be observed in this case.

5.49. A strip of width $h = 1.00$ mm was cut out from a thin lens with an optical power of $\Phi = +2.00$ D along its diameter. Next the lens parts formed were put together. A source S in the form of a luminescent slit emitting monochromatic light with $\lambda_0 = 500$ nm was placed in the focal plane of the bilens formed parallel to the slit (Fig. 5.15). A screen was placed after the bilens at a distance of $b = 1.00$ m from it. Determine:

(a) the width Δx of the interference fringes;

(b) the maximum number N of fringes that can be observed in this case.

5.50. A plane light wave of length λ_0 in a vacuum is incident along a normal on a transparent plate with a refractive index of n. At what thicknesses b of the plane will the reflected wave have (a) the maximum; (b) the minimum intensity?

5.51. Two beams of light have the same wavelength and the same intensity $I_0 = 100\ \mathrm{lm/m^2}$. One beam is emitted by a laser, and the other by a fluorescent lamp. Determine the intensity I of each of the beams after they pass through a plate about 1 mm thick with a refractive index of $n = 1.600$

if the thickness of the plate is: (a) $N\lambda$; (b) $(N + 1/4)\lambda$ (N is an integer, and λ is the wavelength in the plate). Disregard attenuation of the beams because of absorption in the plate.

5.52. Two plane interfaces of three transparent media are parallel to each other (Fig. 5.16). For a certain wavelength, the refractive indices of the first and third media are $n_1 = 1.20$ and $n_3 = 1.40$, respectively.

(a) At what value of the refractive index n_2 of the second medium will the fraction of the reflected light be the same for both surfaces?

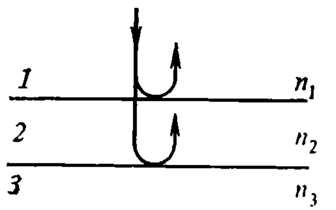

Fig. 5.16

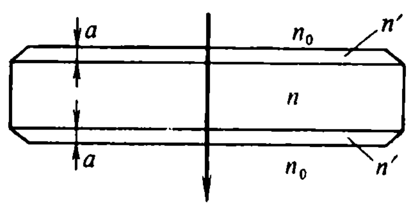

Fig. 5.17

(b) Will the fraction of light reflected from both surfaces upon the reverse path of the ray (from the third medium into the first one) also be the same at the found value of n_2?

5.53. A glass plate is covered at both sides with a film of a transparent substance (Fig. 5.17). For light with a wavelength in a vacuum of $\lambda_0 = 480$ nm, the refractive index of the plate is $n = 1.44$, that of the film is $n' = 1.20$, and that of the air, n_0, is virtually equal to unity. At what minimum thickness a of the films will light of the indicated wavelength pass through the plate without losses on reflection?

5.54. In Fig. 5.18, the letter S stands for a point source emitting light at $\lambda_0 = 600$ nm. Half of the luminous flux incident on half-silvered mirror HSM is reflected in the direction of two glass plates parallel to each other. By rotating micrometer screw MS, the lower plate can be moved, thus varying the gap b between the plates.

Half of the flux reflected by the plates, after passing through the half-silvered mirror, enters telescope T. What will be observed in the field of vision of the telescope if the gap between the plates is $b = 0.5$ mm, and the degree of

monochromaticity of the light characterized by the quantity $\lambda/\Delta\lambda$ is: (a) 500; (b) 5000?

5.55. What will happen to the picture observed in the field of vision of the telescope in case (b) of the preceding problem if, by smoothly rotating micrometer screw MS, (a) we increase; (b) we decrease the gap between the plates?

5.56. In the arrangement shown in Fig. 5.19, a plane light wave with $\lambda_0 = 600$ nm is incident on half-silvered mirror HSM. Half of the luminous flux is reflected along a direction

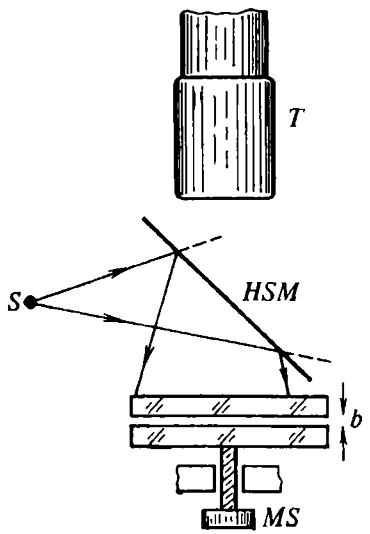

Fig. 5.18

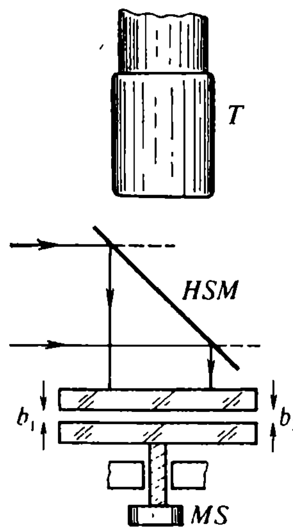

Fig. 5.19

toward glass plates installed at a small angle to each other. By rotating micrometer screw MS, the lower plate can be moved parallel to itself, thus varying gaps b_1 and b_2 between the edges of the plates by the same amount.

Half of the flux reflected by the plates, after passing through the half-silvered mirror, enters telescope T. What will be observed in the field of vision of the telescope if the gaps between the edges of the plates are $b_1 = 497$ μm, $b_2 = 503$ μm, and the degree of monochromaticity of the light $\lambda/\Delta\lambda$ is: (a) 500; (b) 5000; (c) 2500?

5.57. What will happen to the picture observed in the field of vision of the telescope in case (b) of the preceding prob-

lem if, by smoothly rotating micrometer screw MS, (a) we decrease; (b) we increase the gap between the plates?

5.58. A parallel beam of white light is incident at an angle of θ on a film with a thickness of $b = 367$ nm. The refractive index of the film is $n = 1.40$ (the variations in n depending on λ are confined within the limits of 0.01). What will be the colour of the light reflected by the film if θ equals: (a) 30°; (b) 60°?

5.59. A wedge-shaped plate of width $a = 100.0$ mm has a thickness of $b_1 = 0.358$ mm at one edge, and $b_2 = 0.381$ mm at the other. The refractive index of the plate is $n = 1.50$. A beam of parallel rays is incident on the plate at an angle of $\theta = 30°$ to a normal. The wavelength of the incident light is $\lambda_0 = 655$ nm (red light). Determine the width Δx of the interference fringes (measured in the plane of the plate) observed in reflected light for the case when the degree of monochromaticity of the light $\lambda/\Delta\lambda$ is: (a) 5000; (b) 500.

5.60. A vertical wire frame is covered with a soap film. When the film is illuminated with green light with $\lambda_0 = 530$ nm and a degree of monochromaticity of $\lambda/\Delta\lambda = 40$, interference fringes of equal thickness are observed on the upper part of the film. Appraise the thickness b of the film.

5.61. When a wedge-shaped transparent plate is illuminated with green light ($\lambda_0 = 550$ nm), 36 interference fringes of equal thickness are observed on a part of the plate (the remaining part of the plate is illuminated uniformly). What number N of fringes will be observed if the plate is illuminated with red light ($\lambda_0 = 660$ nm) instead of green light, and the degree of monochromaticity of the red light $\lambda/\Delta\lambda$ is 1.20 times less than that of the green light?

5.62. The angle α between the faces of a transparent wedge-shaped plate is 1.03′. The mean thickness of the plate is $b = 3.00$ mm, the length of the plate is $l = 100$ mm. Upon the normal incidence onto the plate of light having a wavelength of $\lambda_1 = 400.00$ nm in it, interference fringes of equal thickness are observed over half the length of the plate. Over what part of the plate will interference fringes be observed if the plate is illuminated with light having a wavelength of $\lambda_2 = 401.00$ nm whose degree of monochromaticity $\lambda/\Delta\lambda$ is the same as that of the initial light?

5.63. A plano-convex lens is placed with its convex side

onto a glass plate. Upon the normal incidence of red light (λ_0 — 610 nm) onto the flat boundary of the lens, the radius of Newton's fifth bright fringe is equal to $r_5 = 5.00$ mm.

Determine:

(a) the radius of curvature R of the convex boundary of the lens;

(b) the optical power Φ of the lens (assume the refractive index of the lens to be 1.50, and the lens to be thin);

(c) the radius r_3 of the third bright fringe.

5.64. How many times will the radius of Newton's m-th ring increase when the length of a light wave grows one-and-a-half times?

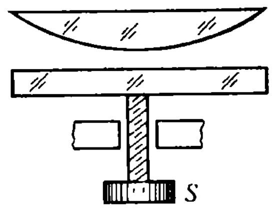

Fig. 5.20

5.65. A plano-convex lens with its convexity facing downward is rigidly fixed in place. A glass plate is under the lens at a small distance from it. The plate can be moved vertically by turning the head of screw S (Fig. 5.20). The pitch of the screw is $h = 100.0$ μm. The lens is illuminated from above with yellow light ($\lambda_0 = 580$ nm), and Newton's fringes are observed in the reflected light.

1. What will happen with the interference pattern if, by smoothly turning the screw, (a) we increase; (b) we decrease the gap between the lens and the plate?

2. What number N of new fringes will appear (and of old ones vanish) if the screw is turned through one revolution?

5.3. Diffraction of Light

5.66. A point source of light with $\lambda = 500$ nm is placed at a distance of $a = 0.500$ m in front of an opaque obstacle with an opening of radius $r = 0.500$ mm. Determine the distance b from the obstacle to the point for which the number m of Fresnel zones uncovered by the opening will be (a) 1; (b) 5; (c) 10.

5.67. A point source of light with $\lambda = 550$ nm is placed at a distance of $a = 1.00$ m in front of an opaque obstacle with an opening of radius $r = 2.00$ mm.

(a) What minimum number m_{min} of uncovered Fresnel zones can be observed in these conditions?

(b) At what value of the distance b from the obstacle to the point of observation is the minimum possible number of uncovered zones obtained?

(c) At what radius r of the opening can only one central Fresnel zone be uncovered in the conditions of the present problem?

5.68. There is a round hole in an opaque obstacle on which a plane light wave impinges. A screen is placed after the hole. What will happen to the intensity at the centre of the diffraction pattern observed on the screen if the latter is removed from the obstacle?

5.69. Proceeding from the definition of Fresnel zones, find the number m of Fresnel zones that are uncovered by an opening of radius r for a point at a distance of b from the centre of the opening if the wave incident on the opening is a plane one.

5.70. A monochromatic plane light wave is incident on an opaque obstacle with an opening of radius $r = 1.000$ mm. When the distance from the obstacle to a screen installed behind it is $b_1 = 0.575$ m, a maximum of the intensity is observed at the centre of the diffraction pattern. When the distance is increased to $b_2 = 0.862$ m, the maximum of intensity gives way to a minimum. Determine the wavelength λ of the light.

5.71. Prove the following equations:

(a) $$\sum_k a_k \cos(\omega_k t + \alpha_k) = \mathrm{Re}\left\{\sum_k a_k \exp[i(\omega_k t + \alpha_k)]\right\};$$

(b) $$\frac{d}{dt}[a\cos(\omega t + \alpha)] = \mathrm{Re}\left\{\frac{d}{dt}[a\exp(i(\omega t + \alpha))]\right\};$$

(c) $$\int a\cos(\omega t + \alpha)\,dt = \mathrm{Re}\left\{\int a\exp[i(\omega t + \alpha)]\,dt\right\};$$

(d) $$\frac{d}{dt}[a_1\cos(\omega_1 t + \alpha_1)] + \int a_2\cos(\omega_2 t + \alpha_2)\,dt$$

$$= \mathrm{Re}\left\{\frac{d}{dt}[a_1\exp(i(\omega_1 t + \alpha_1))] + \int a_2\exp[i(\omega_2 t + \alpha_2)]\,dt\right\};$$

(e) $$\frac{d}{dt}\sum_k a_k\cos(\omega_k t + \alpha_k)$$

$$= \mathrm{Re}\left\{\frac{d}{dt}\sum_k a_k\exp[i(\omega_k t + \alpha_k)]\right\};$$

$$\text{(f)}\ \sum_k \int a_k \cos(\omega_k t + \alpha_h)\, dt$$

$$= \mathrm{Re}\left\{\sum_k \int a_k \exp\left[i\left(\omega_k t + \alpha_k\right)\right] dt\right\}.$$

5.72. Assuming that the oscillation produced at a centre of a diffraction pattern from a round opening of the m-th Fresnel zone can be represented in the form

$$E_m = A_1 \rho^{m-1} \exp\left\{i\left[\omega t + (m-1)\pi\right]\right\}$$

where A_1 is the amplitude of the oscillation produced by the first zone, ρ is a number slightly less than unity (it should be borne in mind that the real part of this expression must be taken), determine the resultant amplitude of the oscillation produced by N Fresnel zones.

5.73. The intensity produced on a screen by a monochromatic light wave in the absence of obstacles is I_0. What will the intensity I at the centre of the diffraction pattern be if the path of the wave is intersected by an obstacle with a round opening uncovering: (a) the first Fresnel zone; (b) half of the first Fresnel zone; (c) one-and-a-half Fresnel zones; (d) a third of the first Fresnel zone?

5.74. How will the intensity change at a point opposite the centre of an opening if half of the opening is covered with a half-plane?

5.75. A large transparent plate is placed in the path of a light wave with $\lambda = 500$ nm. A round, cylindrical hollow facing the direction of wave propagation is made in the plate on an area corresponding to one-and-a-half Fresnel zones for a certain point of observation. The refractive index of the plate is $n = 1.500$. At what smallest depth of the hollow will the intensity of the light at the point of observation be: (a) maximum; (b) minimum; (c) equal to the intensity of the incident light?

5.76. The illuminance of a screen upon diffraction from a round opening is described by the function $E = E(r)$, where r is the distance from the centre of the diffraction pattern. Write an expression for the light flux Φ passing through the opening.

5.77. The radii of the circles bounding the opaque and transparent rings of an amplitude zone plate have the values

$r_m = \alpha\sqrt{m}$, where $\alpha = 1.000$ mm, $m = 1, 2, 3, \ldots$. Determine the principal focal length b of the plate for wavelengths λ equal to: (a) 400 nm; (b) 580 nm; (c) 760 nm. (The focal length of a zone plate is defined to be the distance from the plate to a point on its axis at which a maximum of the intensity is observed when a plane light wave impinges on the plate. The focal length corresponding to the maximum that is largest in value is called the principal one. Non-principal maxima are obtained if 3, 5, 7, ... Fresnel zones can be accommodated in the first zone drawn on the plate.)

5.78. Proceeding from the assumption made in Problem 5.72 and assuming that $\rho = 0.95$, appraise the intensity I at the focal point of the zone plate covering the even Fresnel zones. Express I in terms of the intensity I_0 in the absence of obstacles.

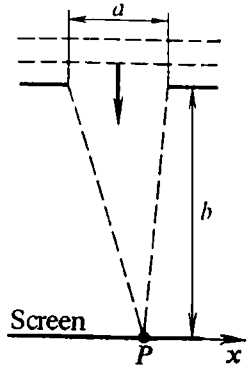

Fig. 5.21

5.79. Solve the preceding problem for a phase zone plate. Compare the result with the answer to the preceding problem.

5.80. A phase zone plate is made from a material with a refractive index of $n = 1.50$. What minimum height h must the projections over the even (or odd) zones of the plate have for a wavelength of $\lambda = 580$ nm?

5.81. An opaque plane with a very long ("infinite") slit of width a is placed in the path of a plane light wave with a wavelength of λ. A screen is placed after the obstacle at a distance of b from it (Fig. 5.21). Let us take point P of observation on the screen and divide the wave surface coinciding with the obstacle into straight Fresnel zones parallel to the edges of the slit (i.e. into zones for which the path difference from the edges to point P is $\lambda/2$). Place the inner boundary of the first zone opposite point P. Two symmetric systems of zones are obtained. We assign unprimed numbers 1, 2, ... to the zones to the right of P, and primed numbers 1′, 2′, ... to the zones to the left of P.

Determine:

1. The number m of unprimed and the number m' of

primed zones opened by the slit for point P on the screen opposite: (a) the middle; (b) the left edge; (c) the right edge of the slit. Compare the result obtained with the answer to Problem 5.69.

2. The coordinate x_m of the outer boundary of the m-th unprimed Fresnel zone (the x-axis is perpendicular to the edges of the slit, the values of x are read off from point P).

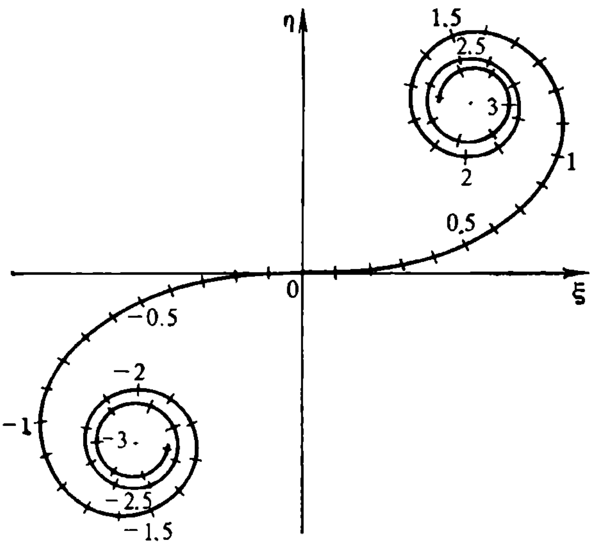

Fig. 5.22

3. The ratio of the values of the width Δx of the first five Fresnel zones.

5.82. Assuming in the preceding problem that $\lambda = 500$ nm, $a = 3.162$ mm, and $b = 1.000$ m, evaluate:

1. The numbers m and m' for point P opposite: (a) the middle; (b) the left edge; (c) the right edge of the slit.

2. The coordinate x_m of the right boundary for the first five Fresnel zones.

5.83. Figure 5.22 shows a Cornu spiral. It allows the method of vector addition of oscillations to be used for determining the amplitude of the light oscillation produced at point P of observation by various sections of a wave surface at a distance of b from point P (Fig. 5.23). The wave surface is divided into infinitely long elementary zones dS of width dx that are perpendicular to the x-axis drawn through P.

The amplitude dA produced by the zone dS is determined by the element dl of the Cornu spiral (the magnitude of dl is proportional to the width dx of the zone). The distance to this element from the origin of coordinates measured along the spiral is characterized by the value of the dimensionless parameter v. The numbers of the spiral are the values of this parameter. The relation between the value of v (determining the position of a point on the Cornu spiral) and the coordinate x (determining the position of the zone dS relative to point P) for a plane wave is established by the expression

$$v = x\sqrt{2/b\lambda}$$

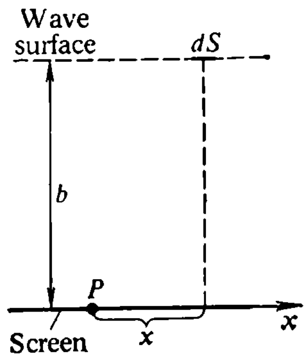

Fig. 5.23

(λ is the wavelength).

At points for which v has the values of 1, 3, 5, . . ., a tangent to the Cornu spiral is parallel to the η-axis; at points for which v has the values of 2, 4, 6, . . ., a tangent is parallel to the ξ-axis.

1. What points of the Cornu spiral correspond to the boundaries between Fresnel zones?

2. What value of the parameter v corresponds to the boundary between the 2nd and 3rd: (a) unprimed; (b) primed Fresnel zones?

5.84. Using the answer to Problem 5.81 and the formula in the condition of the preceding problem:

(a) determine the value of the parameter v corresponding to the outer boundary of the m-th Fresnel zone;

(b) evaluate the values of v for $m = 1, 2, \ldots, 5$.

5.85. In the absence of obstacles, the intensity produced by a plane light wave incident on a screen along a normal is I_0. Use the Cornu spiral to determine the intensity I at point P on the screen produced: (a) only by unprimed (or primed) Fresnel zones; (b) by the 1st unprimed (or primed) zone; (c) by all the primed (or unprimed) zones except the 1st; (d) by the 2nd zone; (e) by the 1st and 2nd zones; (f) by the 2nd and 3rd Fresnel zones.

5.86. A system of diffraction fringes is formed at the boundary of the shadow produced on a screen by a half-plane. Assuming the wavelength to be $\lambda = 580$ nm, the distance between the half-plane and the screen to be $b = 20.0$ cm, and the intensity of the incident wave to be

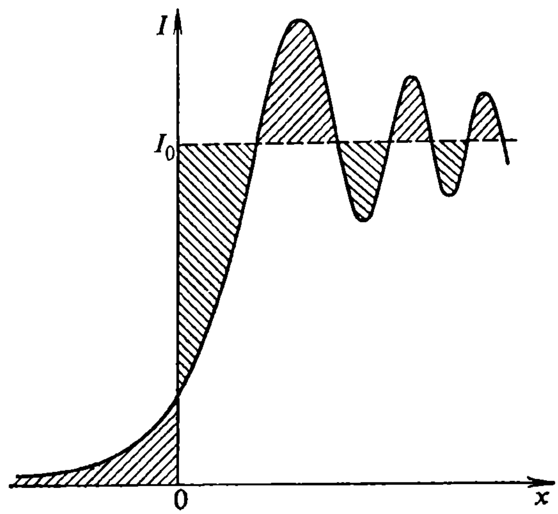

Fig. 5.24

$I_0 = 100$ lm/m^2, determine:
(a) the intensity I_{max} of the 1st diffraction maximum;
(b) the intensity I_{min} of the minimum following it;
(c) the ratio I_{max}/I_{min};
(d) the approximate values of the coordinate x for the middle of the 1st maximum and the middle of the 1st minimum measured from the edge of the geometric shadow.

5.87. Figure 5.24 contains a plot of the intensity of light upon Fresnel diffraction from the edge of a half-plane. What is the ratio of the summary areas indicated by hatching having a different slope?

5.88. A plane light wave with $\lambda = 500$ nm is incident along a normal on a slit of width $a = 2.00$ mm at a distance of $b = 2.00$ m from a screen. In the absence of obstacles, the wave would produce an illuminance of $E_0 = 100.0$ lx on the screen. Determine the illuminance E at point P on the screen arranged: (a) opposite the middle; (b) opposite the edge of the slit.

5.89. In what phase relation is the oscillation produced by all the primed Fresnel zones with the oscillation produced by the second unprimed zone?

5.90. A very long opaque band of width $a = 1.90$ mm was placed at a distance of $b = 1.50$ m from a screen in the path of a plane light wave with $\lambda = 600$ nm incident on the screen. In the absence of the band, the illuminance of the screen is $E_0 = 300$ lx. Determine the illuminance E at point P located: (a) opposite the middle; (b) opposite the edge of the band.

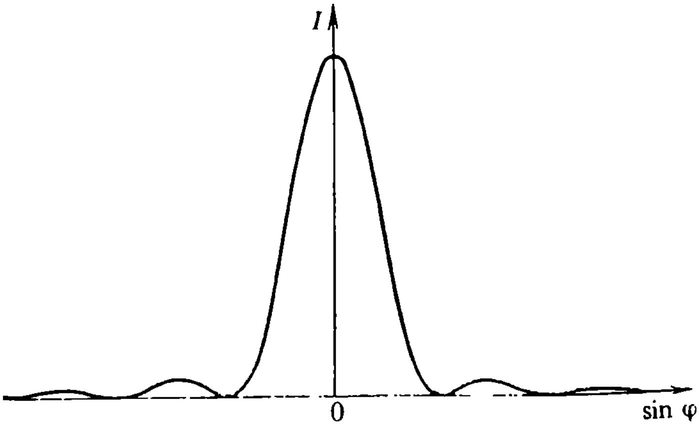

Fig. 5.25

5.91. Figure 5.25 depicts a curve of the light intensity for Fraunhofer diffraction from a slit.

1. What is the meaning of the area confined by the curve?
2. How will the following change when the width of the slit is doubled: (a) the height of the diffraction maxima; (b) the width of the maxima; (c) the position of the minima; (d) the number of observed minima; (e) the area confined by the curve?

5.92. White light falls along a normal on a slit of width $b = 0.10$ mm. Behind the slit there is a lens in whose focal plane a screen is placed. The optical power of the lens is $\Phi = +5.0$ D. Appraise:

(a) the width a of the iridescent fringe at the boundary of the central diffraction maximum observed on the screen;

(b) the ratio of the width a of the fringe to the mean width $\langle \Delta x \rangle$ of the central maximum.

5.93. A plane light wave falls on an opaque flat obstacle containing a slit of width $b = 0.200$ mm. A screen is installed after the obstacle. The wave surfaces, obstacle, and screen are parallel to one another. The distance between the obstacle and the screen is $l = 1.00$ m. The wavelength is $\lambda = 500$ nm. The refractive index of the medium is virtually equal to 1. The conditions of coherence are observed. Find:

(a) the kind of diffraction that is observed in this case;

(b) the width a_0 of the central diffraction maximum;

(c) the distance a_{12} between the middles of the 1st and 2nd diffraction maxima.

5.94. What kind of diffraction will be observed in the conditions of the preceding problem if the width of the slit is increased to 1.0 mm?

5.95. Will the diffraction pattern from a slit move along the screen when the slit is moved parallel to itself if the diffraction is observed: (a) with the aid of a lens; (b) without a lens? Assume the light to fall on the slit along a normal.

5.96. How will the intensity of the light at the middle of a diffraction pattern from a slit behave when the width of the slit is increased in the case of: (a) Fresnel diffraction; (b) Fraunhofer diffraction?

5.97. Construct an approximate graph showing how the intensity I depends on $\sin\varphi$ for a diffraction grating with $N = 5$ slits and with a ratio of the grating period to the slit width of $d/b = 2$.

5.98. Figure 5.26 shows the principal maxima of the intensity produced by a diffraction grating with a large number of slits.

1. What is the meaning of the total area of the maxima?
2. Additional slits are made in the spaces between adjacent slits. How will the following change in this case:

(a) the position of the maxima;

(b) the height of the central maximum;

(c) the width of the maxima;

(d) the total area of the maxima?

5.99. What will happen to a diffraction pattern if every other slit of the diffraction grating is covered?

5.100. Half of a diffraction grating is covered at one edge by an opaque obstacle as a result of which the number of slits is halved. How will the following change:

(a) the positions of the diffraction maxima;
(b) the height of the central maximum;
(c) the width of the maxima;
(d) the total area of the maxima?

The coherence radius of the light falling on the grating is assumed to be much larger than the length of the grating.

5.101. Answer the questions of the preceding problem provided that the coherence radius of the light falling on the grating is much smaller than the grating length.

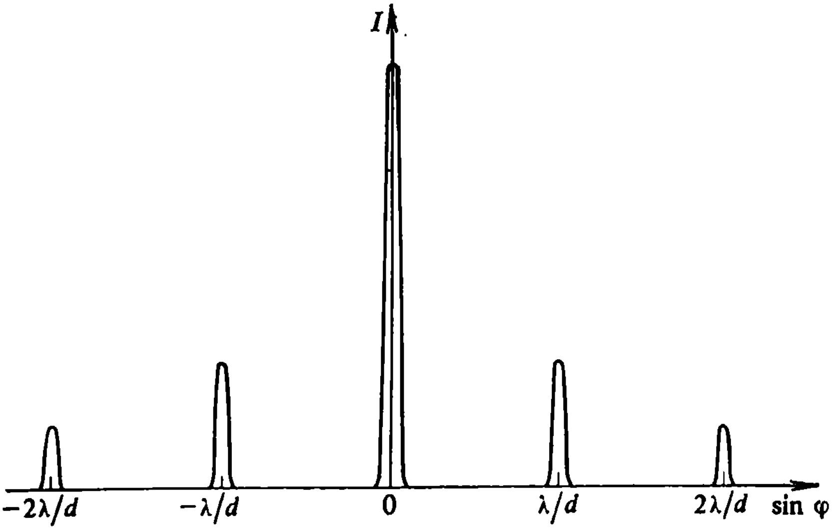

Fig. 5.26

5.102. 1. Determine the value of the ratio $x = b/d$ (d is the period of the diffraction grating, and b is the width of a slit) at which a diffraction maximum of the m-th order will have: (a) the maximum intensity; (b) an intensity equal to zero. The intensity of the light incident on the grating and the period d of the grating are assumed to be known.

2. Find the values of x corresponding to a maximum of the intensity for a maximum: (a) of the first; (b) of the second; (c) of the third order.

3. Find the values of x at which the intensity becomes equal to zero for a maximum: (a) of the first; (b) of the second; (c) of the third order.

5.103. In a spectrum produced by a diffraction grating with a period of $d = 2300$ nm, only two maxima (apart from the central one) are seen at $\lambda = 500$ nm. What is the width b of the slits of this grating?

5.104. Limiting yourself to the case of normal incidence of light on a grating, appraise the maximum possible value of the angular dispersion D of a diffraction grating about which it is known that one of the maxima for $\lambda_1 = 550$ nm is superposed onto one of the maxima for $\lambda_2 = 660$ nm. We have in view the dispersion in a spectrum of the first order.

5.105. The minimum value of the angular dispersion of a diffraction grating is $D = 1.266 \times 10^{-3}$ rad/nm. Find the angular distance $\Delta\varphi$ between the lines with $\lambda_1 = 480$ nm and $\lambda_2 = 680$ nm in the spectrum produced by the grating. The light is assumed to fall normally on the grating.

5.106. The light falling normally on a diffraction grating consists of two sharp spectral lines with wavelengths of $\lambda_1 = 490$ nm (sky-blue light) and $\lambda_2 = 600$ nm (orange light). The first diffraction maximum for a line with a wavelength of λ_1 is at an angle of $\varphi_1 = 10.0°$. Find the angular distance $\Delta\varphi$ between the lines in a spectrum of the second order.

5.107. Will spectral lines with $\lambda_1 = 598$ nm and $\lambda_2 = 602$ nm be resolved by a diffraction grating with $N = 100$ slits in a spectrum: (a) of the first order; (b) of the second order?

5.108. What number N of slits must a diffraction grating have to resolve in a first-order spectrum the lines of the yellow doublet (the double yellow spectral line) of sodium whose wavelengths are 589.0 and 589.6 nm?

5.109. 1. Assuming that light is incident normally on the grating, obtain an exact expression for the angular dispersion D of a diffraction grating depending on λ.

2. Putting the period d of the grating equal to 1000 nm, use the found formula to evaluate the angular dispersion in a spectrum of the first order in the vicinity of the wavelengths: (a) 400 nm; (b) 580 nm; (c) 760 nm.

3. Compare the results obtained with the value of D calculated by the approximate formula $D \approx m/d$ (m is the order of the spectrum).

5.110. 1. Assuming that light falls normally on the grating, obtain an accurate expression for the linear dispersion D_{lin} of a diffraction grating depending on λ,

2. Putting the period d of the grating equal to 1000 nm, and the focal length of the lens to be $f = 1.000$ m, use the found formula to calculate the linear dispersion in a spectrum of the first order near the wavelengths: (a) 400 nm; (b) 580 nm; (c) 760 nm.

3. Compare the results obtained with the value of D_{lin} calculated by the approximate formula $D_{\text{lin}} \approx mf/d$ (m is the order of the spectrum).

5.111. A diffraction grating whose period is $d = 1000$ nm and the length of whose working part is $l = 100.0$ mm is installed in a spectrograph normally to the incident light beam. The focal length of the spectrograph's objective is $f = 1.000$ m.

1. Determine the length Δx of the visible spectrum obtained on a photographic plate installed in the focal plane of the objective.

2. Appraise: (a) the linear dispersion D_{lin}; (b) the resolving power R of the instrument.

5.112. We have a diffraction grating on which a light beam falls normally.

1. Assuming that the number m determining the order of the diffraction maximum changes continuously, obtain a formula for $\delta\varphi/\delta m$ depending on m. This formula, when $m = k + 1/2$ is introduced into it, yields an approximate value of the angular distance between the k-th and $(k + 1)$-th maxima.

2. Assuming that $\lambda/d = 0.3$, calculate the exact value of the angular distance $\Delta\varphi$ between: (a) the first and the second; (b) the second and the third maxima. Compare the result with the approximate values found by the formula obtained in item 1.

5.113. Why must the period d of transparent diffraction gratings be of the order of the wavelength λ and cannot be, for example, equal to 1 mm?

5.114. Light with a wavelength of $\lambda = 600$ nm falls on a diffraction grating with a period of $d = 2500$ nm at an angle of $\varphi_0 = 20.0°$ to a normal. Assuming that the angles measured counterclockwise from the normal are positive, and clockwise are negative (we shall note that φ_0 is positive):

1. Obtain a formula determining the angular positions φ of the principal maxima.

2. Find:

(a) the angle φ at which the central (zero) maximum is obtained;

(b) the angles φ_+ at which the positive maxima are obtained, and the angles φ_- at which the negative maxima are observed;

(c) the number m_+ of observed positive maxima, and the number m_- of observed negative maxima.

3. Compare the total number of maxima with the number of maxima that would be obtained with normal incidence of light on the grating.

5.115. Light with a wavelength of λ falls on a reflecting diffraction grating with a period of d at the glancing angle θ_0. (The glancing angle is the angle supplementing the angle of incidence.)

1. Obtain a formula determining the glancing angles θ at which the principal diffraction maxima are obtained.

2. Determine the angles θ at which diffraction maxima appear: (a) of the first; (b) of the second; (c) of the fifth order for the case when $d = 1.00$ mm, $\lambda = 500$ nm, and $\theta_0 = 1.00°$.

5.116. 1. Obtain an expression for the angular dispersion $D = d\varphi/d\lambda$ of a diffraction grating for the case when light falls on the grating at the angle φ_0 to a normal. Compare the result obtained with the answer to Problem 5.109, item 1.

2. Assuming that the period of a grating is $d = 2250$ nm and the wavelength is $\lambda = 500$ nm, evaluate D in a first-order spectrum for the case: (a) $\varphi_0 = 0$; (b) $\varphi_0 = 30°$; (c) $\varphi_0 = 50°$; (d) $\varphi_0 = 51°$.

5.117. Is it possible to distinguish with the naked eye two posts at a distance of 2 km spaced 1 m apart? Assume the diameter of the pupil to be 4 mm.

5.118. A telescope is used for viewing the Moon's surface. The diameter of the telescope's objective is $d = 4.00$ cm. At what minimum distance $a_{\min}$ between two craters can they be seen separately? Assume that the wavelength of light is 600 nm.

5.119. In a raster picture, the image is formed by dots of various saturation (i.e. various "heaviness"). Beginning from what distance l will the eye stop distinguishing individual dots in the picture and will the latter appear as a continuous transition from lighter places to darker ones if the number of dots per cm^2 is 2500? Assume that the diameter

of the pupil is 4.0 mm and the wavelength is 600 nm.

5.120. The density of table salt (NaCl) is $\rho = 2.163$ g/cm³. Proceeding from the fact that an elementary crystal cell of salt has the form of a cube whose corners alternately accommodate sodium and chloride ions, find the distance d between the atomic planes parallel to the natural faces of the crystal.

5.121. The British physicists W. H. and W. L. Bragg (father and son) were the first to measure in 1913 the wavelength of X-rays. On the basis of the fact that, as was established by crystallographers, table salt belongs to the cubic system, the Braggs calculated the distance d between the atomic planes parallel to the outer faces of a crystal (see the preceding problem). By next measuring the angles at which diffraction maxima appear upon reflection from these planes, the Braggs determined the wavelength λ of the X-ray radiation they had used. Particularly, for the monochromatic radiation emitted by a palladium anticathode (K_α of palladium), maxima of the intensity were obtained at glancing angles of 5°59′, 12°3′, and 18°14′. Find the wavelength λ of this radiation.

5.122. A narrow beam of X-radiation with $\lambda = 0.0214$ nm (K_α of tungsten) falls on a polycrystalline specimen of copper. A photographic plate is installed behind the specimen at a distance from it of $l = 100.0$ mm. Find the radii of the rings formed on the plate at the expense of the diffraction maxima of the first and second orders appearing upon reflection from the atomic planes parallel to the faces of a unit crystal cell. A unit copper cell is cubic face-centered.

5.4. Polarization of Light

5.123. How does the light vector **E** behave at a fixed point of space in the case of an elliptically polarized wave?

5.124. What is the degree of polarization P of light that is a mixture of natural light and plane-polarized light if the ratio of the intensity of the polarized light to that of the natural light is: (a) 1; (b) 10?

5.125. Plane-polarized light of intensity $I_0 = 100$ lm/m² passes consecutively through two perfect polarizers whose planes make the angles $\alpha_1 = 20.0°$ and $\alpha_2 = 50.0°$ with the plane of oscillations in the initial ray (the angles are measured from the plane of oscillations clockwise when looking

along the ray). Determine the intensity I of the light after it leaves the second polarizer.

5.126. What is a quarter-wave plate?

5.127. How can circularly polarized light be obtained?

5.128. Can circularly polarized light be obtained with the aid of a plate whose "thickness" is other than a quarter wave?

5.129. The light vector **E** of a plane wave varies according to the law $\mathbf{E} = \mathbf{E}_0 \cos(\omega t - kx)$, the vector $\mathbf{E}_0$ making the angles α and $(\pi/2 - \alpha)$ with the y- and z-axes, respectively. Write expressions for the components of the vector **E** along the y- and z-axes.

5.130. A plane-parallel layer of a homogeneous anisotropic dielectric in which the components $\mathbf{E}_y$ and $\mathbf{E}_z$ propagate at different speeds is placed in the path of the light wave from the preceding problem. Write expressions for the components $\mathbf{E}'_y$ and $\mathbf{E}'_z$ upon emergence from the layer.

5.131. How does the resultant light vector $\mathbf{E}'$ with the components $\mathbf{E}'_y$ and $\mathbf{E}'_z$ (see Problems 5.129 and 5.130) behave if: (a) $\alpha = 0$; (b) $\alpha = 90°$; (c) $\alpha = 30°$, $\delta = \pi$; (d) $\alpha = 30°$, $\delta = \pi/6$; (e) $\alpha = 45°$, $\delta = \pi/6$; (f) $\alpha = 30°$, $\delta = \pi/4$; (g) $\alpha = 45°$, $\delta = \pi/4$?

5.132. A crystal quarter-wave plate is placed in the path of plane-polarized monochromatic light. What changes will the light emerging from the plate experience when the plate is rotated about the direction of the ray?

5.133. Circularly polarized light whose intensity is I_0 is incident on a perfect polarizer. What will the intensity I of the light after the polarizer be?

5.134. A crystal quarter-wave plate is installed between two crossing perfect polarizers. How will the intensity of the light emerging from the second polarizer change when the plate is rotated about the direction of the ray if natural light of intensity I_{nat} falls on the first polarizer?

5.135. A crystal half-wave plate is placed between two perfect polarizers. Natural monochromatic light of intensity I_{nat} with a wavelength corresponding to the plate is incident on the first (in the direction of the ray) polarizer. The optical axis of the plate makes the angle $\alpha = 30°$ with the vertical. The first polarizer is secured in a position in which its plane is vertical. The second polarizer can rotate. Determine the intensity I of the light emerging from the second polarizer

for cases when the planes of the polarizers: (a) are parallel; (b) are perpendicular to each other.

5.136. Natural light of intensity I_{nat} passes consecutively through polarizer P_1, a crystal half-wave plate, and polarizer P_2 (Fig. 5.27). The axis of the plate is vertical. The axes of polarizers P_1 and P_2 make the angles α_1 and α_2, respectively, with the vertical (the angles are measured from the vertical clockwise when looking along the ray). Considering that the

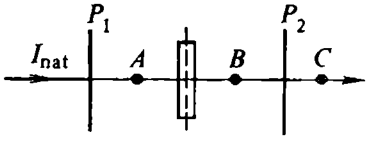

Fig. 5.27

plate absorbs no light and that the polarizers are perfect, determine:

1. The nature of the light at points A, B, C.
2. The intensity I_A and I_B of the light at points A and B.
3. The intensity I_C of the light at point C if $\alpha_1 = 30°$, and the angle α_2 is: (a) $60°$; (b) $150°$.
4. The intensity I_C of the light at point C with arbitrary angles α_1 and α_2.

5.137. In a system similar to that depicted in Fig. 5.27 (see the preceding problem), the half-wave plate has been replaced with a 3/8-wave one. Determine:

(a) The nature of the light at points B and C.

(b) The intensity I_B of the light at point B.

(c) Can the intensity I_C of the light at point C be zero at any values of the angles α_1 and α_2 from the intervals $10° \leqslant \alpha_1 \leqslant 80°$ and $0 \leqslant \alpha_2 \leqslant 90°$?

(d) Can I_C be equal to $I_{nat}/2$?

5.138. An imperfect polarizer passes in its plane $\alpha_1 = 0.90$ part of the intensity of the relevant oscillation, and in a perpendicular plane $\alpha_2 = 0.10$ part of the intensity of the relevant oscillation. Determine the degree of polarization P of the light passing through the polarizer if the light initially was natural.

Note. Natural light can be represented as the superposition of two incoherent waves of the same intensity polarized in

mutually perpendicular planes. With such a representation, the intensity of natural light equals the sum of the intensities of these waves.

5.139. Each of two identical imperfect polarizers provides a degree of polarization of $P_1 = 0.800$. What will be the degree of polarization of the light passing consecutively through both polarizers if the planes of the polarizers are: (a) parallel; (b) perpendicular to each other?

5.140. Natural light passes through a system of two identical imperfect polarizers. Each of them passes in its plane $\alpha_1 = 0.95$ part of the intensity of the relevant oscillation and provides a degree of polarization of $P = 0.90$. What fraction of the initial intensity of the light will be formed by the intensity of the light passing through this system if the planes of the polarizers are mutually perpendicular (the polarizers are crossed)?

5.141. Natural light is passed through two identical imperfect polarizers positioned one after the other. The intensity of the light that has passed through this system with parallel planes of the polarizers ($I_{\parallel}$) exceeds the intensity with mutually perpendicular planes ($I_{\perp}$) $\eta = 9.53$ times. Determine:

(a) the degree of polarization P_1 of the light that has passed through only one of the polarizers;

(b) the degree of polarization $P_{\parallel}$ produced by the system with parallel planes of the polarizers.

5.142. A birefringent wedge-shaped plate whose optical axis is parallel to the edge of the wedge is placed in the path of plane-polarized monochromatic light. The axis makes an angle of 45° with the plane of oscillations in the incident light. What will be the nature of the light after the plate?

5.143. 1. How will the surface of the wedge-shaped plate from the preceding problem appear if it is examined from its back side through a polarizer whose plane is: (a) parallel to the axis of the plate; (b) parallel to the plane of oscillations in the incident light; (c) perpendicular to the plane of oscillations in the incident light?

2. What will happen to the picture being examined if the polarizer is turned through 90° from the position in which its plane coincides with the plane of oscillations in the incident light?

5.144. A wedge-shaped plate cut out from Iceland spar so that the optical axis of the plate is parallel to the edge

of the wedge is between two crossed polarizers. The wedge angle is $\theta = 4.72'$. The axis of the plate makes angles of 45° with the planes of the polarizers. Find the distance Δx between the middles of the bright fringes observed after the second polarizer when light with $\lambda = 486$ nm passes through the system. For this wavelength, the refractive indices of Iceland spar for the ordinary and extraordinary rays are $n_o = 1.668$ and $n_e = 1.491$.

5.145. A vessel filled with nitrobenzene accommodating the plates of a parallel-plate capacitor (Fig. 5.28; such a device is known as a Kerr cell) acquires the properties of a birefringent crystal with its optical axis parallel to the field strength in the capacitor when a voltage is applied across the latter. The difference between the refractive indices of an extraordinary and an ordinary ray is proportional to the square of the field strength E. Consequently, when light passes through the cell, a phase difference δ appears between the component of the light vector parallel to the field and its component perpendicular to the field. This phase difference is proportional to E^2 and the length l of the capacitor along the direction of the ray, which is usually written as $\delta = 2\pi B l E^2$, where B is a characteristic of a substance called the Kerr constant. The latter depends on the wavelength of the light and on the temperature. The Kerr constant for nitrobenzene at room temperature for $\lambda_0 = 600$ nm is $B = 2.2 \times 10^{-12}$ m/V².

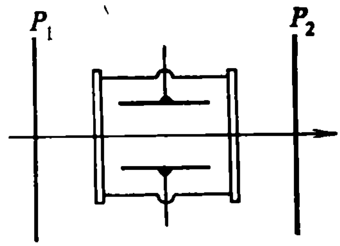

Fig. 5.28

Let us place a Kerr cell between crossed polarizers and arrange it so that the "optical axis" of the cell (i.e. the direction of the electric field in it) makes an angle of 45° with the planes of the polarizers. Assuming that $l = 10.0$ cm, determine:

(a) the minimum value E_{min} of the field strength at which the system will pass the maximum fraction of the light falling on it;

(b) the number of times the cell becomes bright and dark during the time it takes the field strength to increase from zero to 3.38×10^6 V/m.

5.146. An alternating voltage with a frequency of $\nu = 50$ Hz is supplied to the system comprising a Kerr cell and two crossed polarizers described in the preceding problem. The amplitude value of the strength of the electric field produced is $E_m = 3.38 \times 10^6$ V/m.

(a) How many times will such a light shutter interrupt the light in one second?

(b) Will the time intervals between the interruptions be the same?

5.147. White light polarized in a vertical plane passes through a right-hand quartz plate with a thickness of $a =$

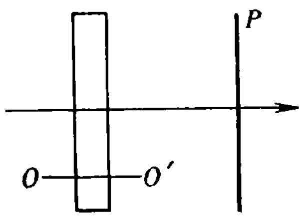

Fig. 5.29

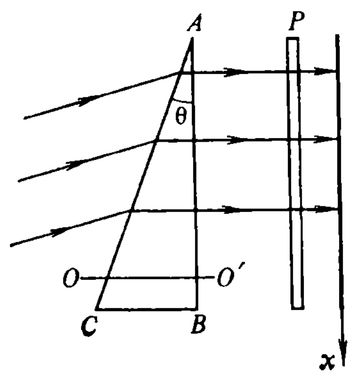

Fig. 5.30*

$= 3.00$ mm cut out at right angles to optical axis OO'. The plate is followed by polaroid P (Fig. 5.29). Within the interval of wavelengths from 500 to 650 nm, the rotational constant α of quartz may approximately (with an accuracy up to 5%) be considered to vary linearly with the wavelength from a value of $\alpha_1 = 31$ ang. deg/mm at $\lambda_1 = 500$ nm to a value of $\alpha_2 = 17$ ang. deg/mm at $\lambda_2 = 650$ nm. Determine what colour will predominate in the light emerging from the polaroid if the plane of the polaroid makes with the vertical an angle of φ equal to: (a) 55°; (b) 64°; (c) 72°; (d) 85° (φ is measured clockwise when looking along the ray).

5.148. We have a prism cut out from quartz so that its optical axis OO' is perpendicular to face AB (Fig. 5.30). The prism angle is $\theta = 30°$. A parallel beam of plane-polarized light with $\lambda = 590$ nm falls on the prism at an angle such that the light propagates in the prism along the optical

axis. If we look at the back side of the prism through polaroid P, we see alternating bright and dark fringes parallel to edge A of the prism. The distance between the middles of the bright (or dark) fringes is $\Delta x = 15.0$ mm. Find:

(a) the rotational constant α of quartz for the wavelength being considered;

(b) the function $I(x)$ describing how the intensity of the light after the polaroid depends on x.

5.5. Interaction of Light Waves with a Substance

5.149. What wavelength is understood in the formula for the group speed $u = v - \lambda\,(dv/d\lambda)$—the wavelength in a vacuum or that in the medium in which the speed of light is v?

5.150. 1. Assume that the phase speed v of light in a medium: (a) varies with the frequency ω of the light according to the law $v = \alpha\omega^q$; (b) varies with the wavelength λ in the given medium according to the law $v = \beta\lambda^p$ (q and p are numbers less than 1, α and β are constants). Find the value of the group speed u.

2. Evaluate u for the case: (a) $q = -1$; (b) $p = -1$.

5.151. For many transparent colourless substances, the dependence of the refractive index n on the wavelength λ_0 in a vacuum can be given approximately by the formula $n = a + b/\lambda_0^2$, where a and b are constants. Table 5.1 gives experimental data obtained for a certain kind of glass.

Table 5.1

λ_0, nm	759.0	589.3	486.0	397.0
n	1.510	1.515	1.521	1.531

1. Find the values of the constants a and b for the given glass according to the largest and smallest values of n.

2. Use the formula given above and the found values of a and b to calculate the refractive index for the two intermediate wavelengths indicated in the table. Compare the result with the tabulated values.

5.152. Proceeding from the assumption that the dependence of the refractive index n on the wavelength λ_0 in a vacuum for a medium is determined by the formula $n = a + b/\lambda_0^2$, where a and b are constants (see the preceding problem),

(a) find an expression (in terms of λ_0) for the group speed u of light in the given medium;

(b) calculate the values of the group speed u (express them in terms of c) for the wavelengths indicated above in Table 5.1 (take the values found in the preceding problem for a and b). Compare them with the values of the phase speed v.

5.153. A free electron is in the field of a monochromatic light wave propagating in a vacuum. The length of the wave is $\lambda = 600$ nm, its intensity is $I = 375$ lm/m^2.

(a) Disregarding in the first approximation the action of the wave field's magnetic component on the electron, find the amplitude a of the electron's oscillations and the amplitude v_m of its speed.

(b) Using the result obtained, determine the ratio of the amplitudes of the magnetic $(F_B)_m$ and electric $(F_E)_m$ forces acting on the electron. Express it in terms of the amplitude of the speed and the speed of light c.

5.154. A rarefied plasma has a free electron concentration equal to n. Having considered the passage of an electromagnetic wave of frequency ω through the plasma, find an expression for the permittivity ε of the plasma depending on ω. Disregard the interaction of the wave with the ions of the plasma, and also the action of the magnetic component of the wave on the electrons (see the preceding problem).

5.155. When light travels in a substance over the distance l, its intensity I is halved. How many times will I diminish when the light travels a distance of $3l$?

5.156. A plane monochromatic light wave propagates in a medium. The medium's absorption coefficient for the given wavelength is $\varkappa = 1.00$ m^{-1} (glasses have an absorption coefficient of this order). By what per cent will the intensity of the light diminish when the wave travels a distance equal to: (a) 5.00 mm (window glass); (b) 10.0 mm (mirror glass); (c) 1.00 m; (d) 4.60 m?

5.157. A transparent plate has a thickness of $a = 10.0$ cm. For a certain wavelength λ, the plate's absorption coefficient

varies linearly from the value $\varkappa_1 = 0.800\ m^{-1}$ at one surface of the plate to $\varkappa_2 = 1.200\ m^{-1}$ at the other surface. Determine the weakening (in per cent) of the intensity of monochromatic light of the given wavelength when it passes through the plate.

5.158. A plane light wave with an intensity of 100.0 lm/m^2 is incident normally on a glass plane-parallel plate. The refractive index on the plate for the given wavelength is $n = 1.500$, its absorption coefficient is $\varkappa = 1.000\ m^{-1}$. The thickness of the plate is $a = 10.00$ cm. The coherence length of the wave is much smaller than a. Find the intensity I of the light that has passed through the plate: (a) without account; (b) with account of multifold reflections.

5.159. Solve the preceding problem assuming that no light is absorbed in the plate. Compare the result obtained with the answer to the preceding problem.

5.160. 1. By what per cent will the intensity of light diminish when it passes through window glass with a thickness of $a = 4.00$ mm at the expense of: (a) absorption; (b) reflections? Assume that the absorption coefficient $\varkappa$ of glass is $1.23\ m^{-1}$, and its refractive index n is 1.52. Disregard the secondary reflections of the light.

2. How many times is the decrease in the intensity because of reflections greater than the decrease because of absorption?

3. What is the total weakening of the light (because of absorption and reflections) in per cent?

5.161. A laboratory uses plates with a thickness of $a_1 = 2.16$ mm and $a_2 = 36.82$ mm made from a certain kind of glass.

Propose a way of determining the absorption coefficient $\varkappa$ of the given kind of glass for a certain λ_0 such that the coefficient of reflection of light by the plates does not have to be known. Obtain the relevant working formula.

5.162. Assume that the first of the plates described in the preceding problem passes 92.5% of the light falling on it, and the second, 88.2%. Find the light absorption coefficient of the glass for the given wavelength.

5.163. How many times is the intensity of molecular scattering of blue light ($\lambda_0 = 460$ nm) greater than that of scattering of red light ($\lambda_0 = 650$ nm)?

5.6. Moving Media Optics

5.164. In an experiment similar to the one by means of which Fizeau determined the drag coefficient for the carrying along of the world's ether by water, the total path of light in water was $2l = 2.00$ m. The wavelength of the light was $\lambda_0 = 600$ nm. Determine the number ΔN of fringes by which the interference pattern is shifted when the water is brought into motion at a speed of $u = 6.00$ m/s. The refractive index of water is $n = 1.33$.

5.165. An aeroplane flies in a direction toward a radar operating on a wavelength of $\lambda = 20.0$ cm. What is the speed v of the aeroplane if the radar registers a frequency of the beats between the signal sent by it and that reflected from the aeroplane of $\Delta\nu = 2778$ Hz?

5.166. In terrestrial conditions, the wavelength of the spectral line H_α emitted by atomic hydrogen is $\lambda_0 = 656$ nm. When measuring the wavelength of this line in the radiation arriving from diametrally opposite edges of the Sun's disk, a difference was discovered equal to $\Delta\lambda_0 = 0.0088$ nm. Using these data, find the period T of the Sun's rotation about its axis.

5.167. What relative change in the frequency $\Delta\omega/\omega$ of the light wave emitted by an atom is observed if the atom: (a) approaches a spectrograph; (b) moves away from a spectrograph at a speed of v equal to the mean speed of thermal motion of the atoms at the temperature T? The mass of the atom is m.

5.168. Identical atoms not interacting with one another (for example, the atoms of a gas close to an ideal one) emit an identical set of sharp spectral lines with the frequencies $\omega_1, \omega_2, \ldots$. The chaotic thermal motion leads to the fact that owing to the Doppler effect, the spectrograph registers a continuous set of frequencies confined within the interval $\delta\omega_i$ in the vicinity of ω_i instead of the frequency ω_i. The broadening of the spectral lines appearing in this way is called Doppler broadening and is designated by $\delta\omega_D$ (or $\delta\lambda_D$). The main part of the atoms move at speeds close to the mean speed $\langle v \rangle$ of thermal motion. It is therefore assumed when calculating $\delta\omega_D$ that all the atoms move at a speed of $\langle v \rangle$.

(a) Write an expression for the relative Doppler width $\delta\omega_D/\omega$ of a spectral line in terms of the mean speed $\langle v \rangle$.

(b) Calculate the Doppler width $\delta\lambda_D$ of a spectral line of length $\lambda_0 = 656$ nm emitted by atomic hydrogen at $T = 2000$ K.

5.169. In observing the radiation emitted by heated argon, the Doppler width of the spectral lines was found to have a relative value of $\delta\omega_D/\omega = 4.9 \times 10^{-6}$. Find the temperature T of the gas.

5.170. In an experiment similar to that run by H. Ives, the radiation emitted by a beam of positive ions appearing in a gas-discharge tube was observed. The observation was performed in a direction at right angles to the direction of motion of the ions. A relative displacement of the spectral lines in the direction of lower frequencies was detected by a value of $\Delta\omega/\omega = 1.3 \times 10^{-5}$. Find the speed v of the ions in the beam.

5.171. The well-known American physicist Robert Wood, who made a great contribution to the development of optics, was very fond of practical jokes. Many legends are associated with his name. According to one of them, Wood once drove his car past a red light. He explained the incident to the traffic officer who stopped him that owing to the Doppler effect, the red light appeared to be green to him. The officer also liked jokes. He therefore agreed to accept Wood's version, but fined him for speeding. Find the speed v of the automobile at which red light with a wavelength of 690 nm would be perceived by its driver as green light with a wavelength of 530 nm.

PART 6

ATOMIC PHYSICS

SYMBOLS

A	mass number; mechanical equivalent of light; work function
A_{F}	relative atomic mass
a	absorptivity; amplitude
B	magnetic induction
b	impact parameter
C	molar heat capacity
c	speed of light in a vacuum
D	symbol of state of atom with $L = 2$
d	diameter; distance; Rydberg correction; symbol of state of electron with $l = 2$
$\mathbf{E}$	electric field strength
E	energy; illuminance; Young's modulus
E_{F}	Fermi level (energy)
E_{k}	kinetic energy
E_{b}	binding energy
$\mathcal{E}$	electromotive force
F	symbol of state of atom with $L = 3$
f	symbol of state of electron with $l = 3$
g	degree of degeneracy (statistical weight); Landé g factor
H	symbol of state of atom with $L = 5$
I	current; intensity of light; luminous intensity; moment of inertia
J	quantum number of angular momentum of atom; quantum yield of photoelectric effect; rotational quantum number
k	Boltzmann constant
L	orbital quantum number of atom
L_{E}	radiance
l	distance; length; orbital quantum number of electron
$\mathbf{M}$	angular momentum (in the preceding parts the symbol $\mathbf{L}$ was used for the angular momentum)
m	mass
N	number of levels; number of particles; number of states
n	number of level; number of particles in unit volume; quantum number
P	probability; symbol of state of atom with $L = 1$
p	pressure; Rydberg correction; symbol of state of electron with $l = 1$
q	electric charge
R	electrical resistance; radiant emittance; radius; Rydberg constant
R^*	radiant emittance of blackbody

r	distance; emissivity; radius
r_0	Bohr radius; distance between nuclei in molecule
S	area; spin quantum number of atom; symbol of state of atom with $L = 0$
s	Rydberg correction; spin quantum number of electron; symbol of state of electron with $l = 0$
T	absolute temperature; half-life
t	time
U	energy; potential difference; voltage
u	energy density
V	relative spectral sensitivity of human eye; volume
$\mathbf{v}$	velocity
v	vibrational quantum number
Z	atomic number of element
α	angle
ε	energy of oscillator; energy of particle
Θ	Debye temperature
θ	angle
λ	decay constant; wavelength
λ_C	Compton wavelength ($\bar{\lambda}_C = \lambda_C/2\pi$)
μ	magnetic moment
ν	frequency
ρ	density
σ	conductivity; screening constant (shielding factor); Stefan-Boltzmann constant
τ	mean lifetime; time
Φ	energy flux
Φ_0	quantum of magnetic flux
φ	angle; potential
ψ	psi-function
ω	cyclic frequency
ω_r	angular velocity
ω_v	natural frequency of vibrations

6.1. Thermal Radiation

6.1. What is the emissivity $r(\omega, T)$ of a perfectly reflecting surface.

6.2. A flux Φ_{inc} of radiant energy is incident on a surface with an absorptivity of $a = 0.5$ in equilibrium with the radiation. What flux Φ propagates from the surface in all directions within the limits of the solid angle 2π? What is responsible for the formation of this flux?

6.3. Determine the wavelength λ_m corresponding to the maximum emissivity of a blackbody at a temperature T equal to: (a) 3 K; (b) 300 K; (c) 3000 K; (d) 5000 K. In what spectral region will the found wavelengths be?

6.4. Upon going over from the temperature T_1 to the temperature T_2, the area confined by the plot of the energy density distribution function for equilibrium radiation by wavelengths increases 16 times. How will the wavelength λ_m at which the maximum of the emissivity of a blackbody is observed change?

6.5. The radiant emittance of a blackbody is $R^* = 250$ kW/m^2. At what wavelength λ_m will the emissivity o this blackbody be maximum?

6.6. Find the mean energy $\langle \varepsilon \rangle$ of a quantum oscillator at the temperature T. The frequency of the oscillator is ω.

6.7. Calculate the mean energy $\langle \varepsilon \rangle_q$ of a quantum oscillator (see the preceding problem) at a temperature of T for: (a) the frequency ω_1 corresponding to the condition $\hbar\omega_1 = kT$; (b) the frequency $\omega_2 = 0.1\omega_1$; (c) the frequency $\omega_3 = 10\omega_1$. Express $\langle \varepsilon \rangle_q$ in terms of kT. Compare the found values with the mean energy $\langle \varepsilon \rangle_{cl}$ of a classical oscillator.

6.8. Find the mean energy $\langle \varepsilon \rangle$ (in eV) of an electromagnetic oscillation at a temperature of 3000 K for wavelengths λ equal to: (a) 500 μm; (b) 50 μm; (c) 5 μm; (d) 0.5 μm (the visible region of the spectrum). Compare the found values of $\langle \varepsilon \rangle$ with the value of kT.

6.9. Find: (a) the temperature dependence of the frequency ω_m at which we observe the maximum of the function $f(\omega, T)$ determining the emissivity of a blackbody;

(b) the value of the product $\lambda_m \omega_m$, where λ_m is the wavelength corresponding to the maximum of the function $\varphi(\lambda, T)$. Compare this value with $2\pi c$.

6.10. The Sun's surface is close in its properties to a blackbody. The maximum of the emissivity corresponds to a wavelength of $\lambda_m = 0.50$ μm (in solar radiation that has passed through the atmosphere and reached the Earth's surface, the maximum corresponds to $\lambda = 0.55$ μm). Determine:

(a) the temperature T of the Sun's surface;

(b) the energy E emitted by the Sun in 1 s in the form of electromagnetic waves;

(c) the mass m lost by the Sun in 1 s because of radiation;

(d) the approximate time τ during which the Sun's mass would diminish because of radiation by 1% if its temperature remained constant.

6.11. Assuming the Sun to have the properties of a blackbody, determine the intensity I of solar radiation near the Earth beyond the limits of its atmosphere (this intensity is known as the solar constant). The temperature of the Sun's surface is $T = 5785$ K.

6.12. The intensity of the visible radiation emitted by the Sun is $I_{vis} = 0.60$ kW/m^2 near the Earth's surface beyond the limits of its atmosphere. Assuming that in the

visible region the radiant energy is distributed uniformly by wavelengths, and using the curve of the relative spectral sensitivity of the human eye in Fig. 5.9. on p. 179, appraise:

(a) the illuminance E of a surface perpendicular to the direction of the rays produced by the solar radiation at the outer boundary of the Earth's atmosphere;

(b) the luminous intensity I of the Sun's light.

The mechanical equivalent of light is $A = 0.0016$ W/lm.

6.13. How does the radiance L_E of a blackbody depend on the temperature T?

6.14. A thin-walled closed envelope has an opening whose linear dimensions are much smaller than those of the envelope. The walls of the envelope are kept at a temperature of $T = 300$ K. Taking into account that a blackbody is a Lambert source, determine the radiance L_E of the opening.

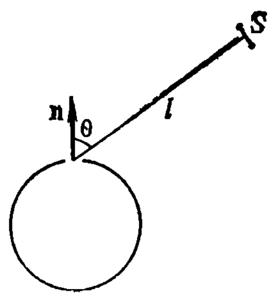

Fig. 6.1

6.15. A spherical envelope 30 cm in diameter has an opening with a diameter of $d = 4.00$ mm. Round area S whose radius is $r = 3.00$ mm is placed at a distance of $l = 500$ mm from the centre of the opening (Fig. 6.1). A line from the centre of the opening to the centre of the area makes the angle $\theta = 45.0°$ with a normal to the opening. The area is perpendicular to this line. The walls of the envelope are kept at a temperature of $T = 2000$ K. Determine the energy flux Φ incident on the area and due to the radiation emerging from the opening.

6.16. A device modelling a blackbody is installed on the body of a space laboratory flying about the Sun in a circular orbit whose radius R equals the mean distance from the Earth to the Sun. The outer surface of the housing of this device is a perfectly reflecting one. The opening in the housing constantly faces the Sun. Disregarding heat exchange through the fastening of the device to the body of the laboratory, determine the equilibrium temperature T that sets in inside the device. Assume that the temperature of the Sun's surface is 5800 K.

6.2. Photons

6.17. Determine the limits (in eV) confining the energy of photons corresponding to the visible part of the spectrum.

6.18. Monochromatic light with a wavelength of $\lambda = 555$ nm, when falling on a surface, produces an illuminance of $E = 100$ lx (such an illuminance in white light is needed to allow one to read without any strain). How many photons impinge on an area of $S = 1$ cm^2 per second?

The mechanical equivalent of light is $A = 0.0016$ W/lm.

6.19. Determine the energy ε (in eV) and the momentum p of a photon with a wavelength λ equal to: (a) 555 nm (visible light); (b) 0.1 nm (X-rays); (c) 0.001 nm (gamma rays). Compare ε with the rest energy of an electron, and p with the momentum of an electron travelling at a speed of $v = 1000$ m/s.

6.20. At what speed v will the momentum p of an electron coincide in magnitude with the momentum of a photon whose wavelength is $\lambda = 0.001$ nm [see item (c) of the preceding problem]?

6.21. A parallel beam of light rays with an intensity of $I = 1.37$ kW/m^2 (see Problem 6.11) falls on a sphere of radius $r = 1.00$ cm having a perfectly smooth surface. Determine, proceeding from corpuscular notions, the force F which the sphere experiences if its surface has:

(a) an absorption coefficient equal to 1;

(b) a reflection coefficient equal to 1.

6.22. The solar constant (i.e. the intensity of solar radiation near the Earth's surface beyond the limits of the Earth's atmosphere) is $I = 1.37$ kW/m^2. About 40% of this energy flux falls to the share of the visible part of the spectrum. Considering this energy to be distributed uniformly by frequencies:

(a) find the function $\varphi(\omega)$ of distribution of the photon flux density by frequencies;

(b) determine the flux density j_{ph} of the "visible" photons incident on the outer boundary of the atmosphere.

6.23. Determine the wavelength λ_{min} corresponding to the short-wave boundary of the X-ray spectrum for the case when a voltage of $U = 50$ kV is applied across the tube.

6.24. An increase in the voltage across an X-ray tube of $\eta = 2$ times is attended by a change in the wavelength cor-

responding to the short-wave boundary of the X-ray spectrum by $\Delta\lambda = 0.025$ nm. Determine the initial voltage U_1 applied across the tube.

6.25. In 1916, R. Millikan when investigating the photoelectric effect from the surface of sodium obtained the data* given in Table 6.1 (ν is the frequency of light in s^{-1},

Table 6.1

ν, 10^{14} s^{-1}	5.49	6.92	7.41	8.22	9.60	11.83
U, V	0.47	1.02	1.20	1.60	2.13	3.02

U is the stopping potential in V). Using these data, determine:

(a) the value of Planck's constant $\hbar$;

(b) the work function A for sodium.

6.26. The work function A for nickel is 4.84 eV. Find the wavelength λ_0 corresponding to the photoelectric threshold.

6.27. The photoelectric threshold for aluminium corresponds to a wavelength of $\lambda_0 = 332$ nm. Find:

(a) the work function A for this metal;

(b) the wavelength λ at which the magnitude of the stopping potential $U = 1.00$ V.

6.28. Up to what potential φ can a zinc ball removed from other bodies be charged by irradiating it with ultraviolet radiation with a wavelength of $\lambda = 200$ nm?

6.29. The cathode of a vacuum photodiode is illuminated uniformly by monochromatic light with $\lambda = 450$ nm. The area of the cathode is $S = 1.00$ cm^2, its illuminance $E = 100$ lx (such an illuminance in white light is needed to allow one to read without strain). Determine the saturation current I_{sat} flowing through the diode. At the indicated wavelength, an energy flux of 0.040 W corresponds to a light flux of 1 lm. Assume that the quantum yield of the photoelectric effect J (i.e. the number of photoelectrons per incident photon) is 0.050.

* These data have been taken from Millikan's original work. A correction for the contact potential difference has been introduced into the values of U.

6.30. A photon with a wavelength of $\lambda = 700$ nm (the visible part of the spectrum) is scattered at an angle of $\theta = \pi/2$ on an electron freely at rest. Determine:

(a) the fraction of its initial energy lost by the photon;

(b) the speed v acquired by the electron.

6.31. Solve a problem similar to the preceding one for the case when $\lambda = 0.100$ nm (X-radiation).

6.32. (a) Determine the kinetic energy E_k acquired by a free particle of mass m initially at rest when a photon with the energy ε is scattered on it at the angle θ.

(b) Simplify the formula obtained for the case when $\varepsilon \ll mc^2$.

6.33. A gamma quantum with the energy $\varepsilon = 1.00$ MeV is scattered at the angle $\theta = 90°$ on a proton freely at rest. Determine:

(a) the kinetic energy E_k imparted by the gamma quantum to the proton;

(b) at what speed v the proton will travel after the "collision".

6.34. In studying the radiation appearing as a result of scattering of an X-ray beam with a wavelength of $\lambda = 0.0714$ nm (K_α of molybdenum) on graphite at an angle of $\theta = 90°$, a diffraction maximum of the first order of the undisplaced component was obtained upon incidence on the crystal of an X-ray spectrograph at a glancing angle of $\varphi = 30.0°$. Through what angle $\delta\varphi$ would the crystal have to be turned for the maximum of the undisplaced component to be replaced with a maximum of the displaced component?

6.35. Using Planck's constant $\hbar$, the speed of light c in a vacuum, and the mass m of a particle, compile an expression for a quantity having the dimension of length. What is this quantity?

6.3. Rutherford Formula. Bohr Atom

6.36. Up to what distance r_{min} can an alpha particle approach a stationary nucleus of a gold atom in a central "collision" if the speed of the particle at a large distance from the nucleus is $v = 3.00 \times 10^7$ m/s?

6.37. Determine the value of the impact parameter b when an alpha particle is scattered through an angle of

$\theta = \pi/2$ by the nucleus of a silver atom. The speed of the alpha particle is $v = 1.00 \times 10^7$ m/s.

6.38. Write the Rutherford formula for the scattering of protons by a substance with the atomic number Z.

6.39. In an experiment similar to Rutherford's one, a flux of alpha particles equal to 2.70×10^5 particle/s was scattered by gold foil with a thickness of $a = 4.00$ μm. The kinetic energy of the particles was $E_k = 8.30$ MeV. The scattered particles were registered by observing the scintillations on a round screen with an area of $S = 1.00$ cm^2. The distance from the screen to the place of intersection of the beam of alpha particles with the foil was $l = 0.200$ m. Determine the mean number of scintillations observed during the time $\Delta t = 1.00$ min when the device registering the particles was installed at an angle θ to the direction of the incident beam equal to: (a) 30°; (b) 60°; (c) 90°; (d) 120°; (e) 150°.

6.40. Find the probability P that an alpha particle in the experiment described in the preceding problem will be scattered into the rear half-sphere, i.e. through an angle of $\theta \geqslant \pi/2$.

6.41. What is the ratio of the probabilities of Rutherford scattering into the rear half-sphere (see the preceding problem) for an alpha particle (P_α) and for a proton (P_p) in identical conditions (i.e. at identical v, Z, n, and a)?

6.42. In an experiment with mercury vapour similar to the Franck-Hertz one, current peaks were observed at potential differences U equal to 4.9 and 9.8 V. When the mercury vapour pressure was lowered, an additional peak appeared at $U = 6.7$ V. How should these results be interpreted?

6.43. Determine the speed v_1 at which an electron travels in the first Bohr orbit in a hydrogen atom.

6.44. A system consists of a hydrogen atom nucleus (a proton) and a muon (a particle having the same charge as an electron and a mass equal to 207 masses of an electron) (such a system is called a muonic or a mesonic atom). Proceeding from the notions of Bohr's theory, determine:

(a) the radius r_1 of the first Bohr orbit of a muon; compare it with the Bohr radius r_0;

(b) the energy E_b (in eV) of the bond of the muon to the proton in the ground state;

(c) the speed v_1 of the muon in the first orbit; compare v_1 with the answer to Problem 6.43;

(d) the number of revolutions which the muon will have time to complete before its decay (the mean lifetime of a muon is $\tau = 2.2\ \mu s$; when this time elapses, the muon decays into an electron, a neutrino, and an antineutrino).

6.45. Within the scope of Bohr's theory, find:

(a) an expression for the binding energy E_b of the electron to the nucleus in the ground state of the hydrogen atom that takes account of the motion of the nucleus;

(b) the relative value δ of the correction to the binding energy obtained by taking the motion of the nucleus into consideration.

6.46. What is the relative magnitude δ of the correction [see item (b) of the preceding problem] for a muonic atom (see Problem 6.44)?

6.47. Using Planck's constant $\hbar$, the mass m_e and the charge e of an electron, compile an expression for a quantity having the dimension of length. What is this quantity?

6.48. Using Planck's constant $\hbar$, the mass m_e and the charge e of an electron, compile an expression for a quantity having the dimension of energy. What is this quantity?

6.49. Determine the magnetic moment μ_1 of an electron in a hydrogen atom in the first Bohr orbit. Compare the result obtained with the Bohr magneton μ_B.

6.50. Determine the magnetic moment μ_1' of a muon in a muonic atom (see Problem 6.44) in the first Bohr orbit. Compare μ_1' with the magnetic moment μ_1 of an electron in a hydrogen atom in the first Bohr orbit (see the preceding problem).

6.51. Find the ratio of the magnetic moment μ_n to the angular momentum M_n for an electron in a hydrogen atom in the n-th Bohr orbit. Compare the result obtained with the answer to Problem 3.165.

6.52. A particle of mass m travels in a centrally symmetric force field $\mathbf{F}(\mathbf{r}) = -k\mathbf{r}$ (k is a positive constant). Assuming that the angular momentum of a particle can only have values that are a multiple of $\hbar$ (as in the Bohr theory), find:

(a) the possible radii r_n of the circular orbits of the particle;

(b) the possible values E_n of the total energy of the particle. Express E_n in terms of the frequency ω with which the particle would vibrate under the action of the force $\mathbf{F}(\mathbf{r})$.

6.4. Spectra of Atoms and Molecules

6.53. (a) Express the frequency ω_m of the leading line of the m-th spectral series of the hydrogen atom in terms of the Rydberg constant R.

(b) Find the ratio of the frequencies of the leading lines of the first four series, taking as unity the frequency ω_2 of the leading line of the Balmer series.

6.54. The ionization potential of a hydrogen atom is $\varphi_i = 13.6$ V. Proceeding from this fact, calculate the value of the Rydberg constant R.

6.55. Proceeding from the fact that the ionization energy of a hydrogen atom is $E_i = 13.6$ eV, determine the first potential φ_1 of excitation of this atom.

6.56. On the basis of the fact that the first excitation potential of a hydrogen atom is $\varphi_1 = 10.2$ V, determine the energy ε (in eV) of a photon corresponding to the first line of the Balmer series.

6.57. The ionization energy of a hydrogen atom is $E_i = 13.6$ eV. Proceeding from this fact, determine the energy ε (in eV) of a photon corresponding to the second line of the Balmer series.

6.58. Proceeding from the fact that the ionization potential of a hydrogen atom is 13.6 V, determine the wavelength λ_1 of the first line and the wavelength λ_∞ of the boundary: (a) of the Lyman series; (b) of the Balmer series; (c) of the Paschen series.

6.59. Knowing that the wavelength of the spectral line H_β is 486.1 nm, find the value of the Rydberg constant R.

6.60. Knowing that the first excitation potential of the hydrogen atom is $\varphi_1 = 10.2$ V, find the wavelength:

(a) of the line H_α;

(b) of the boundary of the Balmer series H_∞.

6.61. The ionization potential of a hydrogen atom is 13.6 V. Proceeding from this fact, determine how many lines of the Balmer series are in the visible part of the spectrum.

6.62. Spectral lines of what wavelengths will appear if a hydrogen atom is transferred to the state $3S$?

6.63. A photon with the energy $\varepsilon = 15.0$ eV ejects an electron from a hydrogen atom at rest in the ground state. At what speed v will the electron travel at a distance from the nucleus?

6.64. The energy of a valence electron in the ground state is $E_1 = -3.8$ eV. What is the ionization potential φ_i of the atom?

6.65. The wavelength $\lambda_\infty = 250$ nm corresponds to the boundary of the principal (i.e. appearing upon a transition to the state with the smallest energy) series of an atom. Find the ionization potential φ_i of the atom.

6.66. The state $3S$ is the ground one for a sodium atom. The Rydberg correction for S-terms is -1.35. Proceeding from these data, evaluate the ionization energy E_i of a sodium atom (express it in eV).

6.67. The state $3S$ is the ground one for a sodium atom. The Rydberg correction for P-terms is $p = -0.87$. The wavelength of the resonance line (due to the transition $3P \to 3S$) is $\lambda = 590$ nm. Proceeding from these data, find the ionization potential φ_i of a sodium atom.

6.68. The state $3S$ is the ground one for a sodium atom. The Rydberg corrections are negative and are in the relation $|s| > |p| > |d|$. The level nP is below the level $(n + 1)S$, and the level nD is below the level $(n + 1)P$. On the basis of these data, list all the possible sequences of the transitions by means of which a sodium atom can pass from the excited state $5S$ to the ground state.

6.69. The wavelengths of the yellow doublet of sodium are 589.00 and 589.59 nm. Proceeding from these data, determine the difference $\Delta\omega$ between the frequencies of the doublets of the sharp series.

6.70. Knowing the wavelengths given in the preceding problem, find for the level $3P$ of a sodium atom the magnitude of the splitting ΔE (in eV) due to spin-orbital interaction.

6.71. Taking into account the hydrogen likeness of the F-terms of the sodium atom, find the difference $\Delta\omega$ between the frequencies of the first and second lines of the fundamental series.

6.72. The ionization potential of a lithium atom is $\varphi_i = 5.39$ V, and the first excitation potential is $\varphi_1 = 1.85$ V. Use these data to find the Rydberg corrections s and p for lithium.

6.73. Using the results obtained in the preceding problem, determine the wavelengths of the spectral lines appearing when lithium atoms pass from the state $3S$ to $2S$.

6.74. The ionization potential of a cesium atom is $\varphi_i = 3.89$ V. The state $6S$ is the ground one. Use these data to find the Rydberg correction s for the S-terms of cesium.

6.75. What speed v is acquired by a hydrogen atom initially at rest when it emits a photon corresponding to the leading line of:

(a) the Lyman series; (b) the Balmer series?

6.76. Find the recoil energy E and the speed v acquired by a free sodium atom initially at rest when it emits a photon corresponding to the $3P \rightarrow 3S$ transition (the yellow line of sodium, see Problem 6.69). What fraction of the energy $\hbar\omega$ of a photon is the recoil energy E?

6.77. Determine the speed v acquired by a free mercury atom initially at rest when it absorbs a photon of resonance frequency (the frequency corresponding to the transition of an atom to the first excited level is known as the resonance frequency). The first excitation potential of the mercury atom is 4.9 V.

6.78. A free sodium atom initially at rest emits a photon. (a) What is the change $\Delta\lambda$ in the wavelength of the photon produced owing to the recoil experienced by the atom in the emission? (b) What is noteworthy in the result obtained?

6.79. A free lithium atom at rest absorbed a photon with a frequency of $\omega = 2.81 \times 10^{15}$ s^{-1}, as a result of which it passed over to the first excited level and began to move at a certain speed. Next the atom returned to the ground state after emitting a new photon in a direction at right angles to that of its motion. At what speed v does the atom move after this?

6.80. A photon having the energy $\varepsilon = 5.4852$ eV ejects a valence electron from a free lithium atom at rest. The electron flies out at right angles to the direction in which the photon flew. At what speed v and in what direction does the ionized atom move? The ionization potential of lithium is $\varphi_i = 5.3918$ V.

6.81. The wavelength of the line K_α is 0.250 73 nm for vanadium ($Z = 23$) and 0.154 43 nm for copper ($Z = 29$).

(a) Proceeding from these data, find the values of the constants C and σ in the equation of Moseley's law: $\sqrt{\omega} = C(Z - \sigma)$. Compare the found value of C with a value equal to $\sqrt{3R/4}$ (R is the Rydberg constant).

(b) Determine the atomic number Z of the element for which the wavelength of the line K_α is 0.193 99 nm. What is this element?

6.82. The wavelength of the line K_α is 0.021 381 nm for tungsten ($Z = 74$) and 0.056 378 nm for silver ($Z = 47$). Proceeding from these data, determine the values of the constants C and σ (see the preceding problem). Compare the values obtained with the result of the preceding problem. Explain the observed discrepancy.

6.83. The wavelength of the line L_α is 0.147 635 nm for tungsten ($Z = 74$) and 0.117 504 nm for lead ($Z = 82$).

(a) Proceeding from these data, find the values of the constants C and σ in the equation of Moseley's law: $\sqrt{\omega} = C(Z - \sigma)$. Compare the found value of C with a value equal to $\sqrt{5R/36}$ (R is the Rydberg constant).

(b) Determine the atomic number Z of the element for which the wavelength of the line L_α is 0.131 298 nm. What is this element?

6.84. The wavelength of the line L_α is 1.7602 nm for iron ($Z = 26$) and 1.2282 nm for zinc ($Z = 30$). Use these data to find the values of the constants C and σ (see the preceding problem). Compare the values obtained with the result of the preceding problem.

6.85. For which of the elements—copper or silver—is the relative change in the frequency $\Delta\omega/\omega$ of the line K_α in Compton scattering on a substance larger? How many times?

6.86. The first excitation potential of the electron shell of a CO molecule is 6.0 V. In the ground electron state of the molecule, the natural frequency of vibrations is $\omega_v = 4.09 \times 10^{14}$ s^{-1}. Find:

(a) the number N of vibrational levels confined between the ground level and the first excited electron one;

(b) the ratio of the energy ΔE_e needed to transfer the molecule to the first excited electron level to the energy ΔE_v needed to transfer the molecule to the first excited vibrational level.

6.87. In the ground electron state of the CO molecule, the natural frequency of vibrations is $\omega_v = 4.09 \times 10^{14}$ s^{-1}, while the equilibrium distance between nuclei is $r_0 = 0.112$ nm. Find:

(a) the number N of rotational levels confined between the ground and first excited vibrational levels;

(b) the ratio of the energy ΔE_v needed to transfer the molecule to the first excited vibrational level to the energy ΔE_r needed to transfer the molecule to the first excited rotational level.

Compare the results obtained with the answer to the preceding problem.

6.88. The distance between the lines of the rotational band of the CN molecule is $\Delta\omega = 7.19 \times 10^{11}\ s^{-1}$. Determine the equilibrium distance r_0 between the nuclei of the molecule.

6.89. We have a diatomic molecule whose moment of inertia is I. Determine the angular speed of rotation ω_r of the molecule in a state with the rotational quantum number J. Compare ω_r with the frequency ω of the spectral line appearing upon a transition from the J-th to the $(J - 1)$-th rotational level.

6.90. The distance between the nuclei of the HCl molecule is $r_0 = 0.127$ nm. Find the angular speed of rotation ω_r of a molecule at the first excited rotational level.

6.91. A gas consisting of CN molecules is in thermodynamic equilibrium at a temperature of $T = 400$ K. The natural frequency of vibrations of a CN molecule is $\omega = 3.90 \times 10^{14}\ s^{-1}$. Determine the ratio of the numer N_{i+1} of molecules at the $(i + 1)$-th vibrational level to the number N_i of molecules at the i-th vibrational level.

6.5. Quantum Mechanics

6.92. Write an expression for the de Broglie wavelength λ of a relativistic particle of mass m: (a) in terms of its speed v; (b) in terms of its kinetic energy E_k.

6.93. At what value of the speed v does the de Broglie wavelength of a microparticle equal its Compton wavelength.

6.94. At what speed v of an electron will its de Broglie wavelength be: (a) 500 nm; (b) 0.1 nm? (For electromagnetic waves, the first wavelength corresponds to the visible part of the spectrum, and the second one to X-rays.)

6.95. In motion along the x-axis, the speed is found to be certain with an accuracy of $\Delta v_x = 1$ cm/s. Appraise the

uncertainty of the coordinate Δx: (a) for an electron; (b) for a Brownian particle of mass $m \sim 10^{-13}$ g; (c) for a pellet of mass $m \sim 0.1$ g.

6.96. A flux of electrons flying parallel to one another and having a speed of $v = 1.00 \times 10^6$ m/s passes through a slit of width $b = 0.0100$ mm. Find the width Δx of the central diffraction maximum observed on a screen at a distance of $l = 1.00$ m from the slit. Compare Δx with the width b of the slit.

6.97. A narrow beam of electrons flying parallel to one another at a speed of $v = 1.00 \times 10^7$ m/s passes through polycrystalline nickel foil and impinges on a screen at a distance of $l = 10.0$ cm behind the foil. Find the radii of the two first diffraction rings obtained on the screen because of the reflection of electrons from crystal planes spaced at a distance of $d = 0.215$ nm.

6.98. Using the uncertainty relation, appraise the minimum energy E_1 that a particle of mass m can have if it is in an infinitely deep one-dimensional potential well of width a.

6.99. Use the uncertainty relation to appraise the minimum energy E_0 of a one-dimensional harmonic oscillator. The mass of the oscillator is m, and its natural frequency is ω.

6.100. Appraise the speed v of an electron in a hydrogen atom proceeding from the fact that the radius r of the atom has a magnitude of the order of 0.1 nm.

6.101. The psi-function $\psi(x, y, z)$ of a particle is given. Write an expression for the probability P of the particle being detected in a region of volume V.

6.102. Find the psi-functions and the values of the energy of a particle of mass m in a one-dimensional infinitely deep potential well of width a $(0 \leqslant x \leqslant a)$. (An infinitely deep well signifies that the potential energy of a particle in the well is zero, and outside the well—infinite.) Compare the result for E_1 with the answer to Problem 6.98.

6.103. A particle moving in a one-dimensional infinitely deep potential well is in the ground state (i.e. in a state with the minimum energy). Evaluate the probability P of the fact that the coordinate x of the particle has a value confined within the limits from ηa to $(1 - \eta)\, a$, where a is the width of the well $(0 \leqslant x \leqslant a)$, and η is a number equal to 0.3676.

6.104. An electron is in an infinitely deep one-dimensional potential well of width $a = 1.00$ cm. Find: (a) the density of the energy levels of the electron dn/dE (i.e. the number of levels per unit energy interval); (b) the value of this density in the vicinity of the level with the number $n = 10^{10}$; (c) the mean value of the energy $\langle E_n \rangle$ of the first $N = 10^{10}$ levels.

6.105. Find the psi-functions and the values of the energy of a particle of mass m in a two-dimensional infinitely deep (see Problem 6.102) potential well whose dimension along the x-axis is a and along the y-axis is b ($0 \leqslant x \leqslant a$, $0 \leqslant y \leqslant b$).

6.106. Find the values of the energy (in eV) of the three lower levels for the particle of the preceding problem, assuming that $m = 0.911 \times 10^{-30}$ kg, and $a = b = 1.00$ nm.

6.107. Find the psi-functions and the values of the energy of a particle of mass m in a three-dimensional infinitely deep (see Problem 6.102) potential well whose dimension along the x-axis is a, along the y-axis is b, and along the z-axis is c ($0 \leqslant x \leqslant a$, $0 \leqslant y \leqslant b$, $0 \leqslant z \leqslant c$).

6.108. A particle of mass m is in an infinitely deep (see Problem 6.102) spherical potential well of radius R. Find:

(a) the psi-functions corresponding to the states for which ψ depends only on r. To perform the calculations, represent the psi-functions in the form $\psi_n(r) = \varphi_n(r)/r$;

(b) the values of the particle's energy E_n in the states described by the functions $\psi_n(r)$.

Note. In addition to states of the kind $\psi(r)$, states are possible in which the psi-functions depend also on the angular coordinates θ and φ.

6.109. The psi-function of a particle has the form $\psi = A \exp(-r/a)/r$, where r is the distance to the particle from the force centre, and a is a constant. Find:

(a) the value of the coefficient A;

(b) the mean distance $\langle r \rangle$ to the particle from the centre.

6.110. The psi-function of a particle has the form $\psi = (1/\sqrt{\pi a \sqrt{2\pi}}) \exp(-r^2/a^2)/r$, where r is the distance to the particle from the force centre, and a is a constant. Find the mean distance $\langle r \rangle$ to the particle from the centre.

6.111. The psi-function of a particle has the form $\psi = A \exp(-r^2/2a^2)$, where r is the distance to the particle from the force centre, and a is a constant. Find:

(a) the value of the coefficient A;

(b) the most probable r_{prob} and mean $\langle r \rangle$ distances to the particle from the centre.

6.112. (a) What is the smallest non-zero angular momentum M_{min} encountered in nature?

(b) List the "objects" having such an angular momentum.

6.113. In Problem 1.172, the angular momentum M of the Earth due to its rotation about its axis was calculated. Express this quantity in units of $\hbar$. (In Part 1 the angular momentum was designated by **L**.)

6.114. The psi-function of the ground state of a harmonic oscillator has the form $\psi_0(x) = \sqrt{\alpha/\sqrt{\pi}} \exp(-\alpha^2 x^2/2)$, where $\alpha = \sqrt{m\omega/\hbar}$ (m is the mass, and ω is the natural frequency of the oscillator). The oscillator's energy in this state is $E_0 = \frac{1}{2}\hbar\omega$. Find:

(a) the mean value of the magnitude of the coordinate $\langle |x| \rangle$; express $\langle |x| \rangle$ in terms of the classical amplitude a (which is related to the oscillator's energy by the expression $E = \frac{1}{2} ma^2\omega^2$) and compare the expression found with that for $\langle |x| \rangle$ of a classical oscillator obtained in Problem 2.78;

(b) the mean value of the oscillator's potential energy $\langle U \rangle$.

6.115. A mathematical pendulum has a mass of $m = 10.0$ mg and a length of $l = 1.00$ cm. Find:

(a) the energy E_0 of the zero-point oscillations of this pendulum;

(b) the classical amplitude a of the pendulum corresponding to the energy E_0.

6.116. The psi-function of the ground state of a hydrogen atom has the form $\psi = A \exp(-r/r_0)$, where r_0 is the Bohr radius (i.e. the radius of the first Bohr orbit). Find:

(a) the value of the constant A;

(b) the density dP/dr of the probability of finding the electron at the distance r from the nucleus;

(c) the most probable distance r_{prob} to the electron from the nucleus;

(d) the mean distance $\langle r \rangle$ to the electron from the nucleus;

(e) the mean value of the potential energy $\langle U \rangle$ of the electron;

(f) the probability P_η of finding the electron at a distance from the nucleus exceeding ηr_0 (η is a number).

6.117. Using the result of item (f) of the preceding problem, calculate the probability of the fact that the electron in the ground state of the hydrogen atom is at a distance from the nucleus exceeding: (a) r_0; (b) $\frac{3}{2}r_0$; (c) $2r_0$; (d) $5r_0$; (e) $10r_0$.

6.118. A certain amount of atomic hydrogen is in thermal equilibrium at a temperature of $T = 3000$ K. What number N of atoms in the ground state falls to one atom in the first excited state?

Take into account the fact that the probability of being in a state with the energy E is proportional, in addition to the Boltzmann factor, to the degree of degeneracy g (or, as is said, to the statistical weight) of the given energy level.

6.6. Quantum-Mechanical Description of the States of Atoms

6.119. What does the square of the orbital angular momentum $\mathbf{M}^2$ of an electron equal in the states: (a) $2p$; (b) $4f$?

6.120. The state of an atom is characterized by the quantum numbers L and S equal to: (a) 2 and 2; (b) 3 and 2; (c) 2 and 3; (d) 1 and 3/2. Write the possible values of the quantum number J at the given values of L and S.

6.121. Which of the following terms have been written incorrectly: (1) $^2S_1$; (2) $^2P_1$; (3) $^3P_{1/2}$; (4) $^3P_3$; (5) $^5D_0$; (6) $^1F_0$; (7) $^8F_{13/2}$?

6.122. The multiplicity of the F-state is five. Write the terms corresponding to this state.

6.123. How many components do the following terms consist of: (a) ^1S; (b) ^2S; (c) ^2P; (d) ^3P; (e) ^4P; (f) ^5D?

6.124. A D-term consists of five components. What can the multiplicity of this term be?

6.125. Each of the states P and D has three components. What are the possible values of the spin quantum number S of these states?

6.126. Find the possible multiplicities $\varkappa$ of terms of the kind: (a) $^\varkappa S_0$; (b) $^\varkappa P_2$; (c) $^\varkappa D_{3/2}$; (d) $^\varkappa F_{1/2}$.

6.127. What terms are possible with the following electron configurations: (a) $2s^2$; (b) $2p3s$; (c) $3p^2$?

6.128. The electron shell of an atom consists of an s-, p-, and d-electron. Write the symbol of the term for the state in which the atom has: (a) the maximum; (b) the minimum total angular momentum for this configuration.

6.129. Write for a system of two equivalent d-electrons the symbols of the terms for the states with (a) the maximum; (b) the minimum possible value of the total angular momentum M_J. What do these values equal?

6.130. Write the symbol of a term corresponding to the state in which the angular momentum of an atom is $M_J = \hbar\sqrt{2}$, the magnetic moment is zero, and the spin quantum number is $S = 2$.

6.131. An atom in a state whose multiplicity is four has an angular momentum of $M_J = (\hbar/2)\sqrt{63}$. What values can the quantum number L of this state have?

6.132. What is the total angular momentum M_J of an atom in a state in which the magnetic moment of the atom is zero, while the orbital and spin quantum numbers have the values $L = 2$ and $S = 3/2$?

6.133. What is the maximum possible total angular momentum M_J of a lithium atom whose valence electron is in a state with $n = 3$? Write the symbol of the term of the corresponding state.

6.134. Solve a problem similar to the preceding one for a sodium atom whose valence electron is in a state with $n = 4$.

6.135. In the case of four equivalent p-electrons, the terms ${}^1S_0$, ${}^3P_2$, ${}^3P_1$, ${}^3P_0$, ${}^1D_2$ do not contradict the Pauli principle. Which of these terms is the ground one?

6.136. There are five equivalent p-electrons above the filled shells and subshells of an atom. Determine the atom's ground term.

6.137. There are three equivalent p-electrons above the filled shells and subshells of an atom. Determine the atom's terms that are in agreement with the Pauli principle. Which of these terms is a ground one?

6.138. Which of the transitions: (1) ${}^2S_{1/2} \to {}^2P_{3/2}$; (2) ${}^2S_{1/2} \to {}^2D_{3/2}$; (3) ${}^2P_{1/2} \to {}^2S_{1/2}$; (4) ${}^2D_{5/2} \to {}^2P_{1/2}$; (5) ${}^2F_{7/2} \to {}^2D_{3/2}$; (6) ${}^2D_{3/2} \to {}^2F_{5/2}$; (7) ${}^2F_{5/2} \to {}^2P_{3/2}$ are forbidden by the selection rules?

6.139. The valence electron of a sodium atom is in a state

with $n = 4$. The values of the other quantum numbers of the electron are such that the atom has the maximum possible value of the angular momentum M_J. Determine the magnetic moment μ of the atom in this state.

6.140. A carbon atom with the electron configuration $1s^2 2s^2 2p3d$ has the maximum possible total angular momentum at such a configuration. What does the magnetic moment μ of the atom equal (in Bohr magnetons) in this state?

6.141. Find three of the simplest terms for which the Landé g factor is zero.

6.142. Express the magnetic moment μ of an atom in terms of the Bohr magneton in the states: (a) $^3S_1$; (b) $^1P_0$; (c) $^1P_1$; (d) $^4D_{1/2}$; (e) $^5F_1$; (f) $^7H_2$.

6.143. Into how many components will the following terms split in a magnetic field: (a) ^1S; (b) ^1P; (c) ^1D; (d) $^2D_{5/2}$?

6.144. Into how many components will a beam of atoms split in an experiment similar to that of Stern and Gerlach if the atoms are in the states: (a) $^2P_{3/2}$; (b) $^3D_1$; (c) $^3F_4$?

6.145. Into how many components will a beam of atoms split in an experiment similar to that of Stern and Gerlach if the atoms are in the states: (a) $^1S_0$; (b) $^4D_{1/2}$?

6.146. In an experiment similar to that of Stern and Gerlach, a beam of chlorine atoms in the state $^2P_{3/2}$ passes through a region with a non-uniform magnetic field with $dB/dx = 100$ T/m. The length of this region is $l_1 = 40.0$ mm. A screen is installed at a distance of $l_2 = 100$ mm from the boundary of the region. The speed of the atoms at the entrance to the region of the field is $v = 600$ m/s. Find the distance a between adjacent traces left on the screen by the beams into which the initial beam splits when passing through the field.

6.147. An atom is in a magnetic field with the induction $B = 1.00$ T. Find the total splitting ΔE (in eV) of the terms: (a) ^1S; (b) ^1P; (c) ^1D; (d) $^2D_{5/2}$.

6.148. Find the numerical value of the normal (Lorentz) shift $\Delta\omega_0$ (the shift of the components of a spectral line in the simple Zeeman effect) corresponding to $B = 1.00$ T.

6.149. Emitting atoms are in a magnetic field with an induction of $B = 1.00$ T. Find the interval $\Delta\omega$ between adjacent Zeeman components for the transitions: (a) $^1P_1 \to {}^1S_0$; (b) $^1D_2 \to {}^1P_1$; (c) $^2P_{1/2} \to {}^2S_{1/2}$; (d) $^2P_{3/2} \to {}^2S_{1/2}$; (e) $^3D_1 \to {}^3P_0$.

6.150. Compare the interval $\Delta\omega$ between the lines of the yellow sodium doublet (see Problem 6.69) with the interval $\Delta\omega'$ between the Zeeman components into which the doublet lines split at $B = 1.00$ T. The doublet lines correspond to the transitions $^2P_{1/2} \rightarrow {}^2S_{1/2}$ and $^2P_{3/2} \rightarrow {}^2S_{1/2}$ [see items (c) and (d) of the preceding problem].

6.151. What number of slits N must a diffraction grating have to resolve the Zeeman components into which the spectral lines of the yellow doublet of sodium split at $B = 1.00$ T (see the preceding problem)? The wavelengths of the spectral lines are 589.0 and 589.6 nm.

6.7. Solid State Physics

6.152. The edge of a cell of a cubic crystal is a. Find the distance l between points with the indices: (a) $[[\frac{1}{2}\,0\,\frac{1}{2}]]$ and $[[\frac{1}{2}\,1\,\frac{1}{2}]]$; (b) $[[0\,0\,0]]$ and $[[1\,1\,1]]$; (c) $[[1\,0\,\frac{1}{2}]]$ and $[[\frac{1}{2}\,1\,\frac{1}{2}]]$.

6.153. Find the angle α between the directions $[2\;3\;6]$ and $[3\;2\;1]$ in a cubic crystal.

6.154. Find the angle α between the planes $(1\;2\;3)$ and $(3\;2\;1)$ in a cubic crystal.

6.155. A string of length l can perform transverse oscillations in a given plane. The speed of propagation of the oscillations is v. Determine the number dN_ω of normal oscillations of the string with frequencies within the interval from ω to $\omega + d\omega$.

6.156. In a rectangular membrane of area S, the speed of propagation of transverse oscillations is v. Determine the number dN_ω of normal oscillations of the membrane with frequencies within the interval from ω to $\omega + d\omega$. Compare the result with the answer to the preceding problem.

6.157. An elastic body of volume V has the shape of a rectangular parallelepiped. The speed of propagation of transverse oscillations in the body is v. Determine the number dN_ω of normal transverse oscillations of the body with frequencies within the interval from ω to $\omega + d\omega$. Compare the result with the answers to Problems 6.155 and 6.156.

6.158. Determine the Debye temperature Θ for a one-dimensional chemically simple crystal, i.e. a train of iden-

tical atoms performing oscillations along the straight line on which they are located. The concentration of the atoms (their number per unit length) is $n = 5.00 \times 10^9$ m^{-1}, the speed of waves in the crystal is $v = 3000$ m/s.

6.159. Determine the Debye temperature Θ for a two-dimensional crystal consisting of atoms of a single species. The atoms can oscillate in the plane on which they are located. The equilibrium positions of the atoms are at the vertices of rectangular crystal cells. The concentration of the atoms (their number per unit area) is $n = 2.50 \times 10^{19}$ m^{-2}, the speed of transverse and longitudinal waves in the crystal is the same and equals $v = 3000$ m/s.

6.160. Determine the Debye temperature Θ for a three-dimensional crystal consisting of atoms of a single species. The equilibrium positions of the atoms are at the vertices of rectangular crystal cells. The concentration of the atoms (their number per unit volume) is $n = 1.25 \times 10^{29}$ m^{-3}. The speed of transverse and longitudinal waves in the crystal is the same and equals $v = 3000$ m/s. Compare the result obtained with the answers to Problems 6.158 and 6.159.

6.161. Find the mean value of the frequency $\langle\omega\rangle$ of normal oscillations:

(a) of the one-dimensional crystal from Problem 6.158;
(b) of the two-dimensional crystal from Problem 6.159;
(c) of the three-dimensional crystal from Problem 6.160.

6.162. Table 6.2 gives the values of the speed $v_\perp$ of transverse waves, the speed $v_\parallel$ of longitudinal waves (in m/s), and the number n of atoms in unit volume (1 m^3) for: (a) beryllium; (b) silver; (c) lead. Determine the Debye temperature for these metals.

Table 6.2

Metal	$v_\perp$	$v_\parallel$	n
Beryllium	8830	12 550	1.23×10^{29}
Silver	1590	3 600	5.86×10^{28}
Lead	700	2 160	3.28×10^{28}

6.163. The speed of transverse elastic waves in aluminium is $v_\perp = 3130$ m/s, and of longitudinal waves is $v_\parallel = 6400$ m/s. Determine the Debye temperature Θ for aluminium.

6.164. Determine the energy U_0 of the zero-point oscillations of a mole of argon cooled to solidification (the Debye temperature is $\Theta = 92$ K).

6.165. At a pressure of $p = 1013$ hPa, argon solidifies at a temperature equal to 84 K. The Debye temperature for argon is $\Theta = 92$ K. It has been established experimentally that at $T_1 = 4.0$ K the molar heat capacity of argon is $C_1 = 0.174$ J/(mol·K). Determine the value of the molar heat capacity C_2 of argon at $T_2 = 2.0$ K.

6.166. The integral $\int\limits_0^{x_m} \frac{e^x x^4\,dx}{(e^x-1)^2}$ in the Debye expression for the heat capacity takes on the value of $4\pi^4/15$ at $x_m \to \infty$. Having this in view, determine the approximate value of the molar heat capacity C of argon ($\Theta = 92$ K) at $T = 4.0$ K. Compare the value obtained with the experimental value given in the preceding problem.

6.167. Find the maximum energy ε_m of a phonon that can be excited in a crystal characterized by a Debye temperature of $\Theta = 300$ K. A photon of what wavelength λ would have the same energy?

6.168. Using the data of Table 6.2 and the answer to Problem 6.162, appraise the maximum value p_m of the momentum of a phonon in silver. A photon of what wavelength λ would have the same momentum?

6.169. The atomic mass of silver is $A_r = 107.9$, its density is $\rho = 10.5$ g/cm^3. Using these data, appraise the maximum value p_m of the momentum of a phonon in silver. Compare the result obtained with the answer to the preceding problem.

6.170. What happens to the energy spectrum of phonons when the volume of a crystal increases twofold (with a constant concentration of the atoms)?

6.171. Does the mean number $\langle n_i \rangle$ of phonons of a strictly definite frequency ω_i excited at a given temperature in a crystalline specimen depend on the number of atoms in this specimen?

6.172. How does the number dn of phonons with frequen-

cies from ω to $\omega + d\omega$ excited at a given temperature in a crystalline specimen depend on the number N of atoms in this specimen?

6.173. How does the total number n of phonons of all frequencies excited at a given temperature in a crystalline specimen depend on the number N of atoms in this specimen?

6.174. What number $\langle n_m \rangle$ of phonons of the maximum frequency is excited on an average at a temperature of $T = 400$ K in a crystal whose Debye temperature is $\Theta = 200$ K?

6.175. Adopting a value of the Debye temperature of $\Theta = 208$ K for silver [see the answer to item (b) of Problem 6.162], determine:

(a) the maximum value ε_m of a phonon's energy;

(b) the mean number $\langle n_m \rangle$ of phonons with the energy ε_m at a temperature of $T = 300$ K.

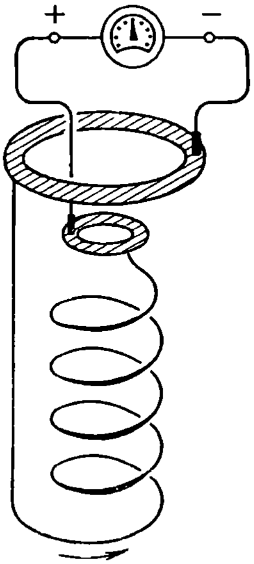

Fig. 6.2

6.176. In an experiment similar to that of Tolman and Stewart, a coil with a diameter of $d = 500$ mm had $N = 400$ turns of copper wire. The coil was connected via sliding contacts to a ballistic galvanometer with its zero at the middle of the scale (Fig. 6.2). The total resistance of the coil, the galvanometer, and the connecting wires had a value of $R = 50.0\ \Omega$. The coil was brought into uniform rotation in the direction indicated by the arrow at a speed of $n = 100$ rps and was then sharply retarded. This caused a charge of $q = 11.0$ nC to pass through the galvanometer and make the pointer deflect to the left. Determine the sign and the magnitude of the specific charge of the current carriers in the copper.

6.177. A copper plate has a length of $l = 60.0$ mm, a width of $b = 20.0$ mm, and a thickness of $a = 1.00$ mm (Fig. 6.3). When a current of $I = 10.0$ A is passed along the plate

a potential difference of $U_{12} = 0.51$ mV is observed between points *1* and *2*, while the potential difference between points *3* and *4* is zero. If, without switching off the current, a uniform magnetic field with the induction $B = 0.100$ T is set up perpendicular to the plate, a potential difference of $U_{34} = 0.055$ μV appears between points *3* and *4*. Using these data, determine the concentration n of the free electrons and their mobility u_0 for copper.

6.178. Does the mean energy $\langle \varepsilon \rangle$ of the free electrons in a crystal depend on the number of atoms forming the crystal?

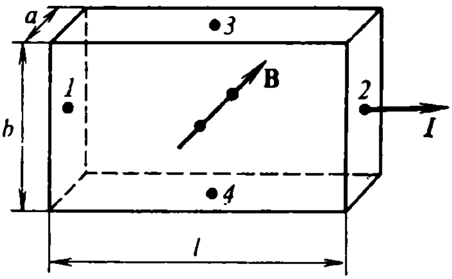

Fig. 6.3

6.179. What happens to the energy spectrum of the free electrons when the number N of atoms forming a crystal increases η times?

6.180. What happens to the interval $\Delta\varepsilon$ between adjacent energy levels of the free electrons in a metal when the volume of the metal increases threefold?

6.181. A crystalline specimen contains 0.17 mol of a chemically simple substance. The width of the allowed zone of energies is $\Delta E = 10$ eV. What is the mean value $\langle \Delta\varepsilon \rangle$ of the interval between adjacent energy levels?

6.182. Write an expression for the interval $\Delta\varepsilon$ between adjacent energy levels of the free electrons in a metal.

6.183. Assuming the volume V of a metal specimen to be 1 cm^3, evaluate by the formula obtained in the preceding problem the interval $\Delta\varepsilon$ (in eV) between adjacent energy levels of the free electrons for values of the energy E equal to: (a) 0.1 eV; (b) 1 eV; (c) 3 eV; (d) 5 eV.

6.184. Assuming that one free electron falls to the share of each copper atom, determine:

(a) the Fermi level $E_F(0)$ at absolute zero for copper;

(b) the mean kinetic energy $\langle E \rangle$ of the free electrons at absolute zero;

(c) the temperature T at which the mean kinetic energy of the electrons in a classical electron gas would equal the mean energy of the free electrons in copper at $T = 0$.

6.185. Assuming that the Fermi level at absolute zero $E_F(0) = 5$ eV, determine the Fermi level at $T = 300$ K. Express E_F in terms of $E_F(0)$.

6.186. What part η of the free electrons in a metal has at absolute zero a kinetic energy exceeding half of the maximum value?

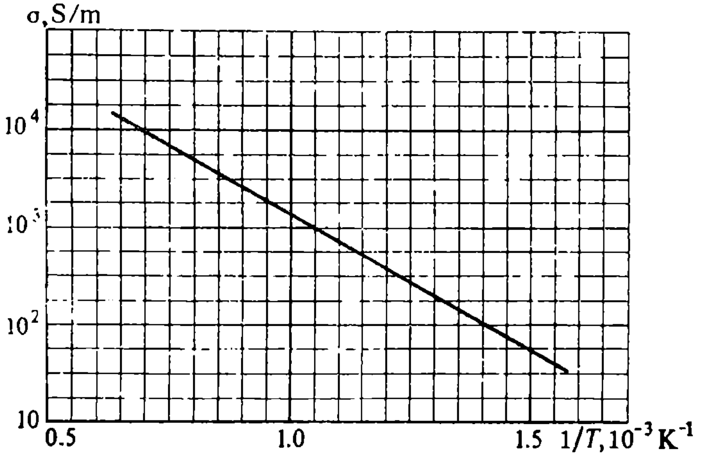

Fig. 6.4

6.187. What part η of the free electrons in a metal has at absolute zero a kinetic energy exceeding the mean energy?

6.188. What is the probability P of the fact that a free electron will be in a state with an energy equal to the Fermi energy E_F?

6.189. What number $\langle n \rangle$ of free electrons occupies on an average a level with an energy equal to the Fermi energy E_F?

6.190. Assuming that the charge of a current carrier equals the double elementary charge, calculate the magnitude of a quantum of the magnetic flux Φ_0.

6.191. Figure 6.4 shows an experimentally obtained relation between the conductivity σ of silicon and a quantity that is the reciprocal of the absolute temperature T. Determine the width of the forbidden energy band ΔE for silicon.

6.192. How many times will the conductivity change when the temperature is increased from 300 to 310 K: (a) in a metal; (b) in an intrinsic semiconductor whose forbidden band width is $\Delta E = 0.300$ eV? What is the nature of the change in both cases?

6.193. What is the work function A of a metal if an increase in its temperature from the value $T = 2000$ K by $\Delta T = 0.0100$ K increases the saturation current of thermionic emission by 0.0100%?

6.194. The gap between the plates of a parallel-plate capacitor is $d = 1.00$ mm. One of the plates is made from platinum (for which the work function is $A = 5.29$ eV), and the other from aluminium (for which $A = 3.74$ eV). The plates are short-circuited by a copper wire. What will the strength E of the electric field between the plates be? How will the field be directed?

6.195. A battery with an e.m.f. of $\mathcal{E} = 10.0$ V is connected between the electrodes of a two-electrode tube (a diode). The material of the cathode is tungsten (for which the work function is $A_W = 4.50$ eV), and that of the anode is nickel (for which $A_{Ni} = 4.84$ eV). What energy E is acquired by an electron along its path from the cathode to the anode? The speed at which the electron flies out of the cathode may be disregarded.

6.196. Two metals have a free electron concentration of $n_1 = 1.00 \times 10^{28}$ m^{-3} and $n_2 = 1.00 \times 10^{29}$ m^{-3}. Determine the internal contact potential difference U_{int} appearing when these metals are brought into contact.

6.8. Binding Energy of a Nucleus. Radioactivity

6.197. Determine the binding energy E_b/A (in MeV) per nucleon for a nucleus of: (a) ${}^{11}_{5}B$; (b) ${}^{20}_{10}Ne$; (c) ${}^{28}_{14}Si$; (d) ${}^{56}_{26}Fe$; (e) ${}^{68}_{30}Zn$; (f) ${}^{137}_{56}Ba$; (g) ${}^{207}_{82}Pb$; (h) ${}^{235}_{92}U$. Plot E_b/A versus the mass number A on graph paper.

6.198. The nucleus of a radium atom ${}^{226}_{88}Ra$ freely at rest experiences an alpha decay. The binding energy of the nucleus ${}^{226}_{88}Ra$ is 1731.6 MeV, of the nucleus ${}^{222}_{86}Rn$ is 1708.2 MeV, and of an alpha particle is 28.3 MeV. Assuming that the daughter nucleus of radon ${}^{222}_{86}Rn$ is formed in an unexcited state, determine:

(a) the speed v_α of the alpha particle formed;

(b) the speed v of the daughter atom.

6.199. Proceeding from the law of radioactive transformation, find:

(a) the half-life T of a radioactive nucleus;

(b) the average lifetime τ of the nucleus;

(c) the relation between T and τ.

Consider the decay constant λ to be known.

6.200. What is larger—the average lifetime τ of a radioactive nucleus or the half-life T? How many times?

6.201. What part η of the atoms of a radioactive substance remains undecayed after the time t equal to three average lifetimes τ of an atom?

6.202. What part η of the atoms of a radioactive substance decays during the time t equal to three half-lives T?

6.203. What is the probability P of the fact that a radioactive atom will decay during the time t equal to the half-life T?

6.204. The average lifetime of the atoms of a radioactive substance is $\tau = 1.00$ s. Determine the probability P of the fact that the nucleus will decay during a time interval t equal to: (a) 1.00 s; (b) 10.0 s; (c) 0.100 s.

6.205. A preparation consists of a substance that experiences a chain of radioactive transformations. The first substance in the series (the parent substance) has such a long half-life that the formation of the daughter substance may be considered to occur at a constant rate of $b = 1.00 \times 10^5$ nuclei/s. The daughter substance has a half-life of $T = 10.0$ days. Assuming that at the instant $t = 0$ we have only the parent substance, find:

(a) the dependence of the number N of daughter substance nuclei contained in the preparation on the time t;

(b) the number N of daughter substance atoms after a time elapses equal to the half-life of the nuclei of this substance;

(c) the number N of daughter substance atoms after a time elapses that considerably exceeds the half-life of this substance.

6.206. The radioactive nuclei of X with a decay constant of λ_1 transform into the radioactive nuclei of Y with a decay constant of λ_2. Assuming that at the instant $t = 0$ there are only N_{X0} nuclei of X,

(a) find the dependence of the number N_Y of nuclei of Y on the time t;

(b) determine the time t_m after which N_Y reaches its maximum value;

(c) investigate the case $\lambda_1 \ll \lambda_2$, $\lambda_1 t \ll 1$. Compare the result obtained for N_Y in this case with the answer to Problem 6.205.

6.207. To determine the age t of an ancient fabric found in one of the Egyptian pyramids, the concentration of radiocarbon atoms ^{14}C in it was determined. It was found to correspond to 9.2 decays per minute per gram of carbon. The concentration of ^{14}C in living plants corresponds to 14.0 decays per minute per gram of carbon. The half-life of ^{14}C is 5730 years. Proceeding from these data, appraise t.

ANSWERS

PART 1. THE PHYSICAL FUNDAMENTALS OF MECHANICS

1.1. The displacement $\Delta\mathbf{r}$ of a point during the time interval from t_1 to t_2.

1.2. The increment of the coordinate x during the time interval from t_1 to t_2.

1.3. No, it cannot.

1.4. Yes, it can, if the vectors **a** and $\Delta\mathbf{a}$ have the same direction.

1.5. $\Delta a = -|\Delta\mathbf{a}|$.

1.6. $\Delta\mathbf{a} = -2\mathbf{a}$, $|\Delta\mathbf{a}| = 2a$, $\Delta a = 0$.

1.7. (a) $|\Delta\mathbf{a}| \approx a\delta\varphi$; (b) $\Delta a = 0$.

1.8. (a) $\Delta\mathbf{v} = 1\mathbf{e}_x + 1\mathbf{e}_y + 1\mathbf{e}_z$ (m/s); (b) $|\Delta\mathbf{v}| = 1.73$ m/s; (c) $\Delta v = 1.57$ m/s.

1.9. $\cos\alpha = \dfrac{a_x b_x + a_y b_y + a_z b_z}{\sqrt{(a_x^2 + a_y^2 + a_z^2)\,(b_x^2 + b_y^2 + b_z^2)}}$.

1.10. $\alpha = 50.8°$.

1.11. $\mathbf{a}\,[\mathbf{bc}] = \mathbf{b}\,[\mathbf{ca}] = abc\cos\alpha$.

1.13. (a) $\langle\mathbf{v}\rangle = (4/\tau)\,R\,(\mathbf{e}_x - \mathbf{e}_y)$; (b) $\langle\mathbf{v}\rangle = -(4/\tau)\,R\mathbf{e}_y$; (c) $\langle\mathbf{v}\rangle = -(4/3\tau)\,R\,(\mathbf{e}_x + \mathbf{e}\ )$; (d) $\langle\mathbf{v}\rangle = 0$; (e) $\langle\mathbf{v}\rangle = -(4/\tau)\,R\mathbf{e}_x$.

1.14. $|\langle\mathbf{v}\rangle| = (2\sqrt{2}/3\pi)\,\langle v\rangle = 0.300\,\langle v\rangle$.

1.15. (a) $\langle v\rangle = 3\pi R/\tau = 4.7$ m/s; (b) $|\langle\mathbf{v}\rangle| = 2R/\tau = 1.00$ m/s; (c) $|\langle\mathbf{a}\rangle| = 6\pi R/\tau^2 = 0.94$ m/s^2.

1.16. $\langle\mathbf{a}\rangle = (2a/\pi)\,(\mathbf{e}_x + \mathbf{e}_y)$; $|\langle\mathbf{a}\rangle| = (2\sqrt{2}/\pi)\,a = 0.900a$.

1.17. (a) The trajectory is a straight line emerging from the origin of coordinates; (b) the trajectory is a line all of whose points are on a sphere of radius r with its centre at the origin of coordinates.

1.18. (a) $s = 500$ m; (b) $|\Delta\mathbf{r}| = 500$ m; (c) the particle travels in one direction along a straight trajectory, therefore $s = |\Delta\mathbf{r}|$.

1.19. (a) $\mathbf{v} = 6t\mathbf{e}_x + 2\mathbf{e}_y$ (m/s), $\mathbf{a} = 6\mathbf{e}_x$ (m/s^2); (b) $v = 6.3$ m/s; (c) $s \approx 63$ m.

1.20. (a) $\Delta\mathbf{r} = 2\mathbf{e}_x + 4\mathbf{e}_y + 8\mathbf{e}_z$ (m); (b) $v = 13$ m/s.

1.21. (a) $v = 5.4$ m/s; (b) $\mathbf{a} = b\,(2\mathbf{e}_x + 3\mathbf{e}_y + 4\mathbf{e}_z)$, $a = 5.4$ m/s^2; (c) $s = 13.5$ m; (d) the particle travels with uniform acceleration along a rectilinear trajectory.

1.22. $R = v^2/a$.

1.23. A circle or a helical line.

1.24. (a) $dv/dt = -0.53$ m/s^2; (b) $R = 3.8$ m.

1.25. (a) $v_x = -(2\pi/T)\,A\sin(2\pi/T)\,t$, $a_x = -(4\pi^2/T^2)\,A \times \cos(2\pi/T)\,t$; (b) $s_1 = 0.293A$; (c) $s_2 = 0.707A$; (d) $s = 4A$.

1.26. $v = A$, $a = A\omega$, $\alpha = \pi/2$; the particle moves uniformly along a circle of radius $R = A/\omega$.

1.27. (a) $\mathbf{r} = A(\cos\omega t\cdot\mathbf{e}_x + \sin\omega t\cdot\mathbf{e}_y)$, $\mathbf{v} = A\omega(-\sin\omega t\cdot\mathbf{e}_x + \cos\omega t\cdot\mathbf{e}_y)$, $\mathbf{a} = -A\omega^2(\cos\omega t\cdot\mathbf{e}_x + \sin\omega t\cdot\mathbf{e}_y) = -\omega^2\mathbf{r}$, $r = A$, $v = A\omega$, $a = A\omega^2$.

(b) $\mathbf{rv} = 0$. This signifies that the vectors $\mathbf{r}$ and $\mathbf{v}$ are mutually perpendicular.

(c) $\mathbf{ra} = -\omega^2A^2 = -ra$. This signifies that the vectors $\mathbf{r}$ and $\mathbf{a}$ are directed oppositely.

(d) $x^2 + y^2 = A^2$—a circle of radius A.

(e) Counterclockwise.

(f) The particle moves uniformly counterclockwise along a circle of radius A whose centre is at the origin of coordinates.

(g) The particle's direction of motion will be reversed.

1.28. (a) $l = \dfrac{v_0^2\sin 2\alpha}{g}$; (b) $h = \dfrac{v_0^2\sin^2\alpha}{2g}$; (c) $\tau = \dfrac{2v_0\sin\alpha}{g}$; (d) $y' = -\dfrac{g}{2v_0^2\cos^2\alpha}x'^2$; (e) $\left|\dfrac{d\mathbf{v}}{dt}\right| = g$, $\dfrac{d\,|\mathbf{v}|}{dt} = 0$; (f) $R_O = \dfrac{v_0^2}{g\cos\alpha}$, $R_{O'} = \dfrac{v_0^2\cos^2\alpha}{g}$.

1.29. $\langle\mathbf{v}\rangle = \mathbf{v}_0 + \mathbf{g}\tau/2$.

1.30. $\alpha = 11.5°$ or $78.5°$.

1.31. $v(s) = \left[v_0^2 + 2\int_0^s f(s)\,ds\right]^{1/2}$.

1.32. $t = \int_0^s \dfrac{ds}{v(s)}$.

1.33. (a) $s = \dfrac{v_0}{b}(1 - e^{-bt})$; (b) $v = v_0e^{-bt}$; (c) $s \approx v_0t$; $v \approx v_0(1 - bt)$.

1.34. The constant b is the reciprocal of the time interval τ_e during which the particle's velocity diminishes e times.

1.35. $s = 7.0 \times 10^2$ m.

1.36. $\sin\alpha_1/\sin\alpha_2 = v_1/v_2 = 1.66$.

1.37. (a) 0.01; (b) 0.001; (c) 0.005; (d) 0.05; (e) 0.002; (f) 4×10^{-6}.

1.38. (a) $\alpha = 5.7°$; (b) $t = l/\sqrt{v^2 - u^2} = 10.0$ s; (c) $s = 201$ m.

1.39. (a) At an angle to line AB whose sine equals u/v.

(b) $t_1 = 2l/\sqrt{v^2 - u^2}$* $= 402$ s (402.02 s), $t_2 = 2lv/(v^2 - u^2) = 404$ s (404.04 s).

(c) $t_1 = \dfrac{2l}{v}\left(1 + \dfrac{1}{2}\dfrac{u^2}{v^2}\right) = 402$ s (402.00 s), $t_2 = \dfrac{2l}{v}\left(1 + \dfrac{u^2}{v^2}\right) = 404$ s (404.00 s).

1.40. (a) $\alpha = 78.5°$; (b) $t = 11.5$ s; (c) $s = 1150$ m.

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* Compare with the answer to Problem 1.38(b).

1.41. $\alpha = 0$.
1.42. $\dot{\mathbf{a}} = [\boldsymbol{\omega}\mathbf{a}]$, $\ddot{\mathbf{a}} = -\mathbf{a}\,(\boldsymbol{\omega}\boldsymbol{\omega})$.
1.43. $\mathbf{v}_1 = v\,(\mathbf{e}_x + \mathbf{e}_y)$, $\mathbf{v}_2 = 2v\mathbf{e}_x$, $\mathbf{v}_3 = v\,(\mathbf{e}_x - \mathbf{e}_y)$.
1.44. (a) $\varphi = 20$ rad; (b) about an axis in the plane x, y and making an angle of 63° with the x-axis.
1.45. $\alpha = 4.7$ rad/s^2.
1.46. $\langle\omega\rangle = 29$ rad/s, $\langle\alpha\rangle = 24$ rad/s^2.
1.47. $s \approx v\omega\,(t_2^2 - t_1^2)/2 = 5.7 \times 10^2$ m.
1.48. $f = 0.25$.
1.49. 1. (a) $F = 6.0$ N; (b) $F = 4.0$ N.
2. The sum of the results in cases (a) and (b) equals the force F_0.
1.50. (a) $F = 7.2$ N; (b) $F = 2.8$ N.
1.51. (a) $F = 4.8$ N; (b) $F = 5.2$ N.
1.52. 1. (a) $a = 5.8$ m/s^2; (b) $F = 0.83$ N.
2. The second block would slide with a greater acceleration than the first one. As a result, a gap would form between the blocks that would grow with time.
1.53. (a) $a = 0.1g = 0.98$ m/s^2; (b) $F_{12} = 0.2Mg = 2.0$ N; (c) $F = 0.9mg = 4.4$ N.
1.54. $F = 970$ kgf.
1.55. $x^2 + y^2 = (mv/qB)^2$.
1.56. (a) $n = 18$ rpm; (b) $F = 1.06mg = 2.1$ N.
1.57. $r \leqslant 1.8$ m.
1.58. (a) $h = 0.46$ m; (b) $t_1 = 0.89$ s; (c) $t_2 = t_1 = 0.89$ s; (d) $v = v_0 = 3.00$ m/s.
1.59. (a) $h = 0.36$ m; (b) $t_1 = 0.70$ s; (c) $t_2 = 0.93$ s; (d) $v = 2.3$ m/s; (e) $A = -1.9$ J.

1.60. (b) $v = \dfrac{mg}{k}\left(1 - \dfrac{1}{\eta}\right)[1 - e^{-(k/m)t}]$.

1.61. (a) $f = 0.38$; (b) $A = -0.0098$ J; (c) $v = 2.7$ m/s.

1.62. (a) $F(t) = \dfrac{3}{2}\dfrac{m}{l}g^2t^2 = 3mg\,\dfrac{l-h}{l}$ (here h is the height of the top end of the chain over the table at the instant t). Consequently, $F(t)$ equals the triple weight of the part of the chain lying on the table at the instant t; (b) $\langle F\rangle = mg$.
1.63. $A = 6b$ J.
1.64. (a); (b); (c) $A = 0$.
1.65. $A = 0$.

1.66. $A = \displaystyle\int_{s_1}^{s_2} ma_\tau(s)\,ds$.

1.67. (a) $\langle P\rangle = 97$ W; (b) $P = \langle P\rangle\sqrt{2} = 137$ W.

1.68. $A = \dfrac{mv^2}{2} + mgh - A_{\text{res}}$.

1.69. (a) $P(t) = mg\,(gt - v_0\sin\alpha)$; (b) $P = 0$; (c) $\langle P\rangle_{\text{rise}} = -\dfrac{1}{2}mgv_0\sin\alpha$; (d) $\langle P\rangle_{\text{fl}} = 0$.

1.70. $P(t) = \dfrac{1}{m}(2t^3 + 3t^5)$.

1.71. (a) $\Delta E = 3$ J; (b) $\Delta E = -2$ J.
1.72. (a) $-\Delta E = -3$ J; (b) $-\Delta E = 2$ J.
1.73. $E_k = 14$ J.
1.74. (a) $A = -10$ J; (b) the kinetic energy receives an increment $\Delta E_k = -10$ J, i.e. diminishes by 10 J; (c) $E_{k,1} \geqslant 10$ J.
1.75. The following relation is obvious: $\mathbf{R}_i = \mathbf{R}_C + \mathbf{r}_i$, where $\mathbf{R}_i$ is the position vector of the i-th point particle in an l-frame, $\mathbf{R}_C$ is the position vector of the centre of mass, $\mathbf{r}_i$ is the position vector of the i-th point drawn from the centre of mass. Differentiation of this relation yields $\mathbf{V}_i = \mathbf{V}_C + \mathbf{v}_i$. By definition, we have

$$E_{k,1} = \frac{1}{2}\sum m_i \mathbf{V}_i^2 = \frac{1}{2}\sum m_i (\mathbf{V}_C + \mathbf{v}_i)^2$$

$$= \frac{1}{2}\sum m_i \mathbf{V}_C^2 + \mathbf{V}_C \sum m_i \mathbf{v}_i + \frac{1}{2}\sum m_i \mathbf{v}_i^2$$

The first term can be written in the form $\frac{1}{2} m\mathbf{V}_C^2$ $(m = \sum m_i)$. The sum $\sum m_i \mathbf{v}_i$ is equivalent to $m\mathbf{v}_C$, where $\mathbf{v}_C$ is the velocity of the centre of mass in the c.m.-frame, i.e. zero; consequently, the second term equals zero. The third term is $E_{k,C}$.

Therefore, $E_{k,1} = E_{k,C} + \frac{1}{2} mV_C^2$, Q.E.D.

1.76. $A = -28$ J.
1.77. $E_k = 24a$.
1.78. (a) $v_1 = v_2$; (b) $t_1 > t_2$.
1.79. The time of sliding is the same for both planes.
1.80. $A = -mgh$.
1.81. $h = \frac{2}{3}R$.
1.82. 1. (a) $h = \frac{5}{2}R$; (b) $F = 0$.

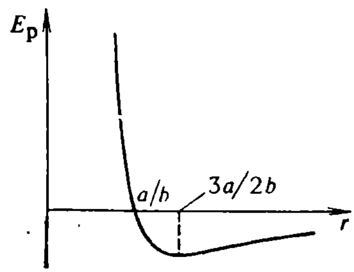

Fig. A.1

1.83. 1. (a) $\nabla r = \mathbf{e}_r$; (b) $\nabla\left(\frac{1}{r}\right) = -\frac{1}{r^2}\mathbf{e}_r$; (c) $\nabla f(r) = \frac{df}{dr}\mathbf{e}_r$ ($\mathbf{e}_r$ is the unit vector of the position vector $\mathbf{r}$ of point P).

Note. For case (b), $\mathbf{e}_l = -\mathbf{e}_r$, for case (c), $\mathbf{e}_l = \mathbf{e}_r$ if $df/dr > 0$, and $\mathbf{e}_l = -\mathbf{e}_r$ if $df/dr < 0$.

1.84. (a) $\mathbf{F} = \frac{\alpha}{r^2}\mathbf{e}_r$, $A = 0.082\alpha$; (b) $\mathbf{F} = -k\mathbf{r}$, $A = -7.5k$.

1.85. (a) $\mathbf{F} = a\left\{-\frac{1}{y}\mathbf{e}_x + \left(\frac{x}{y^2} + \frac{1}{z}\right)\mathbf{e}_y - \frac{y}{z^2}\mathbf{e}_z\right\}$; (b) $A = E_{p,1} - E_{p,2} = -\frac{a}{3}$.

1.86. (a) Yes, it does, when $r = 3a/2b$; (b) see Fig. A.1.

1.87. $E_p = -2E_k$, $E = -E_k$, $E = \frac{1}{2}E_p$.

1.88. At $v_0 > \sqrt{2\alpha/mr_0}$, a hyperbola, at $v_0 = \sqrt{2\alpha/mr_0}$, a para-

bola, and at $v_0 < \sqrt{2\alpha/mr_0}$, an ellipse (in a particular case, a circle).

1.89. When $v_0 = \sqrt{\alpha/mr_0}$, which is equivalent to the condition $E = -m\alpha^2/2L^2$ (E is the total energy, and L is the angular momentum of the particle).

1.90. $h_2 = 1.50$ m.

1.91. $k = mg \sin\alpha/(\eta - 1)\, v^2 = 0.66$.

1.92. (a) $\Delta l = F/k = 0.53$ mm; (b) $v = \sqrt{2(g - F/m)(l_0 + F/k)} \approx \approx \sqrt{2(g - F/m)\, l_0} = 4.2$ m/s; (c) $\Delta l_{max} = \frac{mg}{k}\left\{1 + \sqrt{1 + \frac{2k}{m^2g^2}\left[mgl_0 - F\left(l_0 + \frac{F}{2k}\right)\right]}\right\} = 10$ cm.

1.93. $\mathbf{r}_C = \frac{1}{6}(14\mathbf{e}_x + 11\mathbf{e}_y + 11\mathbf{e}_z)$ (m).

1.94. $l = \frac{3}{4}h$.

1.95. $\mathbf{p} = 0$.

1.96. The centre of mass is stationary.

1.97. It moves with the acceleration $\mathbf{g}$.

1.98. (a) $\Delta\mathbf{p} = m\mathbf{g}\tau$; (b) $\langle\mathbf{p}\rangle = m\mathbf{v}_0 + m\mathbf{g}\tau/2$.

1.99. $\mathbf{p} = m\mathbf{v}_0 + (F/\omega)\{\mathbf{e}_x \sin\omega t - \mathbf{e}_y(1 - \cos\omega t)\}$.

1.100. Both spheres will move at a speed of 2 m/s in the direction in which the first one moved before the collision.

1.101. (a) $\eta = 1/2$; (b) $\eta = 10/11 \approx 0.91$; (c) $\eta = 1/11 \approx 0.091$.

1.102. (a) $m_1 < m_2$; (b) the sphere will stop; (c) the sphere will fly in the opposite direction at virtually the same speed that it had prior to the collision.

1.103. (a) $v_{1x} = -1.40$ m/s, $v_{2x} = 0.60$ m/s.

1.104. 1. (a) $m_1 < m_2$; (b) $m_1 < 3m_2$; (c) $m_1 > m_2$; (d) $m_1 < m_2$.
2. No, it isn't.
3. $\beta < 45°$.
4. (a) $\eta = \frac{2m_1}{m_1 + m_2}$; (b) $\eta = \frac{m_1}{m_1 + m_2}$; (c) and (d) $\eta = \frac{4m_1m_2}{(m_1 + m_2)^2}$.
6. $\eta < 3/4$. 7. $m_1 = m_2$, $\beta = 0$.
8. (a) $\beta = 4°$; (b) $\beta = 45°$.

1.105. $v_1 = 0.146$ m/s, $v_2 = 0.854$ m/s.

1.106. (a) $v_1 = 0.026$ m/s, $v_2 = 0.974$ m/s; (b) $v_1 = 0.342$ m/s, $v_2 = 0.658$ m/s.

1.107. $v = 1.0$ m/s (the direction of $\mathbf{v}$ is opposite that of $\mathbf{v}_0$).

1.108. $r_{min} = \frac{4ke^2}{m_{red}v_0^2} = 0.69 \times 10^{-12}$ m $\left(m_{red} = \frac{m_p m_\alpha}{m_p + m_\alpha}\right.$ is the reduced mass of a proton and an alpha particle$\left.\right)$.

1.109. (a) $v = u\,[1 - e^{-(m/M)\,t}] = 4.5$ m/s; (b) $v_{max} = u = 10.0$ m/s.

1.110. (a) $\mathbf{M} = -2\mathbf{e}_x - 11\mathbf{e}_y + 10\mathbf{e}_z$ (N·m); (b) $M = 15$ N·m; (c) $M_z = 10$ N·m,

1.111. $F = \frac{3M}{f} \frac{d_2^2 - d_1^2}{d_2^3 - d_1^3} = 3.3 \times 10^3$ N.

1.112. $\Delta \mathbf{L} = -40\mathbf{e}_x$ (kg·m²/s).

1.113. $\mathbf{L} = -\frac{mv_0^3 \sin^2\alpha \cos\alpha}{2g} \mathbf{e}_z$.

1.114. 1. (a) $\mathbf{p} = \mathbf{p}_1 + \mathbf{p}_2 \neq 0$; (b) $\mathbf{L}_1 = \{lp_2 + b_1(p_2 - p_1)\}\,\mathbf{n}$, $\mathbf{L}_2 = \{lp_1 + b_2(p_1 - p_2)\}\,\mathbf{n}$ ($\mathbf{L}_1 \neq \mathbf{L}_2$).

2. (a) $\mathbf{p} = 0$; (b) $\mathbf{L}_1 = \mathbf{L}_2 = lp\mathbf{n}$ (compare with the moment of a force couple).

1.116. We use the relations $\mathbf{R}_i = \mathbf{R}_C + \mathbf{r}_i$ and $\mathbf{V}_i = \mathbf{V}_C + \mathbf{v}_i$ (see the answer to Problem 1.75). By definition,

$$\mathbf{L}_O = \sum m_i [\mathbf{R}_i, \mathbf{V}_i] = \sum m_i [(\mathbf{R}_C + \mathbf{r}_i), (\mathbf{V}_C + \mathbf{v}_i)]$$
$$= \sum m_i [\mathbf{R}_C, \mathbf{V}_C] + \sum m_i [\mathbf{R}_C, \mathbf{v}_i] + \sum m_i [\mathbf{r}_i, \mathbf{V}_C] + \sum m_i [\mathbf{r}_i, \mathbf{v}_i]$$

The first term can be written in the form $[\mathbf{R}_C, m\mathbf{V}_C] = [\mathbf{R}_C, \mathbf{p}]$. The second term $[\mathbf{R}_C, \sum m_i \mathbf{v}_i] = [\mathbf{R}_C, m\mathbf{v}_C] = 0$ (because $\mathbf{v}_C$—the velocity of the centre of mass in the c.m.-frame—is zero). The third term $[\sum m_i \mathbf{r}_i, \mathbf{V}_C] = [m\mathbf{r}_C, \mathbf{V}_C] = 0$ (because $\mathbf{r}_C$—the position vector of the centre of mass in the c.m.-frame—is zero). The fourth term is $\mathbf{L}_C$—the angular momentum of the system of point particles in the c.m.-frame. Hence, $\mathbf{L}_O = \mathbf{L}_C + [\mathbf{R}_C, \mathbf{p}]$, Q.E.D.

1.117. (a) $\mathbf{M} = \frac{1}{2} mgh (\sin 2\alpha)\,\mathbf{n}$; (b) $\mathbf{L} = \frac{1}{2} mgh (\sin 2\alpha)\, t \cdot \mathbf{n}$.

1.118. (a) $\mathbf{M} = -mgv_0 (\cos\alpha)\, t \cdot \mathbf{e}_z$; (b) $\mathbf{L} = -\frac{1}{2} mgv_0 \times (\cos\alpha)\, t^2 \cdot \mathbf{e}_z$.

1.119. $|\Delta \mathbf{L}| = \frac{1}{2} mgv_0\tau^2 = 245$ kg·m²/s.

1.120. (a) The dumbbells rotate clockwise at the same angular speed, their centres are stationary; (b) $\omega = 2v/l = 2.00$ rad/s; (c) $\tau = \pi/\omega = 1.57$ s; (d) the dumbbells move translationally at the same speed and in the same directions as at the beginning, away from each other.

1.121. (a) The centres of the dumbbells move in the initial directions, the dumbbells rotate clockwise about their centres; (b) $v_C = v/2 = 0.500$ m/s; (c) $\omega = v/l = 1.00$ rad/s; (d) E diminishes to one-half of its initial value.

1.122. (a) The centres of the dumbbells move downward (in Fig. 1.23) at a speed of v, the dumbbells rotate clockwise at the same speed ω; (b) $\omega = 2v/l = 2.00$ rad/s; (c) $\tau = \pi/\omega = 1.57$ s; (d) the left-hand dumbbell is at rest, the right-hand one moves translationally at the same speed and in the same direction as initially, moving away from the left-hand one.

1.123. (b) The centres of both dumbbells move downward—the left-hand one at a speed of $v'_C = \frac{1}{2} v = 0.500$ m/s, the right-hand one at a speed of $v''_C = \frac{3}{2} v = 1.500$ m/s; (c) both dumbbells rotate at the

angular speed $\omega = v/l = 1.00$ rad/s; (d) E diminishes 4/3 times [compare with the answer to item (d) of Problem 1.121].

1.124. $\langle v \rangle = 2.97 \times 10^4$ m/s ≈ 30 km/s, $v_{max} = 1.017 \langle v \rangle = 3.02 \times 10^4$ m/s, $v_{min} = 0.984 \langle v \rangle = 2.92 \times 10^4$ m/s.

1.125. $\mu = m/2$.

1.126. $\mu \approx m\,(1 - m/M)$.

1.127. (a) The force of gravity mg, the force of normal pressure from the side of the disk equal to $-mg$, and the force of friction directed toward the axis of rotation; the magnitude of this force is $m\omega_0^2 r$. (b) In a frame rotating together with the disk. (c) In a frame rotating about the axis of rotation of the disk at an angular speed other than ω_0.

1.128. $P = 0$.

1.129. $A = 0$.

1.130. $A = 1.5$ J.

1.131. The body will deflect to the east over a distance of $x = 0.69$ mm; $\Delta s = \frac{3}{2} x$.

1.132. (a) $s = \omega R^2/v$; (b) no, it doesn't.

1.133. (a) 1.03 mm to the right (i.e. to the east); (b) 1.03 mm to the right (i.e. to the west).

1.134. $F = 3.8 \times 10^2$ N.

1.135. (a) $\mathbf{v}' = \frac{1}{2}[\mathbf{r}\boldsymbol{\omega}]$; (b) $\mathbf{v} = \frac{1}{2}[\boldsymbol{\omega}\mathbf{r}]$.

1.136. (a) $\tau = (1/\omega) \ln\{l/l_0 + \sqrt{(l/l_0)^2 - 1}\} = 3.0$ s; (b) $F = m\sqrt{4\omega^4 (l^2 - l_0^2) + g^2} = 1.00$ N; (c) $A = m\omega^2 (l^2 - l_0^2) = 0.10$ J.

1.137. $\mathbf{M} = -2ma^2 t\boldsymbol{\omega}$.

1.138. $\mathbf{L}(t) = -mv^2 t^2 \boldsymbol{\omega}$.

1.139. $\mathbf{r} = \mathbf{v}_0 t + [\boldsymbol{\omega}\mathbf{v}_0]/\omega^2$ or $\mathbf{r} = v_0 t \mathbf{e}_x - (v_0/\omega)\,\mathbf{e}_y$.

1.140. $F_1 = 1.76 \times 10^3$ N, $F_2 = 1.18 \times 10^3$ N.

1.141. (a) $\mathbf{F}_1 = -20\mathbf{e}_x$ (N); (b) $\alpha_0 \geqslant 60°$.

1.142. The centre of mass will move along a parabola with the acceleration $\mathbf{g}$.

1.143. $\mathbf{a}_C = 0.11\mathbf{g}$.

1.144. No, they cannot. Owing to the rigidity of the rod, the components of both velocities in the direction of the rod must be identical.

1.145. $I = \mu l^2$, where $\mu = m_1 m_2/(m_1 + m_2)$ is the reduced mass of the particles.

1.146. $I = \frac{1}{2} mR^2$.

1.147. (a) $\langle \rho \rangle_V = \frac{1}{3}(\rho_1 + 2\rho_2) = \frac{7}{3}\rho_1 = 1.17 \times 10^3$ kg/m^3, $\langle \rho \rangle_r = \frac{1}{2}(\rho_1 + \rho_2) = 2\rho_1$, $\langle \rho \rangle_V = \frac{7}{6}\langle \rho \rangle_r$. (b) $I = \frac{13}{10}\rho_1 l \pi R^4 = \frac{39}{70} mR^2 = 1.3 \times 10^{-3}$ kg$\cdot$m^2, $I = \frac{39}{35} I'$.

1.148. $I = \frac{2}{5} mR^2$.

1.149. $I = \frac{3}{10} mR^2$.

1.150. (a) $I_C = \frac{1}{12} ml^2$; (b) $I = \frac{1}{3} ml^2$.

1.151. (a) $I_C = \frac{1}{12} m(a^2 + b^2)$; (b) $I = \frac{1}{3} m(a^2 + b^2)$.

1.152. $I = \frac{1}{6} ma^2$.

1.154. $I = \frac{1}{10} ma^2$ (compare with the answer to Problem 1.149).

1.155. (a) $I_{pyr}/I_{cone} = \pi/3 = 1.047$; (b) $I_{cube}/I_{sph} = \frac{5}{12}(4\pi/3)^{2/3} = 13/12 = 1.083$.

1.156. $I_1 = I_2 = \frac{1}{4} mR^2$, $I_3 = \frac{1}{2} mR^2$.

1.157. $I = m\left(\frac{1}{4} R^2 + \frac{1}{12} h^2\right)$.

1.158. $h/R = \sqrt{3}$.

1.159. $I = 0.106\ \text{kg}\cdot\text{m}^2$.

1.160. $I = 10^{-2}\begin{pmatrix} 0.28 & 0 & 0 \\ 0 & 0.96 & 0 \\ 0 & 0 & 1.12 \end{pmatrix}$ $(\text{kg}\cdot\text{m}^2)$.

1.161. $b_x = b_y = b_z = 6$.

1.162. $\mathbf{b} = \mathbf{a}$.

1.163. $I = \begin{pmatrix} 0.100 & 0 & 0 \\ 0 & 0.100 & 0 \\ 0 & 0 & 0.100 \end{pmatrix} = 0.100 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} =$
$= 0.100E$ $(\text{kg}\cdot\text{m}^2)$ (see Problem 1.162).

1.164. When the body rotates about one of its principal axes of inertia.

1.165. When (a) the relevant body is a spherical top; (b) the vectors ω and $\mathbf{M}$ coincide in direction with one of the body's principal axes of inertia.

1.166. When the body rotates about one of its principal axes of inertia.

1.167. (a) $L = 1.5 \times 10^{-2}\ \text{kg}\cdot\text{m}^2/\text{s}$, $\alpha = 25°$; (b) $E_k = 1.2 \times 10^{-2}$ J.

1.168. (a) $\mathbf{L}(t) = 2mr^2\omega(t)$. (b) No, it doesn't.

1.169. $\mathbf{L} = \frac{2}{5} mR^2\omega$.

1.170. It remains constant.

1.171. $L_1 = mv_0R$, $L_2 = \frac{1}{2} mv_0R$, $L_3 = 0$.

1.172. $L_0 = 7.0 \times 10^{33}\ \text{kg}\cdot\text{m}^2/\text{s}$, $L = 2.7 \times 10^{40}\ \text{kg}\cdot\text{m}^2/\text{s} = 3.9 \times 10^6 L_0$.

1.173. (a) α is maximum at the instants when the axis of the cylinder is at the same level with the axis of rotation, α is minimum at the top and bottom positions of the cylinder; (b) $\alpha_{max} = 2g/3R$, $\alpha_{min} = 0$.

1.174. (a) $a = \frac{1}{6} g = 1.6$ m/s², $F_{12} = \frac{1}{6} Mg = 1.6$ N, $F_2 = \frac{1}{3} Mg = 3.2$ N, $F_m = \frac{5}{6} mg = 4.1$ N; (b) $a = \frac{1}{5} g = 2.0$ m/s², $F_{12} = \frac{1}{5} Mg = 2.0$ N, $F_2 = F_m = \frac{4}{5} mg = 4.0$ N.

1.175. (a) $\alpha_0 = 12ga/(l^2 + 12a^2) = 11$ rad/s², $F_0 = mgl^2/(l^2 + 12a^2) = 0.89mg = 5.3$ N; (b) $\omega = 2\sqrt{6ga/(l^2 + 12a^2)} = 4.6$ rad/s, $F = mg\,(l^2 + 36a^2)/(l^2 + 12a^2) = 1.21mg = 7.1$ N.

1.176. (a) $\omega = 3\sqrt{(g/3l)\{1 + \cos\varphi_0 - (\pi - \varphi_0)\sin\varphi_0\}} = 6.6$ rad/s; (b) $L = ml\sqrt{(gl/3)\{1 + \cos\varphi_0 - (\pi - \varphi_0)\sin\varphi_0\}} = 0.44$ kg·m²/s.

1.177. $L = mh\sqrt{gh/3} = 4.7 \times 10^3$ kg·m²/s, $v = \sqrt{3gh} = 9.5$ m/s.

1.178. (a) $x = 20.8$ cm; (b) the rule will rotate about point O at the angular speed $\omega = 3.00$ rad/s.

1.179. (a) $f \geqslant \frac{2}{7} \tan\alpha = 0.165$; (b) the ball slides along the plane, simultaneously revolving; (c) $v_A = 1.9$ m/s, $v_B = 6.2$ m/s, $v_C = 4.05$ m/s.

1.180. (a) $h = 0.69$ m; (b) $t_1 = 1.34$ s; (c) $t_2 = t_1 = 1.34$ s; (d) $v = v_0 = 3.00$ m/s.

1.181. (a) $h = 0.53$ m; (b) $t_1 = 1.03$ s; (c) $t_2 = 1.40$ s; (d) $v = 2.21$ m/s; (e) $A = -3.1$ J.

1.182. (a) The spool will slide along the plane without rotation with an acceleration of $a_x = 0.51$ m/s²; (b) the spool will be stationary, $a_x = 0$; (c) the spool will roll without slipping to the right, $a_x = 0.36$ m/s²; (d) the spool will roll without slipping to the left, $a_x = -0.30$ m/s².

1.183. (a) $t = \sqrt{3h/g} = 0.39$ s; (b) $F = mg/6 = 1.64$ N.

1.184. $L(t) = (mgR - M_{fr})\,t$.

1.185. $L = (I/R^2 + m)\,vR$, $E_k = \frac{1}{2}(I/R^2 + m)\,v^2$.

1.186. (a) $\omega = \frac{1}{4}\omega_0$; (b) it will diminish to one-fourth of its initial value.

1.187. $\omega = 6m'v/(m + 3m')\,l = 0.62$ rad/s.

1.188. (a) $\omega = 1.22$ rad/s; (b) $v' = -48.9$ m/s.

1.189. $\omega = \{2m'/(2m' + m)\}\,v'/R$.

1.190. $A = \int_{t_1}^{t_2} M_z(t)\,\omega(t)\,dt$.

1.191. $E_k = 100$ J.

1.192. (a) $\omega = 29t$ rad/s; (b) $F = 80$ N; (c) $\tau = 1.9$ s; (d) $v = 7.2$ m/s; (e) $A = -9.3$ J.

1.193. (a) $m' > M_{fr}/gR$; (b) $m' = (\pi/2)\, M_{fr}/gR$.
1.194. $A = -mR^2\omega^2/4$.
1.195. $\langle P \rangle = mR^2 (\omega_2^2 - \omega_1^2)/4t$.
1.196. $N = 3I\omega^2/8\pi fFR = 50$ rev.
1.197. $G = 6.7 \times 10^{-11}$ N$\cdot$m^2/kg^2.
1.198. $F = 6.7 \times 10^{-11}$ N.
1.199. The force will increase $n^{4/3}$ times.
1.200. (a) $F = GmM/a^2 2\sqrt{5}$; (b) $F \approx GmM/b^2$; (c) $F = F' \cdot 2/\sqrt{5} = 0.894F'$.
1.201. (a) $F = GmM/3a^2$; (b) $F = GmM/b^2$; (c) $F = \frac{4}{3} F'$.
1.202. (a) $E_p = -GmM/\sqrt{R^2 + x^2}$; (b) $F_x = -GmMx/(R^2 + x^2)^{3/2}$; (c) $E_p \approx -GmM/x$, $F_x \approx -GmM/x^2$.
1.203. (a) $F = 2\pi Gm\sigma\,(1 - b/\sqrt{b^2 + R^2})$; (b) when $b/R < 10^2$.
1.204. (a) $F = 2Gm\lambda/b$; (b) $M = 2b\lambda$.
1.205. (a) $F = 2\pi Gm\sigma$; (b) F does not depend on b; (c) $F = 2\pi Gm\rho d$.
1.206. $F = 4.2 \times 10^{-7}$ N.
1.207. $F = 4.2 \times 10^{-4}$ N.
1.208. $dS = R^2\, d\Omega$.
1.209. $dS = R^2 \sin\theta\, d\theta\, d\varphi$.
1.210. $d\Omega = \sin\theta\, d\theta\, d\varphi$.
1.211. $F = 0$.
1.212. $F = 0$.
1.213. $F = GmM/2R^2$.
1.214. (a) $E_p\,(r) = -GmM/R = \text{const}$; (b) $E_p\,(r) = -GmM/r$.
1.215. 1. (a) $E_p\,(r) = -\frac{3}{2} GmM\,(R_1 + R_2)/(R_1^2 + R_1R_2 + R_2^2) = \text{const}$; (b) $E_p\,(r) = -GmM/r$. 2. Inside the layer, $F = 0$.
1.216. (a) g, G, the Earth's radius; (b) G, the Earth's mass, the radius of the Earth's orbit, the Earth's period of revolution in its orbit.
1.217. (a) $m_E = gR_E^2/G = 5.97 \times 10^{24}$ kg (R_E is the mean radius of the Earth), $\langle\rho\rangle_E = 5.5 \times 10^3$ kg/m^3; (b) $m_S = 4\pi R_{orb}^3/GT^2 = 1.98 \times 10^{30}$ kg (R_{orb} is the mean radius of the Earth's orbit, and T is the period of the Earth's revolution about the Sun), $\langle\rho\rangle_S = 1.4 \times 10^3$ kg/m^3.
1.218. (a) $F_{S-E} = 3.5 \times 10^{22}$ N; (b) $F_{M-E} = 2.0 \times 10^{20}$ N $= \frac{1}{175} F_{S-E}$.
1.219. $a = 6.00 \times 10^{-4} g = 5.9 \times 10^{-3}$ m/s^2 ≈ 6 mm/s^2.
1.220. $v_1 = \sqrt{gR} = 7.9$ km/s (R is the Earth's radius).
1.221. $v_2 = \sqrt{2gR} = v_1\sqrt{2} = 11.2$ km/s (R is the Earth's radius).
1.222. In case (b).
1.223. $R = \sqrt[3]{GMT^2/4\pi^2} = 4.22 \times 10^7$ m $= 6.6R_E$ (G is the gravitational constant, M is the Earth's mass, T is the period of the Earth's revolution about its axis).

1.224. $R^3 = \alpha T^2$, where α is a coefficient not depending on the mass of a planet and equal to $GM/4\pi^2$ (G is the gravitational constant, and M is the Sun's mass).

1.225. $T_M = 1.88$ years.

1.226. (a) $g(h) = g\{R/(R+h)\}^2$, where R is the Earth's radius; (b) g (100 km) $= 0.969g$, g (1000 km) $= 0.747g$, g (10 000 km) $= 0.151g$.

1.227. (a) $E_p = mgRh/(R+h)$; (b) $E_p \approx mgh$.

1.228. (a) $h = 1.00 \times 10^6$ m; (b) $v = 2.25 \times 10^3$ m/s; (c) $R_c = 0.690 \times 10^6$ m.

1.230. (a) $a(r) = g(r/R)$ (R is the Earth's radius); (b) $v(r) = \sqrt{g(R^2 - r^2)/R}$; (c) $v(0) = \sqrt{gR} = 7.9$ km/s $= v_1$; (d) $\tau = 2\pi\sqrt{R/g} = 84.4$ min $= t_1$; (e) $\langle v \rangle = (2/\pi)\sqrt{gR} = (2/\pi)\, v(0)$.

1.231. (a) $E_p(r) = \begin{cases} mg(r^2 - 3R^2)/2R & \text{for } r \leqslant R, \\ -mgR^2/r & \text{for } r \geqslant R. \end{cases}$

(b) $E_p(0) = -\frac{3}{2} mgR = \frac{3}{2} E_p(R)$.

1.232. (a) The centrifugal force of inertia $\mathbf{F}_{cf}$ and the Coriolis force $\mathbf{F}_C$; (b) $F_{cf} = F_C = 4F_g = 1.42 \times 10^{23}$ N; $\mathbf{F}_{cf}$ is directed away from the Sun, and $\mathbf{F}_C$ toward the Sun.

1.233. $F = m(g_{eq} + \omega^2 R)$, where ω is the angular speed of the Earth, and R is the Earth's radius.

1.235. (a) $t_1 = T/12 = \frac{1}{3}(T/4)$; (b) $t_2 = T/6 = \frac{2}{3}(T/4)$.

1.236. (a) 0.40 m/s; (b) 0.57 m/s; (c) 0.23 m/s.

1.237. (a) $\langle \mathbf{v} \rangle = 0$; (b) $\langle \mathbf{v} \rangle = 0.40\mathbf{e}_x$ (m/s); (c) $\langle \mathbf{v} \rangle = -0.40\mathbf{e}_x$ (m/s).

1.238. $\omega = v_m/A$.

1.239. $A = v_m^2/a_m$, $\omega = a_m/v_m$.

1.240. (a) $A\omega^2 > g$; (b) when the disk breaks away, the platform is moving upward from its middle position ($x > 0$, $\dot{x} > 0$); (c) $h = g/2\omega^2 + A^2\omega^2/2g = 25$ cm.

1.241. $\langle \sin^2 x \rangle = \langle \cos^2 x \rangle = \frac{1}{2}$.

1.242. $A = 0$.

1.243. (a) $x^2/A^2 + p_x^2/m^2A^2\omega^2 = 1$; (c) $S = (2\pi/\omega)\, E$.

1.244. $\omega = (4b/9a^2)\sqrt{2b^3/m}$.

1.245. (a) $l = 0.248$ m $\approx \frac{1}{4}$ m; (b) $T = 2.006$ s ≈ 2 s.

1.246. (a) $l = 0.373$ m $\approx \frac{1}{3}$ m; (b) $T = 1.64$ s $\approx 1\frac{1}{2}$ s.

1.247. $x = (l/2)/\sqrt{3}$, $\omega_{max} = \sqrt{(g/l)\sqrt{3}}$.

1.248. $F = mg\cos(\varphi_m \cos\omega t) + ml\omega^2\varphi_m^2 \sin^2\omega t$.

1.249. (a) The pendulum will remain stationary; (b) the pendulum will rotate uniformly about its point of suspension.

1.250. (a) $T = 2T_0$; (b) $a = 3g$.

1.251. (a) $\omega = \omega_0 \sqrt[4]{(g^2 - 2ga\cos\alpha + a^2)/g^2}$; (b) $\omega = \omega_0 \sqrt[4]{2} = 1.19\omega_0$.

1.252. $T = 2\pi \sqrt{R/g} = 84.4$ min.

1.253. $\omega_0 = \frac{1}{2} \dot{\varphi}_{max}$.

1.254. (a) $T = 0.200$ s, $A = 0.0100$ m; (b) $x = 0.0100\,(1 - \cos 31t)$.

1.255. $\omega = 9.9\ s^{-1}$.

1.256. $T = 1.50$ s.

1.257. $\varphi_m = 4.6°$, $T = 2.0$ s.

1.258. (a) $\omega = \sqrt{k/m} = 19.4\ s^{-1}$, where $k = k_1k_2/(k_1 + k_2)$ (compare with the expression for the reduced mass); (b) $A = 26$ mm.

1.259. (a) $\omega = 16.9\ s^{-1}$; (b) $F_{1m} = 14.8$ N, $F_{2m} = 11.2$ N.

1.260. (a) The centre of mass remains stationary; (b) $\omega = \sqrt{k/\mu}$, where $\mu = m_1m_2/(m_1 + m_2)$ is the reduced mass of the spheres; (c) $v_{max} = A\omega = A\sqrt{k/\mu}$.

1.261. (a) $v_C = m_1v_0/(m_1 + m_2)$; (b) $E_{trans} = E_{tot}m_1/(m_1 + m_2)$, $E_{osc} = E_{tot}m_2/(m_1 + m_2)$, where $E_{tot} = m_1v_0^2/2$ is the total energy of the system; (c) $\omega = \sqrt{k/\mu}$, where $\mu = m_1m_2/(m_1 + m_2)$ is the reduced mass of the system, $A = v_0\sqrt{\mu/k}$.

1.262. (a) $\omega = 10.0\ s^{-1}$; (b) $\varphi_m = 0.100$ rad, $\alpha = -0.93$ rad.

1.263. $T = 1.00$ s.

1.264. (a) $\omega = 8.0\ s^{-1}$; (b) when $\dot{\varphi}^2 \gg (k/m)$, no oscillations appear.

1.265. $x^2/900 + y^2/1600 = 1$ (x and y are in cm).

1.266. $\tau = 20$ s.

1.267. $\tau = 3.3$ s.

1.268. (a) $\beta = 0.100\ s^{-1}$; (b) $\tau = 10.0$ s.

1.269. (a) $\beta = 1.00 \times 10^{-2}\ s^{-1}$; (b) $\lambda = 1.00 \times 10^{-2}$; (c) $Q = 314$; (d) $-\Delta E/E = 2.00 \times 10^{-2}$.

1.270. $Q = 195$.

1.271. $\omega_0 = 103\ s^{-1}$.

1.272. $s = A_0\,(1 + e^{-\lambda/2})/(1 - e^{-\lambda/2}) = 4.0$ m.

1.273. $Q = 4$.

1.274. $\omega_{res} = \frac{1}{2}\sqrt{\omega'^2 - (\ln \eta/\tau)^2} \approx \frac{1}{2}\omega' = 10.0\ s^{-1}$, $A_{res} = 5.3$ mm.

1.275. (a) $A_{dr} = \pi a F_m \sin\varphi$.

1.276. $\omega_{res} = 224\ s^{-1}$.

1.277. $\omega'_{res} = 173\ s^{-1}$.

1.278. No, it doesn't. The speed $v = 3.3 \times 10^8$ m/s of equatorial points of such a body exceeds the speed of light in a vacuum.

1.279. No, it doesn't because the speed at the "equator" of such a ball would be 0.89×10^{11} m/s, i.e. it would exceed the speed of light in a vacuum almost 300 times.

1.280. (a) Rod *2*; (b) rod *1*.

1.281. $v = 0.866c$.

1.282. (a) $\Delta l/l_0 \approx -0.005\cos^2\alpha$; (b) -0.005, -0.0025, 0.

1.283. (a) $\Delta l/l_0 = -(1 - \sqrt{1 - 0.81\cos^2\alpha})$; (b) -0.564, -0.229, 0.

1.284. $l = 0.94$ m, $\alpha = 49°$.

1.285. $V = 0.500V_0$.

1.286. $S = 0.500 S_0$.
1.287. $v_x = c\sqrt{1-(l/l')^2(1-v_x'^2/c^2)} = 0.43c$,
$v_0 = (v_x - v_x')/(1 - v_x v_x'/c^2) = 0.34c$.
1.288. (a) Clock *1*; (b) clock *2*.
1.289. (a) $\tau_1 = (l_0/v)\sqrt{1-v^2/c^2} = 2.89 \times 10^{-8}$ s, $\tau_2' = l_0/v = 3.33 \times 10^{-8}$ s; (b) $\tau_2 = l_0/v = 3.33 \times 10^{-8}$ s, $\tau_1' = (l_0/v)\sqrt{1-v^2/c^2} = 2.89 \times 10^{-8}$ s; (c) $\tau_2 = \tau_2' = (l_0/v)(1+\sqrt{1-v^2/c^2}) = 6.21 \times 10^{-8}$ s.
1.290. $\Delta s = 300$ m.
1.291. $v = 0.995c$.
1.292. $\Delta\tau = \sqrt{\Delta t^2 - (x^2+y^2+z^2)/c^2} = 0.50$ s.
1.293. (a) $v' = 0.300c$, $\alpha' = 55°$; (b) $\mathbf{v} = c\,(0.620\mathbf{e}_x + 0.138\mathbf{e}_y + 0.138\mathbf{e}_z)$, $v = 0.650c$, $\alpha = 17.5°$; (c) $v/v' = 2.17$.
1.294. $v = cp/\sqrt{p^2+m^2c^2} = 0.50c = 1.5 \times 10^8$ m/s.
1.295. $E = \sqrt{E_0^2 + p^2c^2}$.
1.296. $E_k = 0.41mc^2$.
1.297. $v = c\sqrt{3/4} = 0.866c$.
1.298. $p = (E_k/c)\sqrt{1 + 2mc^2/E_k}$.
1.299. $p = mc\sqrt{3}$.
1.300. (a) $\eta = 2/\sqrt{1+3v_0^2/c^2}$; (b) $\eta = 1.971$, 1.512, 1.080, 1.0075; (c) $\eta \approx 1 + \frac{3}{8}(1 - v_0^2/c^2)$.
1.301. $\mathbf{p} = \mathbf{F}t$; $\mathbf{v} = c\mathbf{F}t/\sqrt{m^2c^2+F^2t^2}$.
1.302. $\Delta v = 0.77c$, $\Delta p = 4.5 \times 10^{-22}$ kg·m/s, $\Delta E_k = 8.24 \times 10^{-14}$ J.
1.303. $\mathbf{p} = 10^{-18}\,(10.5\mathbf{e}_x + 2.30\mathbf{e}_y + 2.30\mathbf{e}_z)$ (kg·m/s); $E = 3.4 \times 10^{-9}$ J.
1.304. $h = H/2$.
1.305. (a) $\tau = 53$ min; (b) $v = 9.81 \times 10^{-8}(\tau - t)$ m/s.
1.306. $\tau = (R/r)^2 lR\sqrt{(\pi\rho/2F)\,[1-(r/R)^4]} \approx (R/r)^2 lR\sqrt{\pi\rho/2F} = 7.9$ s.
1.307. $v = 1.7$ m/s.
1.308. $v = 3.00$ m/s.
1.309. (a) The flow is laminar; (b) $dp/dl = 3.1$ Pa/m.
1.310. (a) $Re \approx 600$—the flow is laminar; (b) $\tau = 4\eta lR^2/\rho g r^4 = 1.6 \times 10^4$ s (ρ is the density of water).
1.311. (a) $r = 1.59 \times 10^{-2}$ mm; (b) $Re = 0.19 < 0.25$—the flow over the particle is of a laminar nature.
1.312. (a) Yes, it may; (c) $\tau = 2.3$ s; (d) $t = 0.029$ s.

PART 2. MOLECULAR PHYSICS AND THERMODYNAMICS

2.1. Approximately 6×10^{27} molecules.
2.2. (a) 1.67×10^{-27} kg; (b) 5.31×10^{-26} kg; (c) 3.95×10^{-25} kg.
2.3. $M = 5.49 \times 10^{-7}$ kg.
2.4. 1 amu $= 1.66 \times 10^{-27}$ kg.
2.5. $d = 2.9 \times 10^{-10}$ m $= 2.9$ Å.

2.6. (a) $n = 2.69 \times 10^{25}$ m^{-3}; (b) $\langle a \rangle = 33 \times 10^{-10}$ m $\approx 10d$.
2.7. $n = N_A\rho/M$.
2.8. $n_{Be} = 1.2 \times 10^{29}$ m^{-3}, $n_K = 1.3 \times 10^{28}$ m^{-3}.
2.9. (a) $\nu = nv \cos\theta$; (b) $p = 2nmv^2 \cos^2\theta$.
2.10. When it expands slowly.
2.11. When it is compressed rapidly.
2.12. (a) $A = 600$ J, $A' = -600$ J; (b) $A = -900$ J, $A' = 900$ J.
2.13. $A = 445$ kJ.
2.14. $Q = -1.6$ kJ.
2.15. $Q = 2.07$ kJ.
2.16. $Q_{123} - Q_{143} = 2.00$ kJ.
2.17. $Q = -94$ J.
2.18. It will increase 1.16 times.
2.19. It will increase 3.00 times.
2.20. $T_2 = 383$ K.
2.21. $n = p/kT = 2.69 \times 10^{25}$ $m^{-3} = 2.69 \times 10^{19}$ cm^{-3}. Compare with the answer to Problem 2.6.
2.22. (a) 1.29 kg/m^3; (b) 1.29 g/litre.
2.23. (a) $p_{nitr} = 0.791 \times 10^5$ Pa, $p_{ox} = 0.212 \times 10^5$ Pa, $p_{arg} = 0.009 \times 10^5$ Pa; (b) $M_r = 28.9$.
2.24. $N = \ln(p_0/p)/\ln(1 + v/V)$.
2.25. $p(t) = p_0 \exp(-Ct/V)$.
2.26. $\tau = 127$ s.
2.27. See Fig. A.2.

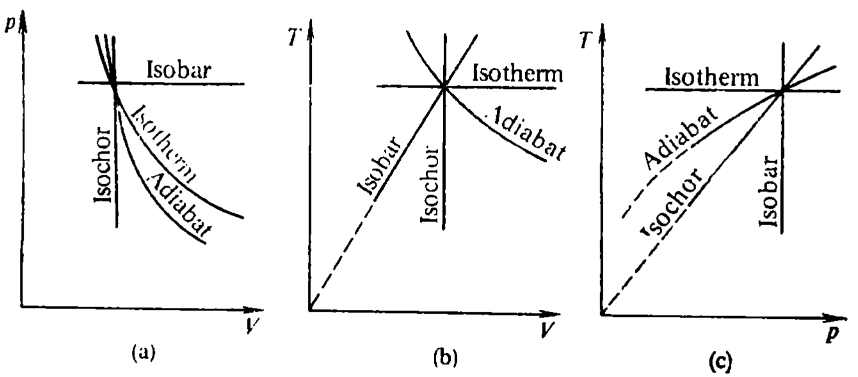

Fig. A.2

2.28. $T_2 > T_1$.
2.29. In *1* and *2* it grows, in *3* it is constant, in *4* and *5* it diminishes.
2.30. See Fig. A.3.
2.31. For all three processes, $\Delta U = [R/(\gamma - 1)]\,\Delta T$.
2.32. (a) $C = \infty$; (b) $C = 0$.
2.33. $A = -3.8$ kJ.
2.34. (a) and (b) $Q = -1.00 \times 10^5$ J.
2.35. 2.51 times.
2.36. (a) $t_2 = 413$ °C; (b) $t_2 = 254$ °C.

2.37. $A = [1/(\gamma - 1)]\{p_1V_1 \ln (p_2V_2^\gamma/p_1V_1^\gamma) + p_1V_1 - p_2V_2\} = 203$ J.
2.38. $\Delta U = 0$.
2.39. $\Delta U = 2.5 \times 10^5$ J.
2.40. $\nu = r^2 \sqrt{\gamma p_0/2mV_0} = 37$ Hz.
2.41. (a) $\Delta U = [1/(\gamma - 1)]\, p\, (V_2 - V_1) = 1.5$ kJ; (b) $A = p\, (V_2 - V_1) = 1.0$ kJ; (c) $Q = [\gamma/(\gamma - 1)]\, p\, (V_2 - V_1) = 2.5$ kJ.

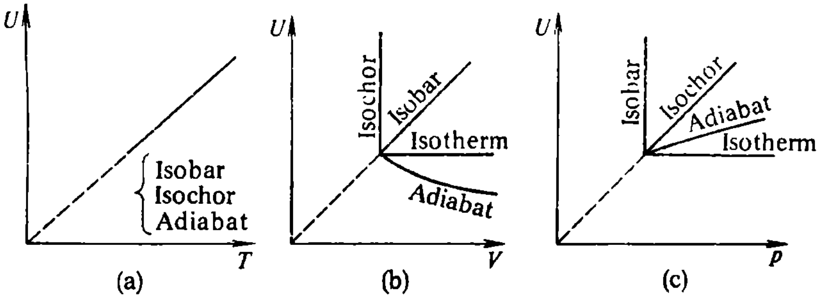

Fig. A.3

2.42. (a) $\Delta U = 0$; (b) $A = 66$ kJ; (c) $Q = 66$ kJ.
2.43. (a) $\gamma = 1.33$; (b) $\Delta U = 2.5$ kJ; (c) $A = 0.85$ kJ.
2.44. (a) $\Delta U = 0$; (b) $A = 2.41$ kJ; (c) $Q = 2.41$ kJ.
2.45. The gas gives up heat to external bodies ($Q_{12} < 0$).
2.46. $Q_{\text{I}} > Q_{\text{II}}$.
2.47. (a) $A_{\text{I}} > 0$, $A_{\text{II}} < 0$; (b) $Q_{\text{I}} > Q_{\text{II}}$.
2.48. (a) $\Delta U = -1.25 \times 10^5$ J. (b) No, it cannot, because the nature of the expansion process is not indicated.
2.49. (a) $A = 108$ kJ; (b) $Q = 108$ kJ.
2.50. $A = 14.8$ kJ.
2.51. (a) $T_2 = 1850$ K; (b) $A = -1.56$ MJ; (c) 16 times.
2.52. $n = 0$.
2.53. $n = -9$.
2.54. (a) No, it isn't; (b) $A = 230$ J.
2.55. Yes, it is.
2.56. (a) $n = -1$; (b) $C = C_V + R/2 = \frac{1}{2}(C_V + C_p)$.

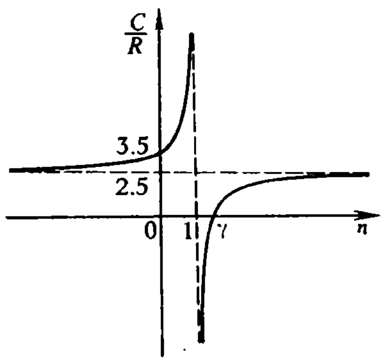

Fig. A.4

2.57. $A_{12} = \nu R\, (T_1 - T_2)/(n-1)$.
2.58. (a) The gas is cooled when it expands; (b) $C = C_V - R$.
2.59. $C = R\, (n - \gamma)/(\gamma - 1)\, (n - 1)$.
2.60. (a) $n < 1$ and $n > \gamma$; (b) $1 < n < \gamma$; (c) $n = \gamma$; (d) $n = 1$.
2.61. (a) $n = 0.92$; (b) $C = 14.4R = 120$ J/(mol·K).
2.62. (a) $C = 12.5R$; (b) $C = 102.5R$; (c) $C = 1002.5R$; (d) $C = -7.5R$.
2.63. See Fig. A.4.

2.64. (a) At $n < 1$; (b) at $n > 1$; (c) at $n = 1$.
2.65. (a) No, it isn't; (b) $Q = 24.5$ kJ.
2.66. $\nu = 2.95 \times 10^{27}$ m^{-2}s^{-1}.
2.67. $\nu = 1.35 \times 10^{27}$ s^{-1}.
2.68. (a) Three translational; (b) three translational and two rotational; (c) three translational and three rotational; (d) three translational, two rotational, and one vibrational; (e) three translational, three rotational, and three vibrational.
2.69. (a) $C_V = \frac{3}{2} R$, $C_p = \frac{5}{2} R$, $\gamma = 1.67$; (b) $C_V = \frac{5}{2} R$, $C_p = \frac{7}{2} R$, $\gamma = 1.40$; (c) $C_V = \frac{7}{2} R$, $C_p = \frac{9}{2} R$, $\gamma = 1.29$; (d) $C_V = 3R$, $C_p = 4R$, $\gamma = 1.33$.
2.70. Of four.
2.71. $U_m = 74.8$ kJ/mol.
2.72. 16 °C.
2.73. $n = 5.00 \times 10^{25}$ m^{-3}.
2.74. $P\,(a \leqslant x \leqslant b) = \int\limits_a^b f\,(x)\,dx.$
2.75. $P\,(a_1 \leqslant x \leqslant a_2;\ \ b_1 \leqslant y \leqslant b_2) = \int\limits_{a_1}^{a_2}\int\limits_{b_1}^{b_2} f_1\,(x)\,f_2\,(y)\,dx\,dy.$
2.76. (a) $A = 1/2a$, $\langle x \rangle = 0$, $\langle |x| \rangle = a/2$, $\langle x^2 \rangle = a^2/3$;
(b) $A = 1/2a$, $\langle x \rangle = a$, $\langle x^2 \rangle = 4a^2/3$;
(c) $A = 1/a$, $\langle x \rangle = 0$, $\langle x^2 \rangle = a^2/6$;
(d) $A = 1/a$, $\langle x \rangle = a$, $\langle x^2 \rangle = 7a^2/6$.
2.77. $P \approx A \exp(-64\alpha) \times 4\pi \times 64 \times 0.0002$.
2.78. (a) $f\,(x) = 1/(\pi \sqrt{a^2 - x^2})$; (b) $\langle x \rangle = 0$; (c) $\langle |x| \rangle = 2a/\pi = 0.637a$; (d) $\langle x^2 \rangle = a^2/2$; (e) $\langle U \rangle = \frac{1}{4} ma^2\omega^2 = \frac{1}{2} E$ (E is the total energy of the oscillator).
2.79. $T = 453$ K.
2.80. $T = 563$ K.
2.81. $dN_u = N\,(4/\sqrt{\pi})\, e^{-u^2} u^2\, du.$
2.82. $Q = \frac{3}{2} R\,(1.01^2 - 1)\,T = 73$ J.
2.83. $v_{\text{prob}} = 390$ m/s, $\langle v \rangle = 440$ m/s, $v_{\text{r.m.s.}} = 478$ m/s.
2.84. (a) $\sum v_x = 0$; (b) $\sum \mathbf{v} = 0$; (c) $\sum \mathbf{v}^2 = 1.61 \times 10^{29}$ m^2/s^2; (d) $\sum v = 2.87 \times 10^{26}$ m/s.
2.85. $\langle |v_x| \rangle = \sqrt{2kT/\pi m}$.
2.86. $\sum mv = M\langle v \rangle = 13$ kg·m/s.
2.87. $\sqrt{\langle \omega^2 \rangle} = \sqrt{8RT/Ml^2} = 2.3 \times 10^{12}$ s^{-1}.
2.88. 1.7%.
2.89. $\eta = \frac{4}{\sqrt{\pi}} \int\limits_1^\infty e^{-u^2} u^2\, du.$

2.90. $\langle v \rangle = 1.00$ km/s.

2.91. 1. $v_{\text{prob}} = 500$ m/s.

2. (a) 3.32×10^{-4}; (b) 1.76×10^{-4}; (c) 2.14×10^{-4}; (d) $0.66 \times \times 10^{-4}$.

2.92. The area numerically equals the number of molecules contained in a cylindrical column of air with a base area of 1 m^2 and a height of h_1.

2.93. $dN_r = n_1 \exp\{-[\varepsilon_p(r) - \varepsilon_p(r_1)]/kT\}\, 4\pi r^2\, dr$.

2.94. $N_A = 6.2 \times 10^{23}$ mol^{-1}.

2.95. (a) $p = 0.56p_0$; (b) $p = 0.33p_0$; (c) $p = 1.26p_0$.

2.96. $\eta = 0.225$.

2.97. (a) $p = 0.97 \times 10^5$ Pa $= 0.97p_0$.

(b) $N = (VN_A p_0/Mgh)\,[1 - \exp(-Mgh/RT)] = 4.9 \times 10^{24}$.

2.98. 1. (a) $h = 5.5$ km; (b) $h = 8.0$ km.

2. (a) 0.09%; (b) 0.16%.

2.99. $p = p_0 \exp(M\omega^2 l^2/2RT) = 1.024p_0 = 1.024 \times 10^5$ Pa.

2.100. $$E = N\,\frac{E_1 \exp(-E_1/kT) + E_2 \exp(-E_2/kT)}{\exp(-E_1/kT) + \exp(-E_2/kT)}.$$

2.101. (a) In $2^5 = 32$ ways;

(b) $\Omega(0, 5) = 1$; $P(0, 5) = 1/32$;

(c) $\Omega(1, 4) = 5$; $P(1, 4) = 5/32$;

(d) $\Omega(2, 3) = 10$; $P(2, 3) = 5/16$.

2.102. Ω remains constant.

2.103. $\Delta S_{12} = 0.96 \times 10^{-23}$ J/K.

2.104. $\Omega_2 = \Omega_1^{\eta}$.

2.105. (a) and (b) $S = 3.18 \times 10^{-3}$ J/K; $\Delta S/S \sim 10^{-20}$.

2.106. (a) 4.787 and 4.771 (−0.33%); (b) 15.1044 and 15.0961 (−0.05%).

2.107. (a) −36%; (b) −14%; (c) −6%; (d) −3.5%; (e) −0.9%.

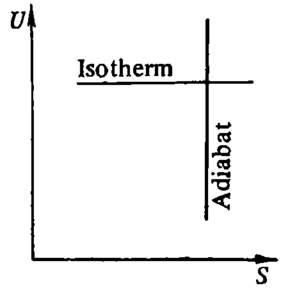

Fig. A.5

2.108. $l \approx 3 \times 10^{-8}$ m.

2.109. (a) $\Omega = 10^{4.1\times 10^{24}}$; (b) $\Omega = 10^{8.2\times 10^{24}}$.

2.110. $10^{3.42\times 10^{24}}$ times.

2.111. If the process is reversible, it remains constant; if the process is irreversible, it increases.

2.112. Yes, it can, if the process is irreversible.

2.113. No, it cannot.

2.114. $S_2 > S_1$.

2.115. See Fig. A.5.

2.116. See Fig. A.6.

2.117. *1*, *2*, and *3*—it grows; *4*—it is constant; *5*—it diminishes.

2.118. See Fig. A.7.

2.119. 1. In both cases $\Delta S = 0$. 2. No, it cannot.

2.120. States *1* and *3* are on the same adiabat.

2.121. $\Delta S_{23} = -1.00 \times 10^{-2}$ J/K.

2.122. $Q = 30.0$ kJ.

2.123. $Q = 1.79$ kJ.

2.124. $A = 47$ kJ.

2.125. $S_2 = 13.6$ J/K.

2.126. $S_2 = 210.6$ J/(mol·K).

2.127. (a) $\Delta S_m = 8.6$ J/(mol·K); (b) $\Delta S_m = 14.4$ J/(mol·K).
2.128. The entropy receives an increment of $\Delta S = 2.00$ J/K.
2.129. The entropy of the body receives the increment $\Delta S = 0.25$ J/K.
2.130. $A = T(S_2 - S_1)$.
2.131. $\Delta S = m\{c \ln (T_2/T_1) + L_f/T_2\} = 2.6 \times 10^2$ J/K (T_1 and

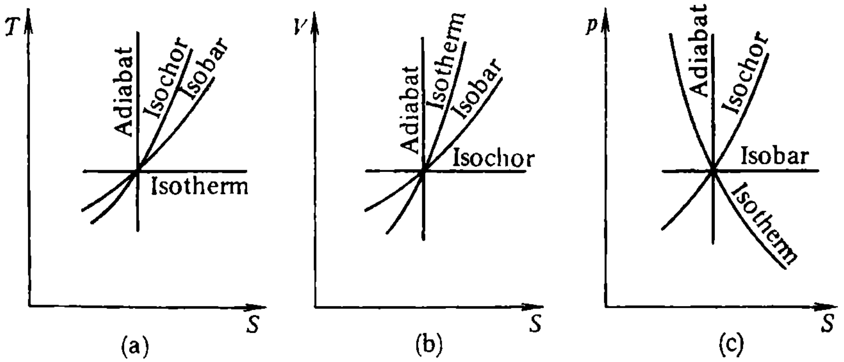

Fig. A.6

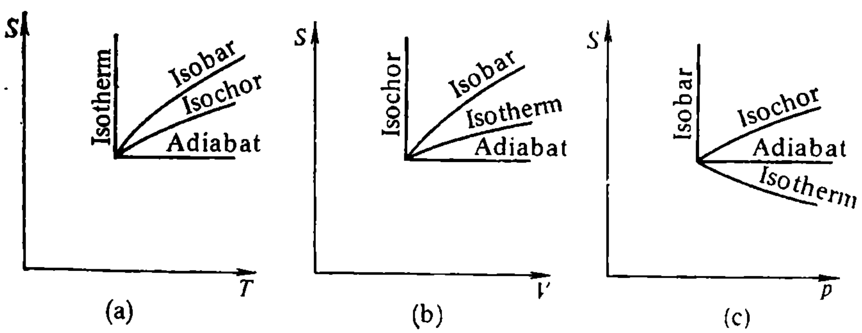

Fig. A.7

T_2 are the initial and final temperatures, respectively, and L_f is the heat of fusion).
2.132. $\Delta S = m\{c \ln (T_2/T_1) - L_v/T_1\} = -7.0$ kJ/K (T_1 and T_2 are the initial and final temperatures, respectively, and L_v is the heat of vaporization).
2.133. $C = \alpha T$.
2.134. $S = C/3$.
2.135. $S_m = [(\gamma + 1)/(\gamma - 1)] R \ln V +$ const.
2.136. $A = [S_1U_2 + (S_1 - 2C_V) U_1] (U_2 - U_1)/2U_1C_V = 9.4$ kJ.
2.137. (a) $\Delta S = 238$ J/K; (b) $\Delta U = 0$.
2.138. (a) $\Delta U_m = 0$; (b) $\Delta S_m = R \ln 2$.

2.139. The bottom and lid of vessel *1* experience the same pressure p_1 due to the molecules of *1*; the bottom and lid of vessel *2* experience the same pressure p_2 due to the molecules of *2*. Therefore the work in moving the vessels apart is zero. The system receives no heat. Consequently, the internal energy and, therefore, the temperature of the system do not change. In a reversible adiabatic process, the entropy also remains unchanged. From the constancy of U, T, and S, there follows the constancy of F. The statement made in the conditions of the problem follows from the constancy of U, S, and F and their additivity.

2.140. $\Delta S = R\left(\nu_1 \ln \dfrac{\nu_1+\nu_2}{\nu_1} + \nu_2 \ln \dfrac{\nu_1+\nu_2}{\nu_2}\right)$.

2.142. (a) $S_\mathrm{m} = 199$ J/(mol·K); (b) $S_\mathrm{m} = 199 + 29.1 \ln (T/T_0)$ J/(mol·K), where $T_0 = 298$ K.

2.143. $\Delta S = -6.1$ kJ/K.

2.144. $\Delta S = -A'/T = 3.0$ J/K, $\Delta F = A' = -900$ J.

2.145. $A = Q - \Delta F - (T_2S_2 - T_1S_1)$.

2.146. $\Delta F_\mathrm{m} = 75$ J/mol.

2.147. $\Delta F = T(S_1 - S_2)$.

2.148. 1. (a) On *1-2* and *3-4* it is constant, on *2-3* it diminishes, on *4-1* it grows; (b) on *1-2* and *4-1* it grows, on *2-3* and *3-4* it diminishes.

2. (a) On *1-2* $A > 0$, on *3-4* $A < 0$, on *2-3* and *4-1* $A = 0$; (b) on *1-2* and *4-1* $Q > 0$, on *2-3* and *3-4* $Q < 0$.

2.149. 1. (a) On *1-2* and *4-1* it grows, on *2-3* and *3-4* it diminishes; (b) on *2-3* and *4-1* it is constant, on *1-2* it grows, on *3-4* it diminishes.

2. (a) On *1-2* and *2-3* $A > 0$, on *3-4* and *4-1* $A < 0$; (b) on *1-2* $Q > 0$, on *4-3* $Q < 0$, on *2-3* and *4-1* $Q = 0$.

2.150. See Fig. A.8.

2.151. See Fig. A.9.

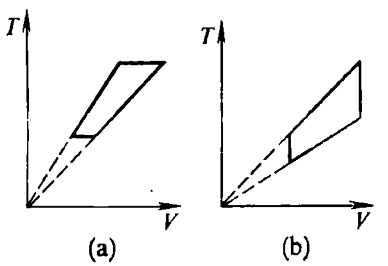

Fig. A.8

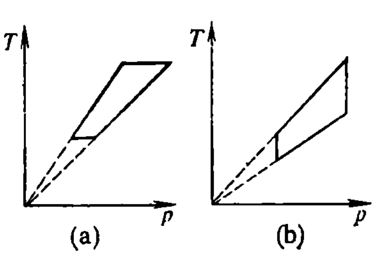

Fig. A.9

2.152. See Fig. A.10.

2.153. See Fig. A.11.

2.154. $A = 113$ kJ.

2.155. The areas *I* and *II* are equal.

2.156. $A_{41} = -26.0$ kJ.

2.157. $\eta = \dfrac{(\gamma - 1)(T_1 - T_2)}{\gamma T_1 - T_2}$.

2.158. $\eta = \dfrac{(\gamma-1)\,(T_1-T_2)}{(2\gamma-1)\,T_1-\gamma T_2}$.

2.159. (a) $\eta = 2/5$; (b) $\eta = 1/5$; (c) $\eta = 1/6$.

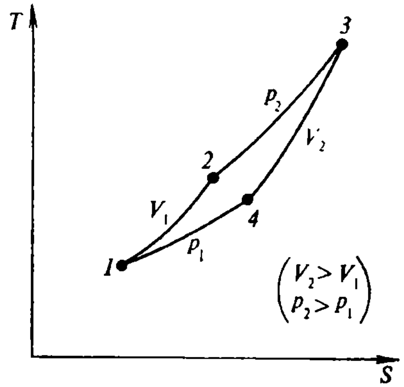

Fig. A.10

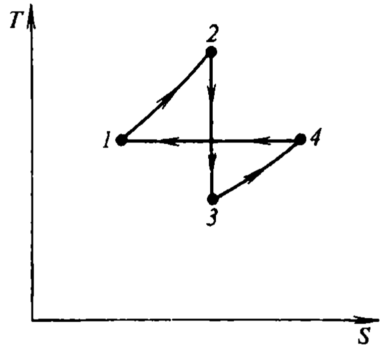

Fig. A.11

2.160. Figure A.12 depicts the cyce being considered (the closed curve) and the corresponding Carnot cycle (the rectangle). We must note that the efficiency of a Carnot cycle depends only on T_1 and T_2 and, consequently, does not depend on the "dimension" of the cycle along the S-axis. This is why we have taken a Carnot cycle having the same length along the S-axis as the one being considered.

A glance at the figure shows that for any cycle the area $aABCb$ (i.e. the amount of heat Q_1 received during the cycle) is less than the area $a12b$ (i.e. Q_1 for the Carnot cycle). On the other hand, the area $aADCb$ (i.e. the amount of heat Q_2' given up during the cycle) is greater than the area $a43b$ (i.e. Q_2' for the Carnot cycle). Hence the ratio Q_2'/Q_1 for any cycle is larger than for the Carnot cycle. It thus follows that $\eta < \eta_{\text{Carnot}}$.

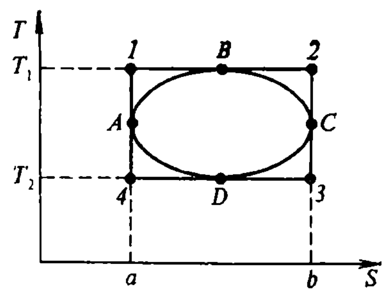

Fig. A.12

2.161. $\eta = 1 - (T_3 + T_4)/(T_1 + T_2)$.

2.162. (a) $p = 13.6 \times 10^6$ Pa $= 0.95 p_{\text{id}}$;
(b) $p = 100 \times 10^5$ Pa $= 0.69 p_{\text{id}}$.

2.163. (a) $p = 25.8 \times 10^5$ Pa $= 0.90 p_{\text{id}}$;
(b) $p = 763 \times 10^5$ Pa $= 2.65 p_{\text{id}}$.

2.164. $a = 0.15$ Pa·m^6/mol^2; $b = 3.33 \times 10^{-5}$ m^3/mol.

2.165. $\Delta T = -5.9$ K.

2.166. $Q = 3.84 \times 10^{-2}$ J.

2.167. $A = RT \ln \dfrac{V_2 - b}{V_1 - b} + a\left(\dfrac{1}{V_2} - \dfrac{1}{V_1}\right)$.

2.168. (a) $\Delta U_{\text{m}} = 0.12$ kJ; (b) $A = 3.19$ kJ $= 0.98 A_{\text{id}}$; (c) $Q = 3.31$ kJ.

2.169. $T(V-b)^{R/C_V}=\text{const}$, $\left(p+\frac{a}{V^2}\right)(V-b)^{R/C_V+1}=\text{const}$ (a and b are the van der Waals constants).

2.170. (a) $C_p - C_V = R/\{1-2a(V-b)^2/RTV^3\} \approx R(1+2a/VRT)$; (b) $C_p - C_V = 1.21R$.

2.171. $C_p - C_V = 1.67R$.

2.172. $\Delta T = T_2 - T_1 = \frac{1}{C_V+R}\left(\frac{RT_1 b}{V_1-b}-\frac{2a}{V_1}\right)$.

2.173. The gas will be heated.

2.174. (a) $\Delta T = +0.21$ K; (b) $\Delta T = 0$; (c) $\Delta T = -0.20$ K.

2.175. (a) $\Delta T = -2.8$ K; (b) $\Delta T = -1.7$ K.

2.176. $S = C_V \ln T + R \ln (V-b) + \text{const}$.

2.177. (a) $\Delta U_m = 0$; (b) $\Delta T = -a/VC_V$; (c) $A_{mol} = -a/V$; (d) $\Delta S_m = C_V \ln (1 - a/VTC_V) + R \ln 2(1+b/V) \approx R(\ln 2 + b/V) - a/VT$.

2.178. Let us consider the reversible isothermal cycle *1-3-4-5-2-4-1* (Fig. A.13). It follows from the conditions $\Delta S = \Delta U = 0$ and $T =$

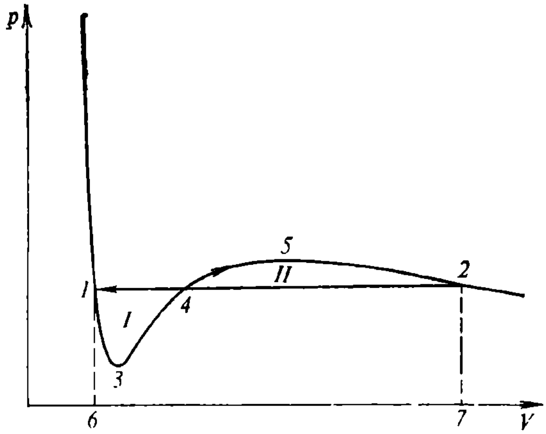

Fig. A.13

$=$ const that $Q=0$ and $A=0$ (Q is the heat received during the cycle, and A is the work done during it). On section *1-3-4-5-2*, the work is positive, on section *2-4-1*, it is negative. The total work equals zero. Therefore, the areas of the figures *6-1-3-4-5-2-7-6* and *6-1-4-2-7-6* must be identical. It thus follows that areas *I* and *II* are identical in magnitude.

We must note that the above reasoning cannot be applied to cycles *1-3-4-1* and *4-5-2-4*. These cycles are irreversible because they include an irreversible transition performed at point *4* from a state belonging to the van der Waals isotherm *1-3-4-5-2* to a state belonging to the straight section *1-4-2* of the real isotherm.

2.179. (a) and (b): yes, it can.

2.180. $d = 2.81 \times 10^{-10}$ m.

2.181. (a) $c = 0.39$ J/(g·K); (b) $c = 0.92$ J/(g·K).
2.182. It increased (a) 2.15; (b) 4.64; (c) 10 times.
2.183. $r_2 = 0.18$ mm.
2.184. $h = 59$ mm.
2.185. $F = 2.5$ kN.
2.186. $F = 5.5$ hN.
2.187. (a) $h = 30.0$ mm; (b) $F = 0.44$ N.
2.188. $xy = 2\alpha/\rho g\varphi$.
2.189. $h = 2\alpha/\rho g r = 10.0$ mm.
2.190. $h = (1/\rho g)\{p(\eta^3 - 1) + (2\alpha/r)\eta(\eta^2 - 1)\} = 6.8$ m (ρ is the density, and α is the surface tension of the water).
2.191. In substances for which the pressure at the triple point exceeds atmospheric pressure.
2.192. $p_{s.v} = C\exp(-LM/RT)$, where C is a constant, and M is the molar mass of the substance.
2.193. (a) $A_{12} = p_{s.v}m(V_2' - V_1')$; (b) $Q_{12} = mL_{12}$; (c) $U_2 - U_1 = mL_{12} - p_{s.v}m(V_2' - V_1')$; (d) $S_2 - S_1 = mL_{12}/T$; (e) $F_2 - F_1 = -p_{s.v}m(V_2' - V_1')$.
2.194. The entire horizontal section of the isotherm in the two-phase region.
2.195. A point on the melting curve.
2.196. (a) See Fig. A.14. (b) A vaporization curve corresponds to the region under the bell-shaped curve.
2.197. $T = 147 + 89\ln r$ (T is in K, and r is in cm).
2.198. (a) $\varkappa = 13.3$ W/(m·K); (b) $T = 200 + 12/r$ (T is in K, and r is in m).
2.199. $\tau = Cl\ln\eta/2\varkappa S = 193$ min.
2.200. (a) $l = 0.76 \times 10^{-7}$ m $\approx 20\langle a\rangle$; (b) $\tau = 1.71 \times 10^{-10}$ s.
2.201. $\nu = 4.2 \times 10^{34}$ m^{-3}s^{-1}.
2.202. $D = 2.6 \times 10^{-4}$ m^2/s, $\eta = 4.6 \times 10^{-5}$ Pa·s.

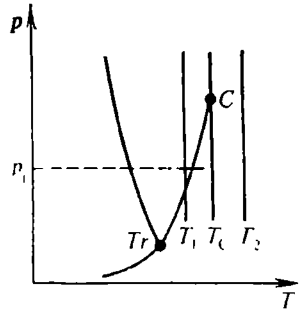

Fig. A.14

2.203. $l \approx 1.3 \times 10^{-7}$ m $\approx 38\langle a\rangle$.
2.204. $D_{12} = 0.70 \times 10^{-4}$ m^2/s.
2.205. (a) and (b) $dQ/dt \approx 0.5$ W/m^2; (c) $dQ/dt \approx 0.03$ W/m^2; (d) $dQ/dt \approx 0.003$ W/m^2.
2.206. $\tau \approx 40$ h.
2.207. $\alpha = 0.50$ rad.
2.208. $\eta = 2\beta ma/\pi R^2$.
2.209. $\nu = 2.4 \times 10^{17}$ m^{-2}s^{-1}.
2.210. $\tau = (4V/S)\ln\eta\sqrt{\pi M/8RT}$.
2.211. $p_2 = 0.129$ Pa.

PART 3. ELECTRICITY AND MAGNETISM

3.1. $\delta \sim 10^{-11}$.
3.2. $q' = q$—the charge is relativistically invariant.
3.3. $q = -9.65 \times 10^4$ C/mol $= -F$, where F is the Faraday constant.

3.4. $q = 3.9 \times 10^5$ C.
3.5. $F_e/F_g = 4.2 \times 10^{42}$.
3.6. $a = 2.5 \times 10^8$ m/s^2.
3.7. $m'_p = 1.86 \times 10^{-9}$ kg $\approx 10^{18} m_p$, where m_p is the true mass of a proton.
3.8. $q_S = 1.70 \times 10^{20}$ C, $q_E = 5.13 \times 10^{14}$ C.
3.9. $q/m = 0.86 \times 10^{-10}$ C/kg $= 1.6 \times 10^{-22}\ e/m_e$.

3.10. $$\mathbf{F} = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N_1} \sum_{k=1}^{N_2} \frac{q_i q'_k}{|\mathbf{r}_i - \mathbf{r}'_k|^3} (\mathbf{r}_i - \mathbf{r}'_k).$$

3.11. $$\mathbf{F} = \frac{1}{4\pi\varepsilon_0} \int\limits_V \int\limits_{V'} \frac{\rho(\mathbf{r})\,\rho(\mathbf{r}')\,(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}\, dV\, dV'.$$

3.12. (a) $\varphi = (1/4\pi\varepsilon_0)(6q/a)$, $\mathbf{E} = 0$; (b) $\varphi = 0$, $\mathbf{E} = 0$.

3.13. $$\varphi = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{q_i}{|\mathbf{r}-\mathbf{r}_i|},\quad \mathbf{E} = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{q_i}{|\mathbf{r}-\mathbf{r}_i|^3}(\mathbf{r}-\mathbf{r}_i).$$

3.14. $$\varphi = \frac{1}{4\pi\varepsilon_0} \int\limits_V \frac{\rho(\mathbf{r})\, dV}{|\mathbf{r}'-\mathbf{r}|},\quad \mathbf{E} = \frac{1}{4\pi\varepsilon_0} \int\limits_V \frac{\rho(\mathbf{r})\,(\mathbf{r}'-\mathbf{r})\, dV}{|\mathbf{r}'-\mathbf{r}|^3}.$$

3.15. $\varphi = R\sigma/\varepsilon_0$, $\mathbf{E} = 0$.
3.16. $\varphi = 6.8 \times 10^5$ V, $\mathbf{E} = 0$.
3.17. $\varphi = R\sigma/2\varepsilon_0$, $E = \sigma/4\varepsilon_0$.
3.18. A system of equally spaced planes parallel to one another.
3.19. $\varphi = -Ex + \text{const}$.
3.20. (a) Yes, it is. (b) $\varphi = -E_1x - E_2y - E_3z + \text{const}$.
3.21. (a) No, it isn't. (b) $\varphi(r) = 2a/\sqrt{r}$.
3.22. (a) The field is centrally symmetric.
(b) $E = |\varphi'|\, 2\sqrt{x^2 + y^2 + z^2}$.
3.23. (a) The field is axisymmetric.
(b) $E = \sqrt{(\partial\varphi/\partial r)^2 + (1/r^2)(\partial\varphi/\partial\theta)^2}$.
3.24. (a) $\mathbf{E} = -2(ax\mathbf{e}_x + ay\mathbf{e}_y + bz\mathbf{e}_z)$,
$$E = 2\sqrt{a^2(x^2 + y^2) + b^2z^2}.$$
(b) An ellipsoid of revolution with the semiaxes $\sqrt{\varphi/a}$, $\sqrt{\varphi/a}$, $\sqrt{\varphi/b}$.
(c) An ellipsoid of revolution with the semiaxes $E/2a$, $E/2a$, $E/2b$.
3.25. (a) $\mathbf{E} = -2(ax\mathbf{e}_x + ay\mathbf{e}_y - bz\mathbf{e}_z)$,
$$E = 2\sqrt{a^2(x^2 + y^2) + b^2z^2}.$$
(b) At $\varphi > 0$—a hyperboloid of revolution of one sheet, at $\varphi = 0$—a right circular cone, at $\varphi < 0$—a hyperboloid of revolution of two sheets.
(c) An ellipsoid of revolution.
3.26. $\varphi = (1/4\pi\varepsilon_0)\, p\cos\theta/r^2$;
$$E = (1/4\pi\varepsilon_0)(p/r^3)\sqrt{1 + 3\cos^2\theta}.$$
3.27. $\mathbf{p}$ does not depend on the choice of the point relative to which it is taken.
3.28. $A = 2pE$.
3.29. (a) and (b) $\mathbf{p} = 0$.

3.30. $F = 2.1 \times 10^{-16}$ N.

3.31. $E = \dfrac{\lambda a}{2\pi\varepsilon_0 r\sqrt{a^2+r^2}}$.

3.32. $\varphi = \dfrac{\lambda}{4\pi\varepsilon_0}\ln\dfrac{r+a}{r-a}$, $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2a\lambda}{r^2-a^2}$. At $r \gg a$—the field of a point charge: $\varphi = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r}$, $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}$ (q is the total charge of the rod).

3.33. $E = \lambda/2\pi\varepsilon_0 r$.

3.34. (a) $\varphi = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{\sqrt{r^2+x^2}}$, $\mathbf{E} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{qx}{(r^2+x^2)^{3/2}}\mathbf{e}_x$. (b) For $x = 0$: $\varphi = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r}$, $\mathbf{E} = 0$; for $|x| \gg r$: $\varphi \approx \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{|x|}$, $\mathbf{E} \approx \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{x^2}\dfrac{x}{|x|}\mathbf{e}_x$ (as for a point charge). (c) $E_m = 1.93 \times 10^4$ V/m, $x_m = \pm 42.4$ mm. (d) The points x_m are points of inflection.

3.35. (a) $\varphi = \dfrac{q}{2\varepsilon_0\pi r^2}(\sqrt{r^2+x^2} - \sqrt{x^2})$, $E_x = \dfrac{q}{2\varepsilon_0\pi r^2} \times \left(\dfrac{x}{\sqrt{x^2}} - \dfrac{x}{\sqrt{r^2+x^2}}\right)$. At $|x| \gg r$—the field of a point charge: $\varphi = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{|x|}$, $E_x = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{x^2}\dfrac{x}{|x|}$; (b) $\varphi = 75$ kV, $E_x = 0.53$ MV/m.

3.36. $\varphi = \dfrac{q}{2\varepsilon_0\pi(b^2-a^2)}(\sqrt{b^2+x^2} - \sqrt{a^2+x^2})$, $E_x = \dfrac{qx}{2\varepsilon_0\pi(b^2-a^2)}\left(\dfrac{1}{\sqrt{a^2+x^2}} - \dfrac{1}{\sqrt{b^2+x^2}}\right)$. At $|x| \gg b$—the field of a point charge.

3.37. $E_x = \dfrac{\sigma}{2\varepsilon_0}\dfrac{x}{|x|}$.

3.38. $a = b\sqrt{3}$, $r = 2b$, $\theta = 60°$.

3.39. (a) $E_x = \dfrac{\sigma}{2\varepsilon_0}\left(\dfrac{x+a}{\sqrt{(x+a)^2}} - \dfrac{x+a}{\sqrt{r^2+(x+a)^2}} - \dfrac{x-a}{\sqrt{(x-a)^2}} + \dfrac{x-a}{\sqrt{r^2+(x-a)^2}}\right)$;

(b) $E_x(0) = \dfrac{\sigma}{\varepsilon_0}\left(1 - \dfrac{a}{\sqrt{r^2+a^2}}\right) \approx \dfrac{\sigma}{\varepsilon_0}\left(1 - \dfrac{a}{r}\right)$;

(c) $E_x(a-0) = \dfrac{\sigma}{\varepsilon_0}\left(1 - \dfrac{a}{\sqrt{r^2+4a^2}}\right) \approx \dfrac{\sigma}{\varepsilon_0}\left(1 - \dfrac{a}{r}\right)$;

(d) $E_x(a+0) = -\dfrac{\sigma}{\varepsilon_0}\dfrac{a}{\sqrt{r^2+4a^2}} \approx -\dfrac{\sigma}{\varepsilon_0}\dfrac{a}{r}$;

(e) $E_x = -\frac{1}{4\pi\varepsilon_0}\frac{2\,(2aq)}{x^3}$, where $q = \pi r^2\sigma$ (as for a dipole with the moment $p = 2aq$).

3.40. (a) $\nabla\,[f\,(x)\,\mathbf{e}_x] = df/dx$; (b) $\nabla\mathbf{r} = 3$; (c) $\nabla\mathbf{e}_r = 2/r$; (d) $\nabla\,[f\,(r)\,\mathbf{e}_r] = \frac{2f\,(r)}{r} + \frac{df\,(r)}{dr}$.

3.41. (a) $\nabla\mathbf{a} = 0$; (b) $\Phi_a = 0$.

3.43. $\Phi_r = 4\pi R^3$.

3.44. $\Phi_a = \int\limits_0^R f\,(r)\,4\pi r^2\,dr$.

3.45. $\Phi_1 = -\Phi_2$.

3.46. Zero.

3.47. $\rho = \varepsilon_0\,(1 + 4y + 9z^2)$.

3.48. $\rho = \varepsilon_0 A\,(2/r - \alpha)\exp\,(-\alpha r)$.

3.49. (a)-(d) $[\nabla\mathbf{a}] = 0$.

3.51. No, it cannot—this field is not a potential one.

3.52. (a) $[\nabla\mathbf{E}] = 2a\mathbf{e}_z$; (b) $C = 2a\pi b^2$.

3.53. (a) $E_x = \frac{\sigma}{2\varepsilon_0}\frac{x}{|x|}$. (b) $\varphi = -\frac{\sigma}{2\varepsilon_0}\,|x| + \text{const}$. (c) No, it isn't.

3.54. Yes, it can, if the magnitude of the charge densities of the plates is different.

3.55. $\mathbf{E}_A = 100\mathbf{e}_x$ (V/m), $\mathbf{E}_B = 300\mathbf{e}_x$ (V/m), $\mathbf{E}_C = -100\mathbf{e}_x$ (V/m).

3.56. (a) $\sigma_1 > \sigma_2$. (b) $\varphi_4 - \varphi_3 = -200$ V.

3.57. $E = \lambda/2\pi\varepsilon_0 r$.

3.58. (a) $E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}$, $\varphi = -\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r}{r_0}$. (b) $E = 3.6$ kV/m, $\varphi = -83$ kV. (c) No, it cannot.

3.59. $F_e = 4.9 \times 10^{-15}$ N, $F_m = 1.9 \times 10^{-22}$ N.

3.60. $F = 8.1$ N/m, $A = 0.112$ J/m.

3.61. (a) $\mathbf{E} = 0$. (b) The potential at all points inside the sphere, including the centre of the sphere, is the same. (c) $\varphi = R\sigma/\varepsilon_0$ (see the answer to Problem 3.15).

3.62. $\mathbf{F} = 0$ (compare with the answer to Problem 1.211).

3.63. $\mathbf{E} = \rho\mathbf{r}/3\varepsilon_0$, $\varphi = (\rho/2\varepsilon_0)\,(R^2 - r^2/3)$.

3.64. The field inside the space is uniform and has a strength of $\mathbf{E} = (\rho/3\varepsilon_0)\,\mathbf{a}$.

3.65. $\mathbf{E} = (\rho_0/2\varepsilon_0)\,(\mathbf{r}/r)$.

3.66. $\mathbf{E} = (\rho_0\mathbf{r}/3\alpha\varepsilon_0 r^3)\,[1 - \exp\,(-\alpha r^3)]$. At large r's: $E \propto 1/r^2$, at small r's: $E \propto r$.

3.67. $\rho = 1.15\ \mu\text{C/m}^3$.

3.68. On the faces perpendicular to the velocity, $\sigma = 1.00\ \mu\text{C/m}^2$, on the remaining faces $\sigma = 1.15\ \mu\text{C/m}^2$.

3.69. $\lambda = 1.11\ \mu$C/m.

3.70. $\mathbf{P} = (1 - 1/\varepsilon)\,\mathbf{D}$.

3.71. (a) $\mathbf{E}$ will diminish ε times; (b) $\mathbf{D}$ will not change; (c) U will diminish ε times.

3.72. (a) $E = 50.0$ V/m, $D = 0.885$ nC/m^2; (b) $P = 0.44$ nC/m^2; (c) $\sigma' = \pm 0.44$ nC/m^2.

3.73. $\langle \rho' \rangle = (P_1 - P_2)/a$.

3.74. (a) $\nabla \mathbf{E} = -E_0 k/(\varepsilon_1 + kx)^2$, where $k = (\varepsilon_2 - \varepsilon_1)/a$; (b) $\Phi_E = SE_0 [2/(\varepsilon_1 + \varepsilon_2) - 1]$; (c) $\rho' = -\varepsilon_0 E_0 k/(\varepsilon_1 + kx)^2$.

3.75. $\rho' = -0.59$ $\mu C/m^3$.

3.76. $\varepsilon(x) = \varepsilon_0 E_0/[(\rho_1'/\alpha) \ln (1 + \alpha x) + D_0/\varepsilon_1]$.

3.77. $E_2 = 5.2$ V/m, $\alpha_2 = 74°$, $\sigma' = 64$ pC/m^2.

3.78. Zero.

3.79. On the part of the plate separated by the surface S: (a) the total extraneous charge is zero; (b) the total bound charge is greater than zero.

3.80. At $|x| \leqslant a$: $\varphi = -\rho x^2/2\varepsilon\varepsilon_0$, $E_x = \rho x/\varepsilon\varepsilon_0$, at $|x| > a$: $\varphi = -[\rho a^2/2\varepsilon\varepsilon_0 + \rho a (|x| - a)/\varepsilon_0]$, $E_x = (\rho a/\varepsilon_0)(x/|x|)$.

3.81. (a) $\mathbf{P} = (1 - 1/\varepsilon) \rho x \mathbf{e}_x$; (b) on both boundary surfaces $\sigma' = (1 - 1/\varepsilon) \rho a$; (c) $\rho' = -(1 - 1/\varepsilon) \rho$.

3.82. (a) $E_x = (x/|x|)(\rho_0/\alpha\varepsilon\varepsilon_0)[1 - \exp(-\alpha |x|)]$; (b) $\rho' = -(1 - 1/\varepsilon) \rho_0 \exp(-\alpha |x|)$.

3.83. $\rho' = 0$.

3.84. $\sigma'_{max} = 3.5$ nC/m^2, $\langle \sigma' \rangle = 1.75$ nC/m^2.

3.85. $\omega = 2\sqrt{3P_r E/l^2\delta}$.

3.86. $F = (1/4\pi\varepsilon_0) q^2/(2a)^2 = 0.36$ mN.

3.87. $\sigma = -qa/2\pi (a^2 + x^2)^{3/2}$, $q_{ind} = -q$.

3.88. In the fifth digit.

3.89. (a) $E_1 = E_2 = E$, $D_1 = D$, $D_2 = \varepsilon D$; (b) $E_1 = E_2 = 2E/(1 + \varepsilon)$, $D_1 = 2D/(1 + \varepsilon)$, $D_2 = 2\varepsilon D/(1 + \varepsilon)$. The density of the E lines in the entire gap is the same, the density of the D lines in part *2* of the gap is ε times greater than in part *1* of it.

3.90. (a) $E_1 = 2\varepsilon E/(1 + \varepsilon)$, $E_2 = 2E/(1 + \varepsilon)$, $D_1 = D_2 = 2\varepsilon D/(1 + \varepsilon)$; (b) $E_1 = E$, $E_2 = E/\varepsilon$, $D_1 = D_2 = D$. The density of the D lines in the entire gap is the same, the density of the E lines in part *2* of the gap is ε times smaller than in part *1* of it.

3.91. $C = 4.4$ nF.

3.92. $C = 6.0$ nF.

3.93. $C = 2\pi\varepsilon\varepsilon_0 l/\ln (r_2/r_1)$.

3.94. $C \approx 1.6$ pF.

3.95. $C = 4\pi\varepsilon\varepsilon_0 r_1 r_2/(r_2 - r_1)$.

3.96. $C = 100$ pF.

3.97. (a) $C = \sum_{k=1}^{N} C_k$; (b) $C = 1/\sum_{k=1}^{N} (1/C_k)$.

3.98. $C = 10$ pF.

3.99. We must connect C_1 and C_2 in parallel and C_3 in series with them.

3.100. $U_1 = 200$ V, $U_2 = 100$ V, $q = 20$ nC, $C = 67$ pF.

3.101. $q_1 = q_2 = q_3 = 55$ μC.

3.102. $q_1 = -24$ μC, $q_2 = -36$ μC, $q_3 = +60$ μC.

3.103. $C_1 = \pi\varepsilon\varepsilon_0/\ln (b/a) = 9$ pF/m.

3.104. $C \approx 2\pi\varepsilon\varepsilon_0 a = C'/2$, where C' is the capacitance of a sphere of radius a, $C = 0.56$ pF.

3.105. $W = 2.3 \times 10^{-25}$ J.

3.106. (a) $W = -2.6 \times 10^{-18}$ J $= -16$ eV; (b) $\sum W = -1.6 \times 10^{6}$ J $= -1.0 \times 10^{25}$ eV.

3.107. (a) $W = (1/4\pi\varepsilon_0)(q^2/a)(\sqrt{2}+4)$.
(b) $W = (1/4\pi\varepsilon_0)(q^2/a)(\sqrt{2}-4)$.
(c) $W = -(1/4\pi\varepsilon_0)\ (q^2/a)\sqrt{2}$.

3.108. $$W=\frac{1}{2}\sum_{\substack{i,k=1\\(i\neq k)}}^{N}\frac{1}{4\pi\varepsilon_0}\frac{q_i q_k}{|\mathbf{r}_i-\mathbf{r}_k|}.$$

3.109. $$W=\frac{1}{8\pi\varepsilon_0}\int\limits_V \rho(\mathbf{r}')\,dV'\int\limits_V\frac{\rho(\mathbf{r})\,dV}{|\mathbf{r}-\mathbf{r}'|}.$$

3.110. (a) $W=\frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r}\right)=4.5$ nJ; (b) $\eta=0.99$; (c) $R=$ $=2.00$ cm.

3.111. (a) $W=\frac{3}{5}\left(\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r}\right)=5.4$ nJ; (b) $W_1=\frac{1}{6}W=0.9$ nJ; (c) $W_2=\frac{5}{6}W=4.5$ nJ.

3.112. $A = 0.9$ nJ.

3.113. $r_{cl} = 2.82\times10^{-15}$ m.

3.114. $W = 27$ mJ.

3.115. (a) $A = q^2(a-b)/8\pi\varepsilon\varepsilon_0 ab = 9$ mJ. (b) Because this formula takes no account of the work done for crowding of the charge on the compressed plate.

3.116. $A = q^2\,\Delta x/2\varepsilon\varepsilon_0 S = 11.3$ μJ.

3.117. (a) w increases ε times; (b) w decreases ε times.

3.118. (a) $R = 1/\sum\limits_{k=1}^{N}(1/R_k)$; (b) $R=\sum\limits_{k=1}^{N}R_k$.

3.119. The resistors R_2 and R_3 connected in parallel have to be connected in series with R_1.

3.120. $R = R_1/2+\sqrt{R_1^2/4+R_1R_2} = 4\ \Omega$.

3.121. $$R=\rho\int\limits_0^l\frac{dx}{S(x)}.$$

3.122. 12 m.

3.123. $R = (\rho/2\pi d)\ln(b/a)$.

3.124. $R = (\rho/4\pi)(1/a-1/b)$. At $b=\infty$, $R=\rho/4\pi a$.

3.125. $I = (\Phi_2-\Phi_1)/\Delta t$; the current flows into the closed surface.

3.126. $I = 1.7$ nA.

3.127. $t = 0.69$ μs.

3.128. (a) $I = (q_0/RC)\exp(-t/RC)$; (b) $q = 0.18$ mC; (c) $Q=$ $=82$ mJ.

3.129. $Q = CU^2/2$.

3.130. $Q = 63$ mJ.

3.131. $n = 6$, $P_{max} = 30$ W.

3.132. (a) $A_1=\frac{1}{4}CU^2=63$ μJ; (b) $A_2=-\frac{1}{2}CU^2=-125$ μJ.

3.133. (a) $A_1 = \frac{1}{2} CU^2 (\varepsilon - 1)/(2\varepsilon + 1) = 36\ \mu J$; (b) $A_2 = -CU^2 (\varepsilon - 1)/(2\varepsilon + 1) = -71\ \mu J$.

3.134. $\rho = 2.33 \times 10^{13}\ \Omega \cdot m$.

3.135. $I = 5.9$ nA.

3.136. 1. (a) $E_1 = 25$ kV/m, $E_2 = 50$ kV/m, $D_1 = 0.44\ \mu C/m^2$, $D_2 = 1.33\ \mu C/m^2$; (b) $\sigma_1 = 0.44\ \mu C/m^2$, $\sigma_2 = -1.33\ \mu C/m^2$, $\sigma = 0.89\ \mu C/m^2$; (c) $\sigma_1' = -0.22\ \mu C/m^2$, $\sigma_2' = 0.88\ \mu C/m^2$, $\sigma' = -0.66\ \mu C/m^2$; (d) $j = 2.5\ \mu A$.

2. (a) $E_1 = 50$ kV/m, $E_2 = 0$, $D_1 = 0.88\ \mu C/m^2$, $D_2 = 0$; (b) $\sigma_1 = 0.88\ \mu C/m^2$, $\sigma_2 = 0$, $\sigma = -0.88\ \mu C/m^2$; (c) $\sigma_1' = -0.44\ \mu C/m^2$, $\sigma_2' = 0$, $\sigma' = 0.44\ \mu C/m^2$; (d) $j = 0$.

3.137. $I = 0.97\ \mu A$.

3.138. $I = UC/\rho\varepsilon\varepsilon_0$.

3.139. $R = \rho/2\pi a = 2R'$, where R' is the resistance between the sphere of radius a and a spherical shell with a very large radius r ($r \gg a$) concentric to it (see the answer to Problem 3.124).

3.140. (a) $q = q_0 \exp(-\sigma t/\varepsilon\varepsilon_0)$; (b) $Q = (q_0^2/8\pi\varepsilon\varepsilon_0)(1/a - 1/b)$.

3.141. Yes, it can, if an e.m.f. equal to IR acts across the section.

3.142. $\varphi_1 - \varphi_2 = -4.5$ V.

3.143. $\varphi_A - \varphi_B = 0$.

3.144. $I_1 = 0.87$ A, $I_2 = -1.31$ A.

3.145. $I_1 = 0.6$ A, $I_2 = -2.9$ A, $I_3 = -2.3$ A.

3.146. (a) $I_1 = I_3 = 1.00$ A, $I_2 = I_4 = -1.00$ A. They will not change. (b) $I_1 = -0.92$ A, $I_2 = 0.04$ A, $I_3 = 0.36$ A, $I_4 = 0.52$ A.

3.147. $I_1 = -6.4$ mA, $I_2 = 1.8$ mA, $I_3 = 4.6$ mA, $I_4 = 0$.

3.148. $U = U_0 Rx/[Rl + R_0 (l - x)\, x/l]$, at $R \gg R_0$, $U = U_0 x/l$.

3.149. $B = 4.8$ mT.

3.150. $I = 24$ kA.

3.151. $F_m = 2.3 \times 10^{-28}\ N = 10^{-6} F_e = (v/c)^2 F_e$.

3.152. (a) $B = 6.3\ \mu T$; (b) $B = 2.2\ \mu T$.

3.153. $B = \frac{1}{2} B_\infty$, where $B_\infty = (\mu_0/4\pi)(2I/b)$ is the magnetic induction at the distance b from the infinite straight current.

3.154. $B = 5.5\ \mu T$.

3.155. $B = 0$.

3.156. $B = \mu_0 I/2\pi b \sqrt{1 + (b/a)^2}$. At $a = \infty$, $B = \mu_0 I/2\pi b$—the field of an infinite straight current.

3.157. $B = \mu_0 (I/2r) \tan(\pi/n)/(\pi/n)$. At the limit when $n \to \infty$, $B = \mu_0 (I/2r)$—the field at the centre of a ring current.

3.158. $B = 8.9\ \mu T$.

3.159. $H = \frac{nI}{2} \left\{ \frac{(l/2) - x}{\sqrt{r^2 + [(l/2) - x]^2}} + \frac{(l/2) + x}{\sqrt{r^2 + [(l/2) + x]^2}} \right\}$. (a) $H = nI$; (b) $H = nI/2$.

3.160. This leads to the appearance of an axial component of the current and correspondingly to the appearance of an additional field similar to that of a straight current.

3.161. $\mathbf{H} = \frac{1}{2} [\mathbf{jr}]$ for $r \leqslant R$, $\mathbf{H} = \frac{1}{2} (R^2/r^2) [\mathbf{jr}]$ for $r \geqslant R$.

3.162. The field inside the space is uniform and has a strength of $\mathbf{H} = \frac{1}{2}[\mathbf{ja}]$.

3.163. $B = 26$ pT.

3.164. $\mathbf{B} = 0$.

3.165. $L = \frac{2}{5} mR^2\omega$, $p_m = \frac{1}{5} qR^2\omega$, $p_m/L = q/2m$.

3.166. $p_m \approx \frac{1}{3}\pi NI (R_1^2 + R_1R_2 + R_2^2) = 2.2$ mA·m², $H \approx \approx \frac{NI}{2(R_2 - R_1)} \ln \frac{R_2}{R_1} = 23$ A/m.

3.167. $B_2 = 1.26$ mT.

3.168. $F = (3\pi/2)\, \mu_0 N^2 I^2 (r/l)^4 = 4 \times 10^{-9}$ N.

3.169. $F = 6$ μN, $A = 0.33$ μJ.

3.170. $I_m = (k/2NS\mu_0 H) \tan^{-1}(l_2/2l_1) = 0.09$ mA (S is the area of the coil).

3.171. $\Delta P = 0.13$ μN.

3.172. $B = \sigma_t \pi d^2/4Ir = 1.8$ kT (the maximum achievable value with the aid of electromagnets having an iron core is under 10 T).

3.173. $\mathbf{j}(r) = (3\alpha/2\pi)\, r\mathbf{e}_z$.

3.174. $H = (U_m r/2d) \sqrt{\sigma^2 + (\varepsilon\varepsilon_0\omega)^2} \cos[\omega t + \tan^{-1}(\varepsilon\varepsilon_0\omega/\sigma)]$.

3.176. $\mathbf{B}$ will increase μ times, $\mathbf{H}$ will not change.

3.177. $\langle H_l \rangle = I/l$.

3.178. $\mathbf{B} = \mathbf{B}_0$; $\mathbf{H} = \mathbf{H}_0/\mu$, where $\mathbf{H}_0$ is the strength of the external magnetic field.

3.179. (a) $\Phi_B = 0$, $\Phi_H = (SB_0/\mu_0)(1/\mu_2 - 1/\mu_1)$.

3.180. (a) $\nabla \mathbf{H} = -(B_0/\mu_0)[k/(\mu_1 + kx)^2]$, where $k = (\mu_2 - \mu_1)/a$; (b) $\Phi_H = (SB_0/\mu_0)[1 - 2/(\mu_1 + \mu_2)]$.

3.181. (a) $\mathbf{H} = [3/\mu_0(2 + \mu)]\, \mathbf{B}_0 = [3/(2+\mu)]\, \mathbf{H}_0$ ($\mathbf{H}_0$ is the strength of the external magnetic field), $\mathbf{B} = [3\mu/(2 + \mu)]\, \mathbf{B}_0$; (b) $\mathbf{B} \approx \approx 3\mathbf{B}_0$.

3.182. $\Phi_B = 0$, $\Phi_H = (SB/\mu_0)(1 - 1/\mu)$.

3.183. $\mu = (\pi d - b) H/(NI - bH) = 38 \times 10^2$.

3.184. $\mu_{max} \approx 9800$ at $H = 65$ A/m.

3.185. (a) $\mu = 3 \times 10^3$; (b) $\Phi = 0.7$ mWb; (c) $W_1 = 0.1$ J, $W_2 = 0.7$ J, $W = 0.8$ J.

3.186. (a) and (c) clockwise; (b) and (d) counterclockwise.

3.187. (a) Counterclockwise. (b) $q = B\pi a^2/R$.

3.188. $F = Q/v$.

3.189. $U = 5.3$ mV.

3.190. (a) $U = 2.0$ nV; (b) $U = -33$ mV.

3.191. $H = 400$ kA/m.

3.192. $L = 1.3$ mH.

3.193. $L = (\mu_0 l/\pi) \ln[(b - a)/a] = 18$ μH.

3.194. $L_1 = (\mu_0/\pi)[1/2 + \ln(b/a)] = 1.4$ μH/m.

3.195. $C_1 = 100$ pF/m, $L_1 = (\mu\mu_0/2\pi) \ln(b/a) = 0.26$ μH/m.

3.196. $L = 0.2$ H.

3.197. $L_{12} = 5.0$ nH.

3.198. $L_{12} = \mu\mu_0 nNS$.

3.199. $L_{12} = (1/2\pi)\, \mu\mu_0 Na \ln(r_2/r_1)$.

3.200. $I = Bv \sin\alpha/R_1 (1 + \sin\alpha) = \text{const}$, the direction is counterclockwise.

3.201. (a) $I = \mu_0 v I_0 \ln(b/a)/2\pi R$, the direction is counterclockwise; (b) $F = [\mu_0 I_0 \ln(b/a)/2\pi]^2 v/R$, $x = (b-a)/\ln(b/a)$; (c) $P = RI^2$.

3.202. $v = mgR(\sin\alpha - k\cos\alpha)/B^2l^2$.

3.203. $a = mg(\sin\alpha - k\cos\alpha)/(m + CB^2l^2) = \text{const}$.

3.204. At $B < B_0$ $\left(B_0^2 = \frac{8R\sqrt{mga(I_0+ma^2)}}{b^4}\right)$ $\alpha = (\alpha_0/\cos\gamma)\, e^{-\beta t}\cos(\omega t + \gamma)$, where $\beta = \frac{B^2b^4}{8R(I_0+ma^2)}$, $\omega = \sqrt{\frac{mga}{I_0+ma^2} - \frac{B^4b^8}{64R^2(I_0+ma^2)^2}}$, $\gamma = \tan^{-1}(-\beta/\omega)$. At $B \geqslant B_0$—aperiodic return of the pendulum to its equilibrium position.

3.205. $\alpha = \alpha_0 \cos\omega t$, where $\omega = \sqrt{\frac{4mga}{4(I_0+ma^2)+CB^2b^4}}$.

3.206. (a) $I = (mg/Bb)\cos\omega t$; (b) $\mathscr{E} = \frac{1}{2}Bb^2\omega + (mgR/Bb)\cos\omega t$.

3.207. (a) $\tau_1 = 0.58$ s; (b) $\tau_2 = 2\tau_1 = 1.16$ s.

3.208. $Q = RL\mathscr{E}^2/2R_0^2(R+R_0) = 6.0\ \mu\text{J}$.

3.209. $W = \pi(b+a)(b-a)^2 \int_0^B H\,dB = 0.7$ J.

3.210. $q = 2.4$ mC.

3.211. (a) $v = 5.9 \times 10^6$ m/s; (b) $v = 1.64 \times 10^8$ m/s.

3.212. $v_{\text{cl}} = 1.88 \times 10^8$ m/s, $v_{\text{rel}} = 1.64 \times 10^8 \text{ m/s} = 0.87 v_{\text{cl}}$.

3.213. (a) Along a parabola. (b) $R = m_e v_0^2 \sin^2\alpha/eE$ (m_e is the mass of an electron, and e is the elementary charge). (c) $\Delta\mathbf{p} = -e\mathbf{E}\tau$. (d) $L = \frac{1}{2} t^2 eEv_0 \sin\alpha$.

3.214. $e' = 8.0 \times 10^{-19}\ \text{C} = 5e$.

3.215. $F = 1.8 \times 10^{-14}$ N.

3.216. (a) $r = 7.3$ cm; (b) $p_m = 4.1 \times 10^{-14}$ J/T, the directions

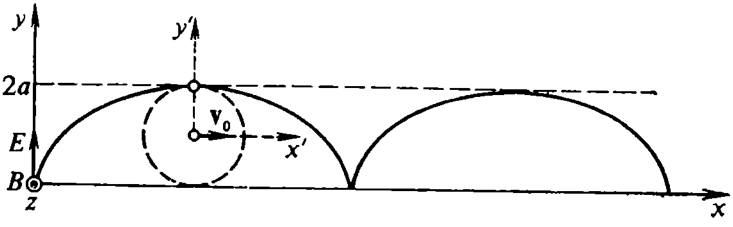

Fig. A.15

of $\mathbf{p}_m$ and $\mathbf{B}$ are opposite; (c) $p_m/L = e'/2m = 2.41 \times 10^7$ C/kg.

3.217. $v = 4.5 \times 10^7$ m/s.

3.218. $l = 21$ mm.

3.219. (a) The motion of the particle is described by the equations:

$$x = v_0 t - a\sin\omega t,\ y = a(1-\cos\omega t),\ z = 0,$$

where $v_0 = E/B$, $a = mE/e'B^2$, $\omega = (e'/m)B$ (Fig. A.15).

If we go over to the reference frame K' in which $x' = x - v_0 t$, $y' = y - a$ (the origin of this frame is displaced along the y-axis by a and moves along the x-axis at the speed v_0), in this frame the motion of the particle is described by the equations: $x' = -a \sin \omega t$, $y' = -a \cos \omega t$. This signifies that in the frame K' the particle travels clockwise in a circle of radius a at the constant angular speed ω. Consequently, the particle moves relative to a stationary frame like a point on the rim of a wheel of radius a travelling along a level road at the speed v_0. The trajectory described by the point in this case is called a cycloid.

(b) The particle's speed will vary within the limits from 0 to $2v_0$ according to the law $v = v_0 \sqrt{2 - 2\cos \omega t}$.

3.220. $-e/m = -1.8 \times 10^{11}$ C/kg.

3.221. $A_{r1} = 4.0$, $A_{r2} = 3.0$. The peaks correspond to the helium isotopes ^{4}He and ^{3}He.

3.222. (a) $W = 17$ MeV, $v = 5.8 \times 10^7$ m/s $= 0.19c$; (b) $\tau = 4.7$ μs; (c) $s = 0.131 \sum_{n=1}^{172} \sqrt{n} \approx 198$ m (use the formula from Appendix 12 to calculate the sum).

3.223. (a) $s = 1.7 \times 10^6$ m $= 1700$ km; (b) $v = 0.999\,95c$.

3.224. $I_0 = U_0 \sqrt{C/L}$.

3.225. $\mathscr{E}_m = 15.1$ mV, $\mathscr{E} = 10.7$ mV.

3.226. (a) $I = 71$ mA; (b) $\varphi = -63°$ (the current is in advance of the voltage); (c) $U_R = 57$ V, $U_L = 28$ V, $U_C = 142$ V; (d) $P = 4.0$ W.

3.227. (a) $Q = 2.4$ kW. (b) It will increase 2 times.

3.228. $U_1 = 1.0$ kV, $I = 10$ A.

3.229. (a)
$$\omega = \sqrt{-\frac{R^2}{L^2} + \frac{1}{LC}\sqrt{1 + 2\frac{C}{L}R^2}} \approx \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = 3.2 \times 10^4 \text{ s}^{-1}.$$

(b)
$$I_1 = U\sqrt{\frac{C}{L}\left(-2 - \frac{C}{L}R^2 + 2\sqrt{1 + 2\frac{C}{L}R^2}\right)} \approx U\frac{C}{L}R = 22 \text{ mA},$$

$$I_2 = U\frac{C}{L}\sqrt{-R^2 + \frac{L}{C}\sqrt{1 + 2\frac{C}{L}R^2}} \approx U\sqrt{\frac{C}{L}} = 7.0 \text{ A},$$

$$I_3 = U\sqrt{\frac{C}{L\sqrt{1 + 2\frac{C}{L}R^2}}} \approx U\sqrt{\frac{C}{L} - \frac{C^2}{L^2}R^2} = 7.0 \text{ A}.$$

3.230. From 186 to 570 m.
3.231. $P = 0.15$ mW.
3.232. $P = 4.2$ mW.
3.233. (a) $Q = 5.00$. (b) $\Delta Q/Q \approx 0.5\%$.
3.234. $(\omega_0 - \omega)/\omega_0 \approx 0.12\%$.
3.235. (a) $W = W_0 \exp(-\omega_0 t/Q)$. (b) 50%.
3.236. $Q \geqslant 7.1$.
3.237. (a) $I_1/I_2 = 19$; (b) $I_1/I_2 = 2.2$.
3.238. (a) $U'_{m1} = 5.0$ mV, $U'_{m2} = 191$ mV; (b) $U'_{m1} = 50$ mV, $U'_{m2} = 196$ mV.

PART 4. WAVES

4.1. This equation can describe with equal right either a longitudinal or a transverse wave.
4.2. The frequency remains unchanged, the wavelength doubles.
4.3. $\Delta x = \lambda/2$.
4.4. See Fig. A.16. At point B, the velocity in both cases is zero.

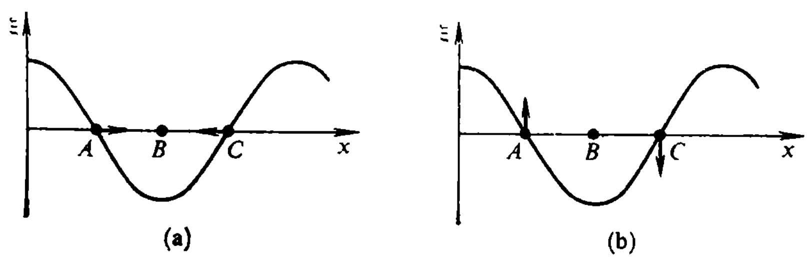

Fig. A.16

4.5. $\delta\varphi = 0.63$ rad.
4.6. The complex amplitude $\hat{A}$ contains data on the ordinary amplitude A and the initial phase of the oscillations α.
4.7. $\hat{A} = 10.6 \exp(0.810i)$.
4.8. $\xi = (A/\sqrt{r}) \cos(\omega t - kr + \alpha)$, where r is the distance from the filament.
4.9. (a) The dimension of α coincides with that of the square of the velocity.
(b) Changes in the quantity f in space and time can have the nature of a plane wave running along the x-axis at a speed equal to $1.20 \times \times 10^4$ m/s.
4.10. An equation of such a kind describes a plane wave of an arbitrary shape (i.e. not necessarily harmonic) propagating along the x-axis at a speed of $v = \omega/k$.
4.11. $v = 3.5 \times 10^3$ m/s.
4.12. 1. (a) Points A and C; (b) points B and D.
2. (a) Zero; (b) it is maximum.

4.13. See Fig. A.17, where ρ_0 is the density of the medium in the absence of a wave.

4.14. See Fig. A.18.

4.15. $\mathbf{j} = (\rho a^2 \omega^3/k) \sin^2 (\omega t - kx + \alpha)\, \mathbf{e}_x$.

4.16. The energy flux carried by an elastic wave through surface S.

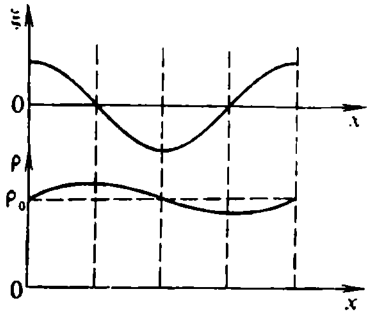

Fig. A.17

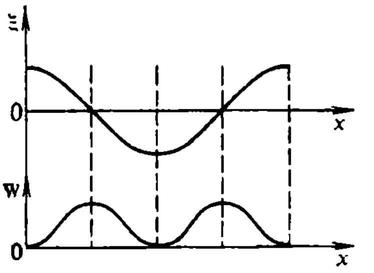

Fig. A.18

4.17. $W = Sj_1 \{1 - \exp[-2\gamma (x_2 - x_1)]\}\, t$.

4.18. (a) $(1/r^2) \exp(-\varkappa r)$ (r is the distance from the centre); (b) $(1/r) \exp(-\varkappa r)$ (r is the distance from the axis).

4.19. These lines are hyperbolas at whose focuses the sources are placed.

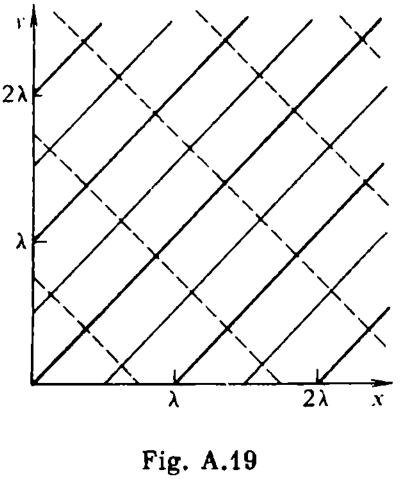

Fig. A.19

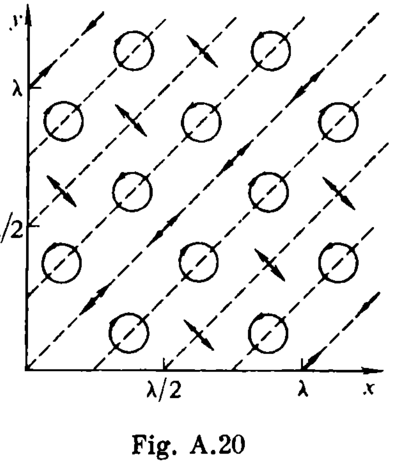

Fig. A.20

4.20. 1. (a) Zero for all surfaces; (b) zero for surfaces *1*, *3*, *5*, *7*, *9*, non-zero for surfaces *2*, *4*, *6*, *8*.

2. Zero for all surfaces.

3. To the right for surfaces *2* and *6*, to the left for surfaces *4* and *8*.

4. To the left for surfaces *2* and *6*, to the right for surfaces *4* and *8*.

4.21. The maxima of the amplitude equal to $2a$ (see the heavy straight lines in Fig. A.19 corresponding to $\alpha = 0$) are arranged along the straight lines $y = x + (\alpha\lambda/2\pi) \pm n\lambda$ $(n = 0, 1, 2, \ldots)$. The minima of the amplitude equal to zero (the thin straight lines in the figure) are arranged along the straight lines $y = x + (\alpha\lambda/2\pi) \pm \pm (n + 1/2)\lambda$ $(n = 0, 1, 2, \ldots)$. The phase has the same value at points satisfying the condition $x + y = \text{const}$. Points on adjacent dashed lines (see the figure) oscillate in counterphase. The dashed lines are spaced at $\lambda/\sqrt{2}$.

4.22. At points on the straight lines $y = x + (\alpha\lambda/2\pi) \pm n\lambda$ $(n = 0, 1, 2, \ldots)$, the particles of the medium oscillate along these lines (see Fig. A.20, drawn for $\alpha = 0$). At points on the straight lines $y = x + (\alpha\lambda/2\pi) \pm (n + 1/2)\lambda$ $(n = 0, 1, 2, \ldots)$, the particles of the medium oscillate at right angles to these lines (and to the z-axis). Finally, at points on the straight lines $y = x + (\alpha\lambda/2\pi) \pm \pm (n + 1/4)\lambda$ $(n = 0, 1, 2, \ldots)$, the particles of the medium travel along circles in the plane x, y. At the other points, the particles travel along ellipses. Motion having this nature occurs in any plane $z = \text{const}$.

4.23. $v = 2.00$ km/s.

4.24. (a) It will increase 2 times; (b) it will increase 3 times.

4.25. $A_a = (1/n\omega_1)\sqrt{8\langle E_k\rangle/m}$.

4.26. $F = 296$ N.

4.27. $F = 8.4$ kN.

4.28. It will decrease $\sqrt{2}$ times.

4.29. $\nu_n = 2.5(2n - 1)$ kHz, where $n = 1, 2, 3, \ldots$.

4.30. $\nu_n = 85(2n - 1)$ Hz, where $n = 1, 2, 3, \ldots$.

4.31. $\nu = 5$ kHz.

4.32. (a) Beats at a frequency of 50 Hz; (b) a sound having a frequency of 50 Hz whose intensity will pulsate with a period of 1 s; (c) nothing.

4.33. $\Delta\nu = 6$ Hz.

4.34. In nitrogen; 1.3 times.

4.35. 1. (a) 306 m/s; (b) 331 m/s; (c) 354 m/s.
2. 0.92:1:1.07.

4.36. $t = 30$ s.

4.37. 1000 times.

4.38. (a) 82 dB; (b) 64 dB; (c) 46 dB; (d) 28 dB; (e) 10 dB.

4.39. (a) 74 dB; (b) 68 dB; (c) 64.5 dB; (d) 62 dB; (e) 60 dB.

4.40. (a) $L_2 = 53$ dB, $r_0 = 3.0$ km; (b) $L_2 = 54$ dB, $r_0 = 100$ km.

4.41. $(\Delta p)_{m1}/(\Delta p)_{m2} = 10^{0.05L_{12}} = 10$.

4.42. (a) 2.9×10^{-5} Pa; (b) 2.9 Pa (compare with the answer to the preceding problem).

4.43. (a) $v_m = 0.63$ m/s; (b) $A/\lambda = 2.9\times10^{-4}$; (c) $v_m/v = 1.9\times10^{-3}$.

4.44. Ahead of the source, $\lambda = 0.5\lambda_0$; behind the source, $\lambda = 1.5\lambda_0$; in directions perpendicular to the direction of motion of the source, $\lambda = \lambda_0$.

4.45. $\nu = 1.03$ kHz.

4.46. Beats at a frequency of $\Delta\nu \approx 2\nu_0(v/u) = 17$ Hz (u is the speed of sound at the given temperature).

4.47. Beats at a frequency of $\Delta\nu = 50$ Hz will be heard at point A.

4.48. The occupants of the second vehicle will hear a sound at 700 Hz, and of the third, at 800 Hz.

4.49. (a) Only receiver R_1. (b) $\Delta\nu = 50$ Hz.

4.50. (a) $\mathbf{k} = \pm(\omega/c)\,\mathbf{e}_x$, $\mathbf{k} = \pm(\omega/c)\,\mathbf{e}_z$; (b) $\mathbf{k} = \pm(2\pi/\lambda)\,\mathbf{e}_x$, $\mathbf{k} = \pm(2\pi/\lambda)\,\mathbf{e}_y$.

4.51. (a) $H_m = 46$ mA/m; (b) $v = 1.7 \times 10^8$ m/s.

4.52. $\mathbf{E} = \mathbf{E}_m \cos(\omega t + kx + \alpha)$, $\mathbf{H} = \mathbf{H}_m \cos(\omega t + kx + \alpha + \pi)$.

4.53. 1. (a) $x_a = \pm n\lambda/2$, $x_n = \pm(n + 1/2)\,\lambda/2$ $(n = 0, 1, 2, \ldots)$; (b) $x_a = \pm(n + 1/2)\,\lambda/2$, $x_n = \pm n\lambda/2$ $(n = 0, 1, 2, \ldots)$. The antinodes of **E** coincide with the nodes of **H**, and vice versa.

2. The phases of oscillations of **E** and **H** differ by $\pi/2$. When **E** is maximum, **H** is zero, and vice versa.

4.54. $\mathbf{S} = 0.38 \cos^2(\omega t + \alpha)\,\mathbf{e}_y$ (W/m²).

4.55. $W = 1.00$ mJ.

4.56. (a) $S = (r\varepsilon_0 U^2/2\tau^2 d^2)\,t$, it is directed inward; (b) $W = \varepsilon_0 U^2 \pi r^2/2d = wV$, where w is the energy density.

4.57. (a) On the side surface, $S = (\mu_0 n^2 I^2 r/2\tau^2)\,t$, it is directed inward; on the ends, $S = 0$; (b) $W = \mu_0 n^2 I^2 \pi r^2 l/2 = wV$, where w is the energy density.

4.58. (a) $E_m = 18.8$ V/m; (b) $\langle w \rangle = 1.57$ nJ/m³; (c) $I = 0.47$ W/m²; (d) $\langle \mathbf{K}_{u.v} \rangle = 5.2 \times 10^{-18}\mathbf{e}_x$ kg/(m²s).

4.59. $p = 1.57$ nPa.

4.60. $\lambda = 6.0 \times 10^6$ m, $P = 8.3 \times 10^{-20}$ W.

4.61. $P = 1.1 \times 10^{-8}$ W.

4.62. $\eta = 0.493 \approx 1/2$.

4.63. $P = 2.0 \times 10^{-16}$ W $= 1.25 \times 10^3$ eV/s.

4.64. (a) $\eta = 80\pi e^3 B/c^2 m^2 = 1.38 \times 10^{-11}$; (b) $\tau = -\ln 0.99 c^2 m^3/40 e^4 B^2 = 0.026$ s; (c) $N = -\ln 0.99 c^2 m^2/80\pi e^3 B = 0.73 \times 10^9$ rev.

4.65. (a) $\eta = 4.1 \times 10^{-18}$; (b) $\tau = 1.6 \times 10^8$ s ≈ 5 years; (c) $N = 2.5 \times 10^{15}$ rev.

PART 5. OPTICS

5.1. (a) 498.3 s $\approx$ 500 s $\approx$ 8.3 min; (b) 1.3 s; (c) 4.6 s; (d) 0.04 s.

5.2. No, it doesn't; colour perception is determined not by the wavelength, but by the frequency of light.

5.3. $\lambda = 605.8$ nm. In the orange region.

5.4. $\lambda = 32.6$ cm.

5.5. $\mathbf{e}' = \mathbf{e} - 2\mathbf{n}\,(\mathbf{en})$.

5.6. $\mathbf{e}'' = (1/\nu)\,\{\mathbf{e} - \mathbf{n}\,[(\mathbf{en}) + \sqrt{(\mathbf{en})^2 + \nu^2 - 1}]\}$.

5.7. (a) $\varphi = (n/n_0 - 1)\,\theta$. (b) In the approximation being considered, the angle of deflection φ does not depend on the angle of incidence α_1.

5.8. (a) $\Phi = 5$ D; (b) $\Phi = -2.5$ D.

5.9. If the refractive index of the medium at both sides of the lens is the same.

5.10. At the centre of the lens.

5.12. See Fig. A.21.

5.13. See Fig. A.22. The sequence of construction is: ray *3* parallel to ray *1*—focal surface F'—ray *4* parallel to ray *2*. Ray *2'* passes through the point of intersection of ray *4* and F'.

5.14. The rays propagate along parallel directions.

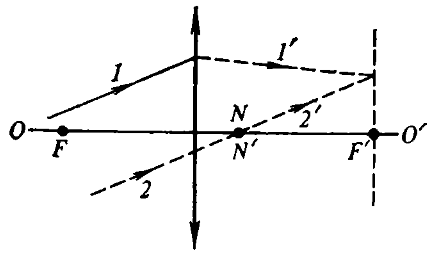

Fig. A.21

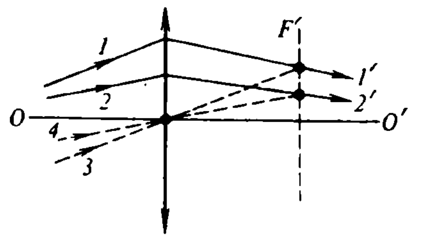

Fig. A.22

5.15. 1. In the common focal plane $F_1' - F_2'$.

2. (a) Virtual; (b) erect.

3. When the focal lengths of the lenses are equal in magnitude.

5.16. (b) The image will move toward plane H'. When the object is in plane H, the image will get into plane H', and the size of the image will become equal to that of the object.

5.17. The construction is shown in Fig. A.23. N and N' are the

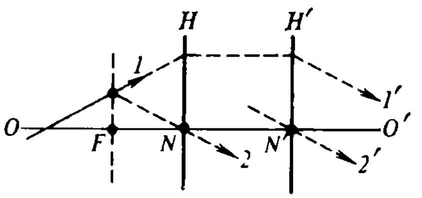

Fig. A.23

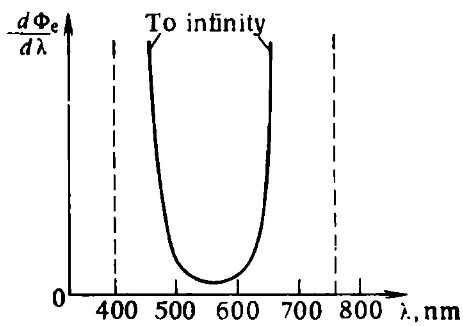

Fig. A.24

nodal points of the system, *2* and *2'* are auxiliary conjugate rays parallel to each other. Ray *1'* is parallel to ray *2'*.

5.18. $E = 1.0 \times 10^2$ lx.

5.19. 0.21 W.

5.20. 214 lm.

5.21. This curve would have the same shape as the curve of the relative spectral sensitivity of the eye depicted in Fig. 5.9.

5.22. (a) See Fig. A.24. (b) No, it is not.

5.23. $p = 1.6$ nPa.

5.24. $E_m = 12.6$ V/m $= 4.2 \times 10^{-4}$ cgse, $H_m = 0.050$ A/m $=$ $= 6.3 \times 10^{-4}$ Oe.

5.25. $I = 100.0$ cd.
5.26. $E = 100$ lx.
5.27. $\Phi = (2\pi/3)\, Eh^2 = 19 \times 10^2$ lm.
5.28. 1. $E = Ih/(h^2 + r^2)^{3/2}$.
2. (a) 50 lx; (b) 27 lx.
3. $\Phi = 2\pi I\,(1 - h/\sqrt{h^2 + R^2}) = 33$ lm.
4. $\eta = 0.053$.
5.29. $I\ (\theta) = (42\ \text{cd})/\cos^3\theta$; $E = 42$ lx.

5.30. $$M = \int_0^{2\pi} d\varphi \int_0^{\pi/2} L\,(\theta,\ \varphi) \cos\theta \sin\theta\, d\theta.$$

5.31. $\Phi = 66$ lm.
5.32. $A = a\sqrt{N}$.
5.33. The intensity pulsates with a period equal to 10 s.
5.34. $I = I_1 + I_2$.
5.35. $A = a \sin (N\delta/2)/\sin (\delta/2)$.
5.36. (a) $t_{\text{coh}} \approx 4 \times 10^{-14}$ s; (b) $l_{\text{coh}} \approx 0.01$ mm; (c) $\rho_{\text{coh}} \approx$ ≈ 0.3 mm; (d) $V_{\text{coh}} \approx 0.003$ mm^3.
5.37. $\rho_J \approx 0.3$ mm $\approx 5\rho_E$.
5.38. $\rho_{\text{coh}} \approx 2.5$ m.
5.39. $\Delta x = \lambda/\varphi$.
5.40. The wavelength in a vacuum.
5.41. (a) For points whose distance from the middle of the screen is much smaller than l, $\delta\Delta = bd/l$. (b) $b_{\max} \approx \lambda l/4d$.
5.42. In red light.
5.43. $a = \lambda_0$, where λ_0 is the wavelength of light in a vacuum.
5.44. The pattern will be shifted in the direction of the covered slit by 10 fringes.
5.45. (a) $\lambda_0 = 650$ nm; (b) $\lambda_0 = 450$ nm.
5.46. $\Delta x = 0.25$ mm.
5.47. (a) $\varphi = \lambda\ (r + b)/2r\,\Delta x = 9.5'$; (b) $N = 5$.
5.48. (a) $\theta = (a + b)\,\lambda/2a\,(n - 1)\,\Delta x = 14.3'$; (b) $N = 7$.
5.49. (a) $\Delta x = \lambda/\Phi h = 0.25$ mm; (b) $N = 7$.
5.50. (a) $b = (\lambda_0/2n)(m + 1/2)$ $(m = 0,\ 1,\ 2,\ \ldots)$; (b) $b =$ $= (\lambda_0/2n)\, m$ $(m = 1,\ 2,\ 3,\ \ldots)$.
5.51. For the fluorescent lamp beam in both cases $I = 90$ lm/m^2. For the laser beam in case (a) $I = 100$ lm/m^2, in case (b) $I = 80$ lm/m^2.
5.52. (a) $n_2 = \sqrt{n_1 n_3} = 1.30$. (b) Yes, it will.
5.53. $a = 0.100$ µm.
5.54. (a) A uniformly illuminated field of vision; (b) alternating light and dark fringes.
5.55. (a) The fringes will increase in diameter; the outer fringes will move out of the field of vision of the telescope; new fringes will appear at the centre, one for each movement of a plate over a distance of $\lambda/2$; (b) the fringes will diminish in diameter; the inner fringe will contract to a point and vanish; new fringes will appear in the field of vision of the telescope from outside.
5.56. (a) A uniformly illuminated field of vision; (b) alternating light and dark fringes parallel to the line of intersection of the planes bordering on the plates; (c) very slightly distinguishable fringes whose distinctness will increase in the direction from gap b_2 to gap b_1.

5.57. The fringes will be displaced (a) to the right; (b) to the left (with respect to Fig. 5.19). Movement of a plate by $\lambda/2$ is attended by displacement of the pattern by one fringe.
5.58. (a) red ($\lambda_0 = 640$ nm); (b) green ($\lambda_0 = 538$ nm).
5.59. (a) $\Delta x = 1.0$ mm; (b) no fringes are observed.
5.60. $b \approx 10$ μm.
5.61. $N = 30$.
5.62. Over 3/4 the plate's length.
5.63. (a) $R = 9.1$ m; (b) $\Phi = +0.055$ D; (c) $r_3 = 3.3$ mm.
5.64. 1.225 times.
5.65. 1. (a) The fringes will decrease in diameter; the inner fringe will contract to a point and vanish; new fringes will appear at the outer boundary of the pattern; (b) the fringes will increase in diameter; the outer fringes will move out of the limits of the pattern; new fringes will appear at the centre.
2. $N = 345$.
5.66. (a) $b = \infty$; (b) $b = 125$ mm; (c) $b = 56$ mm.
5.67. (a) $m_{min} = 8$. (b) $b = 10.0$ m. (c) $r < 0.74$ mm.
5.68. Maxima and minima of the intensity will replace one another successively.
5.69. $m = r^2/b\lambda$.
5.70. $\lambda = 580$ nm.
5.72. $A = \dfrac{A_1 - (-1)^N \rho A_N}{1+\rho} \approx \dfrac{A_1 - (-1)^N A_N}{2}$ (ρ differs very slightly from unity).
5.73. (a) $I = 4I_0$; (b) and (c) $I = 2I_0$; (d) $I = I_0$.
5.74. The intensity will diminish to 1/4 of its initial value.
5.75. (a) 375 nm; (b) 875 nm; (c) 750 nm.
5.76. $\Phi = \int\limits_0^\infty E(r)\, 2\pi r\, dr$.
5.77. (a) $b = 2.50$ m; (b) $b = 1.72$ m; (c) $b = 1.32$ m.
5.78. $I = 4I_0/(1-\rho^2)^2 = 421 I_0$.
5.79. $I = 4I_0/(1-\rho)^2 = 1600 I_0$.
5.80. $h = 580$ nm.
5.81. 1. (a) $m = m' = a^2/4b\lambda$; (b) $m = a^2/b\lambda$, $m' = 0$; (c) $m = 0$, $m' = a^2/b\lambda$.
2. $x_m = \sqrt{mb\lambda}$.
3. 1:0.414:0.318:0.268:0.236.
5.82. 1. (a) $m = m' = 5$; (b) $m = 20$, $m' = 0$; (c) $m = 0$, $m' = 20$.
2. $x_1 = 0.71$ mm, $x_2 = 1.00$ mm, $x_3 = 1.22$ mm, $x_4 = 1.41$ mm, $x_5 = 1.58$ mm.
5.83. 1. Points at which tangents to the curve are parallel to the ξ-axis.
2. (a) $v = 2$; (b) $v = -2$.
5.84. (a) $v = \sqrt{2m}$; (b) 1.41, 2.00, 2.45, 2.83, 3.16.
5.85. (a) $I = 0.25I_0$; (b) $I = 0.38I_0$; (c) $I = 0.025I_0$; (d) $I = 0.07I_0$; (e) $I = 0.18I_0$; (f) $I = 0.004I_0$.
5.86. (a) $I_{max} = 137$ lm/m²; (b) $I_{min} = 78$ lm/m²; (c) $I_{max}/I_{min} = 1.76$; (d) $x_{max} = 0.3$ mm, $x_{min} = 0.45$ mm.

5.87. The summary area of the sections hatched with a slope to the left equals the summary area of the sections hatched with a slope to the right.

5.88. (a) 156 lx; (b) 20 lx.

5.89. These oscillations are shifted in phase relative to each other by 0.75π.

5.90. (a) 30 lx; (b) 92 lx.

5.91. 1. The area characterizes the luminous flux passing through unit length of the slit.

2. (a) The height of the maxima will increase 4 times; (b) the width of the maxima will diminish to one-half of its initial value; (c) the coordinates of all the minima will be halved, as a result of which the 2nd minimum will be in the place of the 1st one, the 4th in the place of the 2nd one, etc.; (d) the number of minima will be doubled; (e) the area will increase 2 times.

5.92. (a) $a \approx 0.7$ mm; (b) $a/\langle\Delta x\rangle \approx 0.3$.

5.93. (a) Fraunhofer diffraction; (b) $a_0 = 5.0$ mm; (c) $a_{12} = 2.5$ mm.

5.94. Fresnel diffraction.

5.95. (a) No, it will not, the centre of the pattern is opposite the centre of the lens; (b) yes, it will, the centre of the pattern is opposite the middle of the slit.

5.96. (a) If the slit was originally narrow enough, the intensity will first grow monotonically, then pulsate with a constantly diminishing spread, oscillating about the value I_0, which is observed in the absence of obstacles; (b) I will grow monotonically.

5.98. 1. The total area of the maxima characterizes the luminous flux passing through unit length of the slits.

2. (a) The 1st maximum will take the place of the 2nd one, the 2nd—that of the 4th one, and so on; (b) the height of the central maximum will increase 4 times; (c) the width of the maxima will remain unchanged; (d) the total area of the maxima will double.

5.99. The maxima will become twice as dense; the height of the central maximum will diminish to one-fourth of its initial value; the width of the maxima will remain unchanged; the total area of the maxima will be halved (compare with Problem 5.98).

5.100. (a) The positions of the maxima will not change; (b) the height of the central maximum will decrease to one-fourth of its initial value; (c) the width of the maxima will double; (d) the total area of the maxima will be halved.

5.101. (a) The positions of the maxima will not change; (b) the height of the central maximum will be halved; (c) the width of the maxima will remain unchanged; (d) the total area of the maxima will be halved.

5.102. 1. (a) $x = (2k + 1)/2m$, where $k = 0, 1, \ldots$ (the values of k at which $x > 1$ are excluded); (b) $x = k/m$, where $k = 1, 2, \ldots, m - 1$.

2. (a) $x = 1/2$; (b) $x = 1/4$ and $3/4$; (c) $x = 1/6$, $3/6$, and $5/6$.

3. (a) The intensity does not become equal to zero at any values of x except for 0 and 1 having no meaning; (b) $x = 1/2$; (c) $x = 1/3$ and $2/3$.

5.103. $b = 1150$ nm.

5.104. $D \approx 3 \times 10^5$ rad/m ≈ 1 ang. min/nm.

5.105. $\Delta\varphi = 22.0°$.
5.106. $\Delta\varphi = 4.8°$.
5.107. (a) No, they will not; (b) yes, they will.
5.108. $N \approx 1000$ slits.
5.109. 1. $D = m/d\sqrt{1-(m\lambda/d)^2}$.
2. (a) 1.09×10^{-3} rad/nm; (b) 1.23×10^{-3} rad/nm; (c) 1.54×10^{-3} rad/nm.
5.110. 1. $D_{\text{lin}} = fm/d\,[1-(m\lambda/d)^2]^{3/2}$.
2. (a) 1.30 mm/nm; (b) 1.85 mm/nm; (c) 3.64 mm/nm.
5.111. 1. $\Delta x = 733$ mm.
2. (a) $D_{\text{lin}} \sim 1$ mm/nm (see Problem 5.110); (b) $R = 10^5$.
5.112. 1. $\delta\varphi/\delta m = (\lambda/d)/\sqrt{1-(m\lambda/d)^2}$.
2. The exact values are: (a) 19.4°; (b) 27.3°. The values calculated by the formula obtained in item 1 are: (a) 19.2°; (b) 25.9°.
5.113. At a large period, the diffraction maxima are arranged very densely (see the answer to item 1 of Problem 5.112). At $d = 1$ mm, the angular distance between adjacent maxima will be of the order of $\lambda/d \approx 0.5 \times 10^{-3}$ rad $\approx 2'$.
5.114. 1. $d(\sin\varphi - \sin\varphi_0) = \pm m\lambda$.
2. (a) $\varphi = \varphi_0 = 20.0°$; (b) $\varphi_+ = 35.6°$, $55.3°$; $\varphi_- = 5.9°$, $-7.9°$, $-22.2°$, $-38.2°$, $-59.1°$; (c) $m_+ = 2$, $m_- = 5$.
3. With normal incidence, the total number of maxima is larger by 1.
5.115. 1. $d(\cos\theta_0 - \cos\theta) = \pm m\lambda$.
2. (a) 2.07°, (b) 2.75°, (c) 4.17°.
5.116. 1. $$D = \frac{|m|}{d\sqrt{1-\left(\sin\varphi_0 + m\dfrac{\lambda}{d}\right)^2}}$$
where $-\dfrac{d}{\lambda}(1+\sin\varphi_0) \leqslant m \leqslant \dfrac{d}{\lambda}(1-\sin\varphi_0)$.
2. (a) 1.6 ang. min/nm; (b) 2.2 ang. min/nm; (c) 10 ang. min/nm; (d) 43 ang. min/nm.
5.117. Yes, it is.
5.118. $a_{\min} = 7.0$ km.
5.119. $l = 1.1$ m.
5.120. $d = 0.282$ nm.
5.121. $\lambda = 0.0588$ nm.
5.122. 11.9 mm and 24.2 mm.
5.123. The vector **E** rotates about the direction of the ray, simultaneously changing in magnitude so that its tip describes an ellipse.
5.124. (a) $P = 0.50$; (b) $P = 0.91$.
5.125. $I = I_0\cos^2\alpha_1\cos^2(\alpha_2 - \alpha_1) = 66$ lm/m².
5.126. It is a uniaxial birefringent plate cut out parallel to an optical axis that produces a phase difference of $\pm\pi/2$ between an ordinary and an extraordinary ray. The thickness d of this plate satisfies the condition: $d = [(m + 1/4)\lambda_0]/(|n_o - n_e|)$, where m is an integer or zero, λ_0 is the length of a light wave in a vacuum, n_o and n_e are the refractive indices of an ordinary and extraordinary rays.
5.127. By passing plane-polarized light through a quarter-wave plate installed so that its optical axis makes an angle of 45° with the plane of oscillations of the plane-polarized light.

5.128. No, it cannot.

5.129. $\mathbf{E}_y = E_0 (\cos\alpha)\,\mathbf{e}_y \cos(\omega t - kx)$, $\mathbf{E}_z = E_0 (\sin\alpha)\,\mathbf{e}_z \cos(\omega t - kx)$.

5.130. $\mathbf{E}'_y = E_0 (\cos\alpha)\,\mathbf{e}_y \cos[\omega(t-\tau) - kx]$, $\mathbf{E}'_z = E_0 (\sin\alpha)\,\mathbf{e}_z \times \cos[\omega(t-\tau) - kx + \delta]$ (τ is the delay time of the wave $\mathbf{E}'_y$ due to its lower speed in the dielectric than in a vacuum, and δ is the phase difference appearing when the light wave passes through the layer at the expense of the different speed of propagation of the components).

5.131. (a) and (b) $\mathbf{E}'(t) = \mathbf{E}(t-\tau)$, where $\mathbf{E}$ is the initial vector (see Problem 5.129), τ is the delay time (see Problem 5.130); (c) $\mathbf{E}'_y(t) = \mathbf{E}_y(t-\tau)$, $\mathbf{E}'_z(t) = -\mathbf{E}_z(t-\tau)$—the planes of oscillations of the vectors $\mathbf{E}$ and $\mathbf{E}'$ are symmetric relative to the y-axis; (d) and (e) the vector $\mathbf{E}'$ rotates about the x-axis, describing with its tip an ellipse whose axes do not coincide with the y- and z-axes; (f) the tip of the vector $\mathbf{E}'$ describes an ellipse whose axes coincide with the y- and z-axes; (g) the vector $\mathbf{E}'$ rotates uniformly about the x-axis, describing a circle with its tip.

5.132. The light will become plane-polarized four times during one revolution, and circularly polarized four times (at intermediate positions). In the remaining time, the light will be elliptically polarized with a continuous change in the shape of the ellipse from a segment of a straight line to a circle and back.

5.133. $I = I_0/2$.

5.134. The intensity will change within the limits from zero to $I_{\text{nat}}/4$, reaching a minimum four times a revolution and a maximum four times.

5.135. (a) $I_{\parallel} = I_{\text{nat}}/8$; (b) $I_{\perp} = 3I_{\text{nat}}/8$.

5.136. 1. At all three points, the light is plane-polarized.

2. $I_A = I_B = I_{\text{nat}}/2$.

3. (a) $I_C = 0$; (b) $I_C = I_{\text{nat}}/2$.

4. $I_C = (I_{\text{nat}}/2)\cos^2(\alpha_1 + \alpha_2)$.

5.137. (a) At point B the light is elliptically polarized, or at $\alpha_1 = 0$ or $\pi/2$—plane-polarized; at point C—plane-polarized light. (b) $I_B = I_{\text{nat}}/2$. (c) No, it cannot. (d) Yes, it can, if $\alpha_1 = \alpha_2 = 0$ or $\pi/2$.

5.138. $P = 0.80$.

5.139. (a) $P_{\parallel} = 2P_1/(1 + P_1^2) = 0.976$; (b) $P_{\perp} = 0$.

5.140. $\eta = \alpha_1^2 (1 - P)/(1 + P) = 0.048$.

5.141. (a) $P_1 = \sqrt{(\eta - 1)/(\eta + 1)} = 0.900$; (b) $P_{\parallel} = \sqrt{\eta^2 - 1}/\eta = 0.994$.

5.142. Opposite the places of the plate for which the optical path difference Δ of an ordinary and an extraordinary ray is $m\lambda$ (m is an integer), the light will be plane-polarized with its plane of oscillations coinciding with the plane of oscillations in the incident light. Opposite the places for which $\Delta = (m + 1/2)\lambda_0$, the light will be plane-polarized with its plane of oscillations perpendicular to the plane of oscillations in the incident light. Opposite the places for which $\Delta = (m \pm 1/4)\lambda_0$, the light will be circularly polarized. At the other places, the light will be elliptically polarized.

5.143. 1. (a) The surface will be illuminated uniformly; (b) and (c) the surface will be mottled with alternating bright and dark fringes

2. The bright and dark fringes will exchange places.

5.144. $\Delta x = 2.0$ mm.

5.145. (a) $E_{min} = 1.51 \times 10^6$ V/m; (b) it becomes bright three times and dark three times (with account taken of the darkening at $E = 0$).

5.146. (a) 500 times. (b) No, they will not.

5.147. (a) red (636 nm); (b) orange (604 nm); (c) yellow (575 nm); (d) green (529 nm).

5.148. (a) $\alpha = 21$ ang. deg/mm; (b) $I(x) = I_m \cos^2 (\pi x/\Delta x)$, where I_m is a constant.

5.149. The wavelength in the medium in which the speed of light is v.

5.150. 1. (a) $u = v/(1 - q)$, (b) $u = (1 - p)\, v$.

2. (a) $u = v/2$; (b) $u = 2v$.

5.151. 1. $a = 1.502$; $b = 4.56 \times 10^3$ nm^2.

5.152. (a) $u = c\lambda_0^2/(a\lambda_0^2 + 3b)$; (b) see Table A.1.

Table A.1

λ_0, nm	759.0	589.3	486.0	397.0
u/c	0.655	0.649	0.641	0.629
v/c	0.662	0.660	0.658	0.653
u/v	0.990	0.983	0.975	0.964

5.153. (a) $a = eE_m/m\omega^2 = 4.9 \times 10^{-19}$ m, $v_m = a\omega = 1.53 \times 10^{-3}$ m/s (E_m is the amplitude of the electric field strength, m is the mass of an electron). (b) $(F_B)_m/(F_E)_m = v_m/2c = 0.26 \times 10^{-11}$.

5.154. $\varepsilon(\omega) = 1 - \dfrac{ne^2}{\varepsilon_0 m\omega^2}$ (m is the mass of an electron).

5.155. 8 times.

5.156. (a) 0.50%; (b) 1.00%; (c) 63%; (d) 99.0%.

5.157. 9.5%.

5.158. (a) 83.4 lm/m^2; (b) 83.5 lm/m^2.

5.159. (a) 92.2 lm/m^2; (b) 92.3 lm/m^2.

5.160. 1. (a) 0.49%; (b) 8.33%. 2. 17 times. 3. 8.78%.

5.161. $\varkappa = \ln (\tau_1/\tau_2)/(a_2 - a_1)$, where τ is the transmission coefficient of light by the plate (the ratio of the intensity of the light passing through the plate to the intensity of the incident light).

5.162. $\varkappa = 1.37$ m^{-1}.

5.163. Four times.

5.164. $\Delta N = 0.10$.

5.165. $v = 1000$ km/h.

5.166. $T = (\lambda/\Delta\lambda)\, 4\pi R/c = 25$ days (R is the Sun's radius).

5.167. $\Delta\omega/\omega = \pm(1/c)\sqrt{8kT/\pi m}$; the plus is for case (a), the minus for case (b) (k is the Boltzmann constant).

5.168. (a) $\delta\omega_D/\omega = 2\langle v\rangle/c$. (b) $\delta\lambda_D = 0.020$ nm.

5.169. $T = 1.0 \times 10^3$ K.

5.170. $v = 1.5 \times 10^6$ m/s.
5.171. $v = 0.26c = 78\ 000$ km/s.

PART 6. ATOMIC PHYSICS

6.1. $r(\omega, T) \equiv 0$.
6.2. $\Phi = \Phi_{inc}$; 0.5Φ is formed at the expense of the radiation, and 0.5Φ at the expense of reflection of half the incident flux.
6.3. (a) 0.97 mm (the far infrared region bordering with the microwave radio-frequency range); (b) 9700 nm (the infrared region); (c) 970 nm (the near infrared region); (d) 580 nm (the visible part of the spectrum).
6.4. λ_m will diminish to 1/2 of its initial value.
6.5. $\lambda_m = 2.00$ μm.
6.6. $\langle\varepsilon\rangle = \hbar\omega/[\exp(\hbar\omega/kT) - 1]$.
6.7. (a) $\langle\varepsilon\rangle_q = 0.582kT = 0.582\langle\varepsilon\rangle_{cl}$; (b) $\langle\varepsilon\rangle_q = 0.950kT = 0.950\langle\varepsilon\rangle_{cl}$; (c) $\langle\varepsilon\rangle_q = 0.000\ 454kT = 0.000\ 454\langle\varepsilon\rangle_{cl}$.
6.8. (a) $\langle\varepsilon\rangle = 0.257$ eV $= 0.993kT$; (b) $\langle\varepsilon\rangle = 0.247$ eV $= 0.954kT$; (c) $\langle\varepsilon\rangle = 0.154$ eV $= 0.600kT$; (d) $\langle\varepsilon\rangle = 0.0178$ meV $= 0.000\ 687kT$.
6.9. (a) $\omega_m = 2.821(k/\hbar)T = 3.71 \times 10^{11}T$; (b) $\lambda_m\omega_m = 1.08 \times 10^9$ m/s $= 0.57 \times 2\pi c$.
6.10. (a) $T = 5.8 \times 10^3$ K; (b) $E = 3.9 \times 10^{26}$ W; (c) $m = 4.3 \times 10^9$ kg/s; (d) $\tau \approx 10^{11}$ years.
6.11. $I = 1.37$ kW/m².
6.12. (a) $E = 1.1 \times 10^5$ lx (the exact value is 1.36×10^5 lx); (b) $I = 2.5 \times 10^{27}$ cd (the exact value is 3.0×10^{27} cd).
6.13. L_E is proportional to T^4.
6.14. $L_E = 1.5 \times 10^2$ W·m²/sr.
6.15. $\Phi = 2.9 \times 10^{-4}$ W.
6.16. $T = 395$ K.
6.17. From 1.6 to 3.1 eV.
6.18. 4.5×10^{13} photon/(cm²·s).
6.19. (a) $\varepsilon = 2.23$ eV $= 4.4 \times 10^{-6}m_ec^2$, $p = 1.2 \times 10^{-27}$ kg·m/s $= 1.3m_ev$; (b) $\varepsilon = 12.3$ keV $= 2.4 \times 10^{-2}m_ec^2$, $p = 0.66 \times 10^{-23}$ kg·m/s $= 0.7 \times 10^4m_ev$; (c) $\varepsilon = 1.23$ MeV $= 2.4m_ec^2$, $p = 0.66 \times 10^{-21}$ kg·m/s $= 0.7 \times 10^6m_ev$.
6.20. $v = 0.92c$.
6.21. (a) and (b) $F = 1.43$ nN.
6.22. (a) $\varphi(\omega) = \alpha/\omega$, where $\alpha = 2.4 \times 10^{21}$ photon/(m²·s); (b) $j_{ph} = \alpha \ln(\omega_2/\omega_1) = 1.5 \times 10^{21}$ photon/(m²·s), where ω_1 and ω_2 are the frequencies corresponding to the boundaries of the visible spectrum.
6.23. $\lambda_{min} = 0.025$ nm.
6.24. $U_1 = 25$ kV.
6.25. (a) $\hbar = 1.04 \times 10^{-34}$ J·s; (b) $A = 1.8$ eV (later investigations give a value somewhat greater than 2 eV).
6.26. $\lambda_0 = 260$ nm.
6.27. (a) $A = 3.7$ eV; (b) $\lambda = 260$ nm.
6.28. $\varphi = 2.5$ V.

6.29. $I_{sat} = 7\ \mu A$.

6.30. (a) $\Delta E/E = \lambda_C/(\lambda + \lambda_C) = 0.35 \times 10^{-5}$ (λ_C is the Compton wavelength of an electron); (b) $v = 1.5$ km/s.

6.31. (a) $\Delta E/E = 0.024$; (b) $v = 1.02 \times 10^7$ m/s $= 0.034c$.

6.32. (a) $E_k = \varepsilon\alpha \sin^2(\theta/2)/[1 + \alpha \sin^2(\theta/2)]$, where $\alpha = 2\varepsilon/mc^2$; (b) $E_k = (\varepsilon^2/mc^2)(1 - \cos\theta)$.

6.33. (a) $E_k = (\varepsilon^2/m_pc^2)(1 - \cos\theta) = 1.07$ keV; (b) $v = c\sqrt{2E_k/m_pc^2} = 1.5 \times 10^{-3}c = 4.5 \times 10^5$ m/s.

6.34. $\delta\varphi = 1°8'$.

6.35. $\lambda_C = \hbar/mc$—the Compton wavelength of the given particle.

6.36. $r_{min} = 1.2 \times 10^{-14}$ m.

6.37. $b = 3.26 \times 10^{-14}$ m.

6.38. In the SI:

$$\frac{dN}{N} = na\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{Ze^2}{2m_pv^2}\right)^2 \frac{d\Omega}{\sin^4(\theta/2)}$$

where n is the number of atoms in unit volume of the foil, a is the thickness of the foil, m_p is the mass of a proton, and v is the speed of a proton.

In the Gaussian system, the expression is the same, but without the factor $1/4\pi\varepsilon_0$.

6.39. (a) 100; (b) 7.2; (c) 1.8; (d) 0.8; (e) 0.5.

6.40. $P = 1.4 \times 10^{-4}$.

6.41. $P_p = 4P_\alpha$.

6.42. The first excitation potential of mercury atoms is $\varphi_1 = 4.9$ V, the second excitation potential is $\varphi_2 = 6.7$ V.

6.43. In the SI: $v_1 = (1/4\pi\varepsilon_0)e^2/\hbar = 2.2 \times 10^6$ m/s. In the Gaussian system: $v_1 = e^2/\hbar = 2.2 \times 10^8$ cm/s.

6.44. (a) $r_1 = 2.56 \times 10^{-13}$ m; (b) $E_b = 2.82$ keV; (c) $v_1 = 2.2 \times 10^6$ m/s (compare with the answer to the preceding problem); (d) 3.0×10^{12} revolutions.

6.45. (a) In the SI: $E_b = (1/4\pi\varepsilon_0)m_{red}e^4/2\hbar^2 = 13.58$ eV, where $m_{red} = m_em_p/(m_e + m_p)$ is the reduced mass of the electron-proton system. In the Gaussian system, the expression is the same, but without the factor $1/4\pi\varepsilon_0$; (b) $\delta = -1/(1 + m_p/m_e) = -0.000\,54 = -0.054\%$.

6.46. $\delta = -1/(1 + m_p/m_\mu) = -0.101 = -10.1\%$.

6.47. $r_0 = \hbar^2/m_ee^2$, i.e. the Bohr radius.

6.48. $m_ee^4/\hbar^2 = -2E_1$, where E_1 is the energy of a hydrogen atom in the ground state.

6.49. $\mu_1 = e\hbar/2m_e = 0.927 \times 10^{-23}$ J/T $= \mu_B$.

6.50. $\mu_1' = e\hbar/2m_\mu = (1/207)\mu_1$.

6.51. $\mu_n/M_n = e/2m_e$.

6.52. (a) $r_n = \sqrt{n\hbar/\omega m}$; (b) $E_n = n\hbar\omega$ ($n = 1, 2, \ldots$).

6.53. (a) $\omega_m = R(2m + 1)/m^2(m + 1)^2$. (b) $\omega_1:\omega_2:\omega_3:\omega_4 = 5.4:1:0.35:0.16$.

6.54. $R = e\varphi_1/\hbar = 2.07 \times 10^{16}$ s^{-1}.

6.55. $\varphi_1 = 3E_1/4e = 10.2$ V.

6.56. $\varepsilon = 5e\varphi_1/27 = 1.9$ eV.

6.57. $\varepsilon = 3E_1/16 = 2.6$ eV.

6.58. (a) $\lambda_1 = 122$ nm, $\lambda_\infty = 91$ nm; (b) $\lambda_1 = 657$ nm, $\lambda_\infty = 365$ nm; (c) $\lambda_1 = 1876$ nm, $\lambda_\infty = 821$ nm.

6.59. $R = 2.068 \times 10^{16}$ s^{-1}.
6.60. (a) $\lambda_\alpha = 660$ nm; (b) $\lambda_\infty = 370$ nm.
6.61. Four lines.
6.62. 657 nm and 122 nm.
6.63. $v = 7.0 \times 10^5$ m/s.
6.64. $\varphi_i = 3.8$ V.
6.65. $\varphi_i = 2\pi\hbar c/e\lambda_\infty = 5.0$ V.
6.66. $E_i = 5.0$ eV (the experimental value is 5.1 eV).
6.67. $\varphi_i = 5.1$ V.
6.68. (1) $5S \to 4P \to 4S \to 3P \to 3S$; (2) $5S \to 4P \to 3D \to 3P \to 3S$; (3) $5S \to 4P \to 3S$; (4) $5S \to 3P \to 3S$.
6.69. $\Delta\omega = 3.2 \times 10^{12}$ s^{-1}.
6.70. $\Delta E = 2.1 \times 10^{-3}$ eV.
6.71. $\Delta\omega = 4.7 \times 10^{14}$ s^{-1}.
6.72. $s = -0.41$, $p = -0.04$.
6.73. $\lambda_1 = 820$ nm and $\lambda_2 = 680$ nm.
6.74. $s = -4.13$.
6.75. (a) $v = 3\hbar R/4m_{\text{H}}c = 3.25$ m/s; (b) $v = 5\hbar R/36m_{\text{H}}c = 0.60$ m/s.
6.76. $E = 1.64 \times 10^{-29}$ J $= 0.49 \times 10^{-10}\hbar\omega$, $v = 2.9$ cm/s.
6.77. $v = 0.78$ cm/s.
6.78. (a) $\Delta\lambda = \pi\hbar/m_{\text{Na}}c = \lambda_C/2 = 2.9 \times 10^{-17}$ m (λ_C is the Compton wavelength of an atom), (b) $\Delta\lambda$ does not depend on the energy of the emitted photon.
6.79. $v = 1.2$ m/s.
6.80. $v = 14.2$ m/s; at an angle of 89.0° to the direction in which the photon flew.
6.81. (a) $C = 1.252 \times 10^8$ s$^{-1/2}$ $= 1.005\sqrt{3R/4}$, $\sigma = 1.11$, (b) $Z = 26$, iron.
6.82. $C = 1.336 \times 10^8$ s$^{-1/2}$ $= 1.07\sqrt{3R/4}$, $\sigma = 3.7$. With an increase in Z, the graph of the function $\sqrt{\omega} = f(Z)$ deviates more and more from a straight line, gradually bending upward.
6.83. (a) $C = 5.398 \times 10^7$ s$^{-1/2}$ $= 1.007\sqrt{5R/36}$, $\sigma = 7.8$, (b) $Z = 78$, platinum.
6.84. $C = 5.098 \times 10^7$ s$^{-1/2}$ $= 0.95\sqrt{5R/36}$, $\sigma = 5.7$.
6.85. For silver $(46/28)^2 = 2.7$ times.
6.86. (a) $N = 22$; (b) $\Delta E_e/\Delta E_v = 22$.
6.87. (a) $N = 32$; (b) $\Delta E_v/\Delta E_r = 553$.
6.88. $r_0 = 0.117$ nm.
6.89. $\omega_r = (\hbar/I)\sqrt{J(J+1)} = \omega\sqrt{(J+1)/J}$.
6.90. $\omega_r = 5.8 \times 10^{12}$ rad/s.
6.91. $N_{i+1}/N_i = 1/1716$.
6.92. (a) $\lambda = (2\pi\hbar/mv)\sqrt{1 - v^2/c^2}$; (b) $\lambda = 2\pi\hbar/\sqrt{2mE_k(1 + E_k/2mc^2)}$.
6.93. $v = c/\sqrt{2} = 0.707c$.
6.94. (a) $v = 1.46 \times 10^3$ m/s; (b) $v = 0.73 \times 10^7$ m/s $= 0.024c$.
6.95. (a) $\Delta x = 0.5$ cm; (b) $\Delta x \sim 10^{-14}$ cm; (c) $\Delta x \sim 10^{-27}$ cm.
6.96. $\Delta x = 1.5$ mm $= 150b$.
6.97. $r_1 = 35$ mm, $r_2 = 82$ mm.
6.98. $E_1 \sim \hbar^2/ma^2$.

6.99. $E_0 \approx \hbar\omega$ (the exact value is $\hbar\omega/2$).

6.100. $v \sim$ [illegible] $\approx 10^6$ m/s $\sim 0.01c$ (compare with the answer to Problem 6.43).

6.101. $P = \int\limits_V |\psi(x, y, z)|^2 \, dV$.

6.102. $\psi_n = \sqrt{2/a}\,\sin(n\pi x/a)$, $E_n = (\pi^2\hbar^2/2ma^2)\,n^2$, where n is the quantum number taking on values of 1, 2, 3, ...

6.103. $P = 1 - 2\int\limits_0^{a/4} \frac{2}{a}\sin^2\frac{\pi x}{a}\,dx = 0.500$.

6.104. (a) $dn/dE = (a/\pi\hbar)\sqrt{m_e/2E}$;
(b) [illegible] $= 6.83 \times 10^{[illegible]}$ lev/J $= 1.23 \times 10^{[illegible]}$ lev/eV;

$$\text{(c)}\ \langle E_N\rangle = \frac{\int\limits_0^{E_N} E\left(\frac{dn}{dE}\right)dE}{\int\limits_0^{E_N}\left(\frac{dn}{dE}\right)dE} = \frac{1}{3}E_N = 2.0 \times 10^{-14}\ \text{J} = 0.12\ \text{MeV}.$$

6.105. $\psi_{n_1, n_2}(x, y) = \sqrt{4/S}\,\sin(n_1\pi x/a)\cdot\sin(n_2\pi y/b)$, $E_{n_1, n_2} = (\pi^2\hbar^2/2m)(n_1^2/a^2 + n_2^2/b^2)$, where $S = ab$ is the area of the well, n_1 and n_2 are quantum numbers taking on the values 1, 2, 3, . . . independently of each other.

6.106. $E_{11} = 0.75$ eV, $E_{12} = E_{21} = 1.89$ eV, $E_{22} = 3.02$ eV.

6.107. $\psi_{n_1, n_2, n_3}(x, y, z) = \sqrt{8/V}\,\sin(n_1\pi x/a)\cdot\sin(n_2\pi y/b)\times\sin(n_3\pi z/c)$, $E_{n_1, n_2, n_3} = (\pi^2\hbar^2/2m)(n_1^2/a^2 + n_2^2/b^2 + n_3^2/c^2)$, where $V = abc$ is the volume of the well, n_1, n_2, n_3 are quantum numbers taking on the values 1, 2, 3, . . . independently of one another.

6.108. (a) $\psi_n = \sqrt{1/2\pi R}\,(1/r)\sin(n\pi r/R)$; (b) $E_n = \pi^2\hbar^2 n^2/2mR^2$, where $n = 1, 2, 3, \ldots$.

6.109. (a) $A = 1/\sqrt{2\pi a}$; (b) $\langle r\rangle = a/2$.

6.110. $\langle r\rangle = a/\sqrt{2\pi}$.

6.111. (a) $A = 1/\sqrt{\pi a^3}$ [illegible]; (b) $r_{prob} = a$, $\langle r\rangle = 2a$ [illegible].

6.112. (a) $M_{min} = \hbar\sqrt{3}/2 = 0.913 \times 10^{-34}$ kg·m²/s.
(b) An electron, proton, neutron, and other elementary particles with a spin equal to 1/2.

6.113. $M = 6.7 \times 10^{67}\hbar$.

6.114. (a) [illegible] $= 0.564 =$ [illegible] $= 0.886$ [illegible]; (b) [illegible] $= \hbar\omega/4 =$ [illegible].

6.115. (a) $E_0 = 1.6 \times 10^{-33}$ J; (b) $a = 0.88 \times 10^{-16}$ m.

6.116. (a) $A = 1/\sqrt{\pi r_0^3}$; (b) [illegible] $\exp(-2r/r_0)$; (c) $r_{prob} = r_0$; (d) $\langle r\rangle = 3r_0/2$; (e) [illegible]; (f) $P_\eta = (2\eta^2 + 2\eta + 1)\exp(-2\eta)$.

6.117. (a) $P_1 = 0.677$; (b) $P_{1.5} = 0.423$; (c) $P_2 = 0.238$; (d) $P_5 = 0.002\ 76$; (e) $P_{10} = 4.45 \times 10^{-7}$.

6.118. $N = (g_1/g_2) \exp(\hbar\omega/kT) = 3.53 \times 10^{16}$ (ω is the frequency of the leading line of the Lyman series).
6.119. (a) $\mathbf{M}^2 = 2\hbar^2$; (b) $\mathbf{M}^2 = 12\hbar^2$.
6.120. (a) 0, 1, 2, 3, 4; (b) and (c) 1, 2, 3, 4, 5; (d) 1/2, 3/2, 5/2.
6.121. The first, second, third, fourth, and sixth.
6.122. $^5F_1$, $^5F_2$, $^5F_3$, $^5F_4$, $^5F_5$.
6.123. (a) and (b) one; (c) two; (d) and (e) three; (f) five.
6.124. 5, 6, 7, ...
6.125. For the P-state, S can have the values 1, 2, 3, . . .; for the D-state, $S = 1$.
6.126. (a) $\varkappa = 1$; (b) $\varkappa = 3, 5, 7$; (c) $\varkappa = 2, 4, 6, 8$; (d) $\varkappa = 6, 8$.
6.127. (a) $^1S_0$; (b) $^1P_1$, $^3P_2$, $^3P_1$, $^3P_0$; (c) $^1S_0$, $^3P_2$, $^3P_1$, $^3P_0$, $^1D_2$.
6.128. (a) $^4F_{9/2}$; (b) $^2P_{1/2}$.
6.129. (a) $^3F_4$ and $^1G_4$, $M_J = \hbar\sqrt{20}$; (b) $^1S_0$ and $^3P_0$, $M_J = 0$.
6.130. $^5F_1$.
6.131. $L = 2, 3, 4, 5$.
6.132. $M_J = \hbar\sqrt{3/4}$.
6.133. $M_J = (\hbar/2)\sqrt{35}$. $^2D_{5/2}$.
6.134. $M_J = (\hbar/2)\sqrt{63}$. $^2F_{7/2}$.
6.135. $^3P_2$.
6.136. $^2P_{3/2}$.
6.137. $^4S_{3/2}$, $^2P_{3/2}$, $^2P_{1/2}$, $^2D_{5/2}$, $^2D_{3/2}$. The term $^4S_{3/2}$ is a fundamental one.
6.138. The second, fourth, fifth, and seventh.
6.139. $\mu = (4/7)\mu_B\sqrt{63}$.
6.140. $\mu = (\mu_B/2)\sqrt{125}$.
6.141. $^4D_{1/2}$, $^5F_1$, $^7H_2$.
6.142. (a) $\mu = \mu_B\sqrt{2}$; (b) $\mu = 0$; (c) $\mu = \mu_B 2\sqrt{2}$; (d), (e), (f) $\mu = 0$.
6.143. (a) It does not split; (b) into three; (c) into five; (d) into six.
6.144. (a) Into four; (b) into three; (c) into nine.
6.145. (a) and (b) There will be no splitting.
6.146. $a = \mu_B g \dfrac{dB}{dx} \cdot \dfrac{l_1}{mv^2}\left(\dfrac{1}{2} l_1 + l_2\right) = 0.28$ mm.
6.147. (a) $\Delta E = 0$; (b) $\Delta E = 1.16 \times 10^{-4}$ eV; (c) $\Delta E = 2.32 \times 10^{-4}$ eV; (d) $\Delta E = 3.48 \times 10^{-4}$ eV.
6.148. $\Delta\omega_0 = 0.879 \times 10^{11}$ s^{-1}.
6.149. (a) and (b) $\Delta\omega = 0.88 \times 10^{11}$ s^{-1}; (c) and (d) $\Delta\omega = 0.59 \times 10^{11}$ s^{-1}; (e) $\Delta\omega = 0.44 \times 10^{11}$ s^{-1}.
6.150. $\Delta\omega = 55\Delta\omega'$.
6.151. $N \approx 5 \times 10^4$ slits.
6.152. (a) $l = a$; (b) $l = a\sqrt{3}$; (c) $l = a\sqrt{5/4}$.
6.153. $\alpha = 46.6°$.
6.154. $\alpha = 44.4°$.
6.155. $dN_\omega = (l/\pi v)\, d\omega$.
6.156. $dN_\omega = (S/2\pi v^2)\, \omega\, d\omega$.
6.157. $dN_\omega = (V/\pi^2 v^3)\, \omega^2\, d\omega$.
6.158. $\Theta = \pi\hbar v n/k = 360$ K (k is the Boltzmann constant).
6.159. $\Theta = (\hbar/k)\, 2v\sqrt{\pi n} = 406$ K.
6.160. $\Theta = (\hbar/k)\, v\sqrt[3]{6\pi^2 n} = 447$ K.
6.161. (a) $\langle\omega\rangle = (1/2)\omega_m = k\Theta/2\hbar = 2.4 \times 10^{13}$ s^{-1}; (b) $\langle\omega\rangle =$

$= (2/3)\omega_m = 2k\Theta/3\hbar = 2.7 \times 10^{13}$ s^{-1}; (c) $\langle\omega\rangle = (3/4)\omega_m = 3k\Theta/4\hbar = 2.9 \times 10^{13}$ s^{-1}.

6.162. (a) 1420 K; (b) 208 K; (c) 76 K.
6.163. $\Theta = 408$ K.
6.164. $U_0 = (9/8)R\Theta = 860$ J (R is the gas constant).
6.165. $C_2 = (T_2/T_1)^3 C_1 = 0.022$ J/(mol·K).
6.166. $C \approx 0.16$ J/(mol·K).
6.167. $\varepsilon_m = 0.026$ eV, $\lambda = 0.048$ mm.
6.168. $p_m \sim 10^{-24}$ kg·m/s, $\lambda = 0.66$ nm.
6.169. $p_m \sim 10^{-24}$ kg·m/s.
6.170. The energy levels become two times more dense.
6.171. No, it doesn't.
6.172. dn is proportional to N.
6.173. n is proportional to N.
6.174. $\langle n_m\rangle = 1/[\exp(\Theta/T) - 1] = 1.54$.
6.175. (a) $\varepsilon_m = 0.018$ eV; (b) $\langle n_m\rangle = 1.0$.
6.176. $e/m = -1.8 \times 10^{11}$ C/kg.
6.177. $n = 1.1 \times 10^{29}$ m^{-3}, $u_0 = 3.2 \times 10^{-3}$ m^2/(V·s).
6.178. No, it doesn't.
6.179. The density dn/dE of the levels increases η times.
6.180. $\Delta\varepsilon$ diminishes to one-third of its initial value.
6.181. $\langle\Delta\varepsilon\rangle = 1.0 \times 10^{-22}$ eV
6.182. $\Delta\varepsilon = (2\pi\hbar)^3/4\pi V(2m)^{3/2}\sqrt{E}$.
6.183. (a) 4.7×10^{-22} eV; (b) 1.5×10^{-22} eV; (c) 0.85×10^{-22} eV; (d) 0.66×10^{-22} eV.
6.184. (a) $E_F(0) = 7.0$ eV; (b) $\langle E\rangle = 4.2$ eV; (c) $T = 33 \times 10^3$ K.
6.185. $E_F = E_F(0)(1 - 2.2 \times 10^{-5})$.
6.186. $\eta = 0.65$.
6.187. $\eta = 0.54$.
6.188. $P = 0.5$.
6.189. $\langle n\rangle = 1$.
6.190. $\Phi_0 = 2.07 \times 10^{-15}$ Wb $= 2.07 \times 10^{-7}$ Mx.
6.191. $\Delta E = 1.1$ eV.
6.192. (a) It will diminish 1.03 times; (b) it will increase 1.21 times.
6.193. $A = 3.1$ eV.
6.194. $E = 1.55$ kV/m. The field will be directed from the aluminium plate to the platinum one.
6.195. $E = e\mathscr{E} + A_W - A_{Ni} = 9.7$ eV.
6.196. $U_{int} = (\hbar^2/2me)(3\pi^2)^{2/3}(n_2^{2/3} - n_1^{2/3}) = 6.2$ V.
6.197. E_b/A (in MeV) = (a) 6.93; (b) 8.03; (c) 8.45; (d) 8.79; (e) 8.76; (f) 8.39; (g) 7.87; (h) 7.59.
6.198. (a) $v_\alpha = 1.5 \times 10^7$ m/s; (b) $v = 2.7 \times 10^5$ m/s.
6.199. (a) $T = \ln 2/\lambda$; (b) $\tau = 1/\lambda$; (c) $T = \tau \ln 2$.
6.200. τ is 1.44 times larger than T.
6.201. $\eta = \exp(-3) = 0.050$.
6.202. $\eta = 0.875$.
6.203. $P = 1/2$.
6.204. (a) $P = 0.63$; (b) $P = 0.999\,95$; (c) $P = 0.095$.
6.205. (a) $N = (bT/\ln 2)[1 - \exp(-t \ln 2/T)]$; (b) $N = 6.2 \times 10^{10}$; (c) $N \approx bT/\ln 2 = 1.25 \times 10^{11}$.
6.206. (a) $N_Y = N_{X_0}[\lambda_1/(\lambda_2 - \lambda_1)][\exp(-\lambda_1 t) - \exp(-\lambda_2 t)]$; (b) $t_m = \ln(\lambda_1/\lambda_2)/(\lambda_1 - \lambda_2)$.
6.207. $t = 3.5 \times 10^3$ years.

HINTS ON HOW TO SOLVE PROBLEMS

1.24. Use the expression for the cosine of the angle between two vectors (see the answer to Problem 1.9).

1.31. Use the transformation $\frac{dv}{dt}=\frac{dv}{ds}\frac{ds}{dt}=v\frac{dv}{ds}$.

1.33. Use the formula $\exp(\pm x) \approx 1 \pm x$ that holds for $x \ll 1$.

1.60. (b) To solve the differential equation obtained for v, go over to the variable x related to v by the expression $x = v - mg/k$.

1.63. Use the formula $A = \mathbf{F}\,\Delta\mathbf{r} = F_x\,\Delta x + F_y\,\Delta y + F_z\,\Delta z$.

1.115. Write expressions for the angular momentum relative to two arbitrary points O and O' displaced relative to each other by the length $\mathbf{b}$, and convince yourself that these expressions equal each other.

1.118. (a) Take advantage of the fact that the modulus of the moment of a vector equals the product of the magnitude of the vector and the arm; (b) use the relation $\Delta\mathbf{L} = \int \mathbf{M}\,dt$.

1.122. Go over to a c.m.-frame that moves relative to the l-frame downward (in the figure) at the speed v. In the c.m.-frame, the events will occur in exactly the same way as in Problem 1.120. After considering the collision processes, return to the l-frame.

1.123. See the hint to Problem 1.122.

1.136. See Problems 1.31 and 1.32.

1.137. Take advantage of the fact that $\cos\alpha = \boldsymbol{\omega}\mathbf{L}/\omega L$, and express the numerator and the denominator of this formula in terms of the vector components.

1.203. Use the result of Problem 1.202.

1.206. See Problem 1.205.

1.208. A solid angle Ω is defined to be a part of space confined within a conical surface. A solid angle is measured by the ratio of the area S cut out by it on a sphere with its centre at the vertex of the conical surface to the square of the radius R of the sphere: $\Omega = S/R^2$.

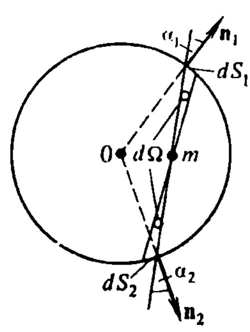

Fig. A.25

1.211. Consider the action of an infinitely thin homogeneous spherical layer on point particle m. Divide the layer into elementary areas

dS_1 and dS_2 opposite each other (Fig. A.25) corresponding to an identical solid angle $d\Omega$. The magnitude of the i-th area is $dS_i = r_i^2\, d\Omega/\cos\alpha_i$. A glance at the figure shows that $\alpha_1 = \alpha_2$. Consequently, dS_i is proportional to r_i^2. Hence, it is easy to conclude that the forces with which the portions dS_1 and dS_2 of the layer act on m balance each other.

1.214. It is expedient to perform the calculations in spherical coordinates, representing an element of mass of the layer in the form $dM = (M/4\pi) \sin\theta\, d\theta\, d\varphi$ and placing the particle on the polar axis.

1.225. Use the result of Problem 1.224.

1.228. Use the laws of energy and angular momentum conservation, and also the results of Problems 1.226 and 1.227. For simplification of the calculations, it is good to introduce the notation: $x = R/(R + h)$, $a = 2gR/v_0^2$.

1.230. Take into account the relations: $\dfrac{dv}{dt} = \dfrac{dv}{dr}\dfrac{dr}{dt} = \dfrac{dv}{dr}\, v$, $dt = \dfrac{dr}{v}$.

1.275. (b) The force of friction is $F_{fr} = -r\dot{x}$, where $r = 2\beta m$. Consequently, $A_{fr} = \int\limits_0^T F_{fr}\dot{x}\, dt = -r\int\limits_0^T \dot{x}^2\, dt = -\frac{1}{2} ra^2\omega^2 T = -\pi a\,(ra\omega) = -\pi a\,(2\beta\omega am)$.

It follows from the formula $\tan\varphi = 2\beta\omega/(\omega_0^2 - \omega^2)$ that $\sin\varphi = 2\beta\omega/\sqrt{(\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2} = 2\beta\omega\,(am/F_m)$. Hence, $2\beta\omega am = F_m \sin\varphi$. Introduction of this value into the expression for work yields: $A_{fr} = -\pi a F_m \sin\varphi = -A_{dr}$.

2.45. Use the equation of the first law of thermodynamics.

2.46. See the hint to Problem 2.45.

2.88. Go over to the variable $u = v/v_{prob}$ in the Maxwell distribution and perform the calculations by the formula $\Delta n/n = f(u)\,\Delta u$, assuming that $u = 1$ and $\Delta u = 0.02$.

2.123. Use the relation $d'Q = T\, dS$.

2.145. Use the equation of the first law of thermodynamics and the relation $F = U - TS$.

2.177. Use the answer to Problem 2.176 and the formula $\ln(1 \pm x) \approx \pm x$ (for $x \ll 1$).

2.192. Owing to the low density of the vapour, we may consider that it behaves like an ideal gas.

2.201. $\nu = (\langle v\rangle/\lambda)\,(n/2)$ (n is the number of molecules in unit volume).

2.207. We separate a ring of radius r and width dr on the upper disk. The ring experiences the force of friction $dF = \eta\,(dv/dz)\, dS = \eta\,(r\omega/a)\, 2\pi r\, dr$. The moment of this force is $dM = r\, dF$. The total moment of the force of friction applied to the upper disk is

$$M = \int dM = 2\pi\eta\,(\omega/a)\int\limits_0^R r^3\, dr = \pi\eta\omega R^4/2a.$$

2.208. See the hint to Problem 2.207.

3.17. To find E, locate the origin of the spherical system of coordinates at the centre of the hemisphere, and divide the surface of the hemisphere into rings of the area $dS = 2\pi R^2 \sin\theta\, d\theta$.

3.26. Take into account that the "length" l of the dipole is much smaller than r; disregard terms containing powers of the ratio l/r higher than the first one. After calculating φ, find the radial component E_r of the field strength and the component E_θ perpendicular to it.

3.35. Use the result of Problem 3.34 and the principle of field superposition.

3.36. Use the result of Problem 3.35 and the principle of field superposition.

3.39. See the hint to Problem 3.36.

3.64. Use the vector expression for the field strength inside a volume-charged sphere and the principle of field superposition.

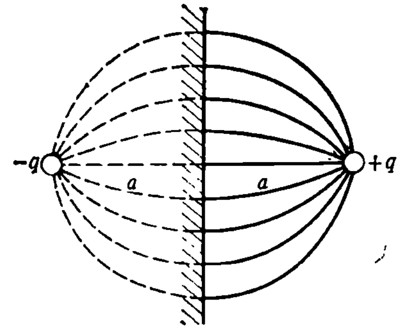

Fig. A.26

3.86. It follows from the condition of equilibrium of the charges on a conductor that the field inside the metal, which is a superposition of the field of the charge q and the field of the surface charges σ induced on the wall, equals zero. Consequently, the field set up by the charges σ in the metal coincides with the field that would be set up by the charge $-q$ placed at the same point where the charge q is. Because of symmetry, the field set up by the charges σ outside the metal coincides with the field that would be set up by the charge $-q$ located at a point inside the metal that is a mirror image of the point at which the charge q is (Fig. A.26). The fictitious charge $-q$ is called the image of the charge q, and the method of solution with the use of such fictitious charges—the method of images.

3.109. Use the formula $W = \frac{1}{2}\int\limits_V \varphi(\mathbf{r}')\, \rho(\mathbf{r}')\, dV'$ and the answer to Problem 3.14.

3.120. Since the circuit is infinite, all the units beginning with the second one can be replaced with a resistor whose resistance equals the required resistance R.

3.138. This problem can be solved in two ways.

1. Assume that the capacitor charged to the voltage U and disconnected from the current source is submerged into the medium being considered. Write for the instant immediately following the submersion an expression for the current flowing through an arbitrary closed surface enveloping one of the capacitor plates. Next, use Gauss's theorem for $\mathbf{E}$ for the same surface.

2. Divide the space between the plates into very small (at the limit—infinitely small) volumes confined by equipotential surfaces and $\mathbf{E}$ lines (coinciding with the $\mathbf{j}$ lines). Write expressions for the resistance and capacitance of the parallel-series system of an infinitely

large number of resistors (capacitors) appearing as a result of this division.

3.139. Use the results of Problems 3.104 and 3.138.

3.140. The current through a surface of radius r $(a < r < b)$ is $I(r) = 4\pi r^2 j(r) = 4\pi r^2 \sigma E(r)$, where $E(r) = (1/4\pi\varepsilon\varepsilon_0)\, q(r)/r^2$ [$q(r)$ is an extraneous charge confined inside a sphere of radius r]. The current $I(r)$ can be written in the form $-dq(r)/dt$. Replacing $I(r)$ and $E(r)$ with their values, we arrive at the equation

$$\dot{q}(r) = -(\sigma/\varepsilon\varepsilon_0)\, q(r)$$

Its solution has the form $q(r) = q_0 \exp(-\sigma t/\varepsilon\varepsilon_0)$. This expression does not depend on r. It thus follows that there is a surplus charge only on the inner plate. Hence, the charge on the inner plate varies according to the law $q = q_0 \exp(-\sigma t/\varepsilon\varepsilon_0)$.

Differentiating $q(r)$ with respect to t, we obtain an expression for the current

$$I(r) = -\dot{q}(r) = (\sigma q_0/\varepsilon\varepsilon_0) \exp(-\sigma t/\varepsilon\varepsilon_0)$$

that is also independent of r.

We find the amount of heat evolved by the formula

$$Q = \int_0^\infty RI^2\, dt$$

where we should take the value found in Problem 3.124 for R (after introducing σ instead of ρ).

3.162. Use the answer to Problem 3.161 and the principle of field superposition.

3.166. Consider the current to be distributed uniformly in a radial direction.

3.175. Consider the circulation of the vector **H** in an arbitrary circuit on a spherical surface of radius r $(a < r < b)$; take into account the displacement currents.

3.185. From the theorem on the circulation of the vector **H**, the following equation relating H and B in iron is obtained:

$$B = \frac{\mu_0 NI}{b} - \frac{\mu_0 (\pi d - b)}{b} H = a - kH$$

In addition, the relation $B = f(H)$ exists between B and H in iron. It is depicted graphically in Fig. 3.32. The required values of H and B satisfy both equations simultaneously. By solving these simultaneous equations graphically [i.e. by finding the coordinates of the point of intersection of the straight line $B = a - kH$ with the curve $B = f(H)$], we obtain $H = 0.33$ kA/m and $B = 1.3$ T. We shall let the reader perform the further calculations.

3.195. To find L_1, evaluate the energy of the magnetic field associated with the cable.

3.208. Use the expression for the energy of a current.

3.209. The formula $w = \frac{1}{2} HB$ for the density of magnetic energy is suitable only with a linear relation between H and B. Otherwise

the energy density is found by the formula $w = \int_0^B H\, dB$. The value of this integral can be determined by counting the number of squares under the curve $H = f(B)$ in Fig. 3.32. The value of H in the core is found using the circulation theorem.

3.229. Use a vector diagram. Taking the U-axis as the initia6 one, construct vectors of the currents flowing in the branch C and the branch L, R. The sum of these vectors will give the current vector I_1.

3.238. Adopting as the initial axis that of the voltage U_{cir} applied across the circuit, construct a joint diagram for the currents I_C, I_L, $I_{R'}$, and the voltages U_{cir}, $U_{R'}$, U. Apply the law of cosines to the triangle formed by I_C, I_L, and $I_{R'}$, and also to the triangle formed by U_{cir}, $U_{R'}$, and U.

Divide the solution into a number of stages (the finding of R, the finding of ω_1 and ω_2, the finding of I_C, I_L, $I_{R'}$, and so on). Introduce into the result obtained in each stage the numerical values of the quantities for which these values are already known. Use the expressions obtained in this way in the following stages.

4.5. The phase difference at points at a distance of Δx from each other is determined by the expression $\delta\varphi = 2\pi\, \Delta x/\lambda$. When calculating $\ln \eta$, take advantage of the fact that η differs only slightly from unity.

4.25. When calculating the integral, do not forget that the mean value of the square of the sine during a period is 1/2.

4.28. See the formula for v in the conditions of Problem 4.26.

4.31. See Problem 4.30.

4.33. See the formula for v in the conditions of Problem 4.26.

4.40. Solve an equation of the kind $\log r_0 = a - br_0$ graphically by plotting graphs of the functions $\log r_0$ and $a - br_0$ on graph paper. An expedient scale is 50 mm for unity along the axis of ordinates, and 50 mm for 1000 m along the axis of abscissas.

This equation can also be solved by the method of successive approximations. Let us represent the equation in the form of $r_0 = (a - \log r_0)/b$. It is evident from the nature of the problem that r_0 exceeds 200 m. Taking as a zero approximation the value of r_0 equal to 200 m, we introduce its logarithm into the right-hand side of the equation. As a result, we obtain a value of r_0 equal to $r_0^{(1)}$. Introducing its logarithm into the right-hand side of the equation, we obtain the value $r_0^{(2)}$, and so on. Already $r_0^{(4)}$ differs from the exact value by about 1%, and $r_0^{(5)}$, by 0.3%.

4.55. Take advantage of the fact that t is larger than the period T of the wave by many orders of magnitude.

4.56. Take into account the magnetic field produced by the displacement current.

4.57. Take into account the electric field produced by the varying magnetic field.

4.61. The density of the energy flux emitted by the dipole in a direction making the angle θ with its axis is proportional to $\sin^2 \theta$.

5.6. Use the formula $[\mathbf{a}[\mathbf{bc}]] = \mathbf{b}(\mathbf{ac}) - \mathbf{c}(\mathbf{ab})$. Take into account that $(\mathbf{e'n}) < 0$.

5.24. Use the graph shown in Fig. 5.9 (p. 179).

5.34. The oscillation of the light vector in the resultant wave has the form $\mathbf{E} = \mathbf{A}_1 \cos \omega t + \mathbf{A}_2 \cos (\omega t + \alpha)$. The resultant intensity is $I = \langle E^2 \rangle = \langle [A_1^2 \cos^2 \omega t + A_2^2 \cos^2 (\omega t + \alpha) + 2\mathbf{A}_1\mathbf{A}_2 \cos \omega t \times \times \cos (\omega t + \alpha)] \rangle = A_1^2/2 + A_2^2/2 = I_1 + I_2$ $(\mathbf{A}_1\mathbf{A}_2 = 0)$.

5.41. (b) If $\delta\Delta = \lambda/2$, the maxima from the top edges of the slits will be superposed onto the minima from the bottom edges, and vice versa, owing to which the interference pattern will be blurred. Assuming that the pattern is still distinguishable if $\delta\Delta = \lambda/4$, we obtain the result given in the answer.

5.47. (b) N equals the maximum odd integer that does not exceed the value of the expression $4br\varphi^2/\lambda\,(r + l)$.

5.48. (a) See the answer to Problem 5.7.

(b) N equals the maximum odd integer that does not exceed the value of the expression $4ab\,(n - 1)^2\theta^2/\lambda\,(a + b)$.

5.49. (a) See Problem 5.39.

(b) N equals the maximum odd integer that does not exceed the value of the expression $bh^2\Phi^2/\lambda$.

5.51. Take into account that in the case of a laser beam the waves reflected from both surfaces of the plate are coherent and will interfere.

5.72. The complex amplitude $\hat{A}$ of the resultant oscillation is real in the given case and equals the sum of the terms of the geometric progression $\sum\limits_{m=1}^{N} A_1 (-\rho)^{m-1}$.

5.86. Use the Cornu spiral (Fig. 5.22 on p. 192) and the formula given in the conditions of Problem 5.83 that relates the parameter v to the coordinate x.

5.88. Use the Cornu spiral (Fig. 5.22 on p. 192).

5.102. The intensity produced by the grating at the angle φ is determined by the formula

$$I_{\mathrm{gr}} = I_0 \frac{\sin^2 (\pi b \sin \varphi/\lambda)}{(\pi b \sin \varphi/\lambda)^2} \frac{\sin^2 (N\pi d \sin \varphi/\lambda)}{\sin^2 (\pi d \sin \varphi/\lambda)}$$

where I_0 is the intensity produced by one slit at the angle $\varphi = 0$, and N is the number of slits. For the m-th maximum, $\sin \varphi/\lambda = m/d$. The intensity I_0 is proportional to the square of the slit width, i.e. is proportional to x^2. With a view to this, we obtain that I_{gr} is proportional to $\sin^2 (m\pi x)$. The maximum of this quantity is reached at values of x satisfying the condition $m\pi x = (k + 1/2)\,\pi$; the zero value is obtained when $m\pi x = k\pi$.

5.122. There are four atoms to one face-centered cell. The distance between the atomic planes is half the edge of a cell.

5.139. Express P and P_1 in terms of the fraction α_1 of the intensity passed by one polarizer in its plane, and the fraction α_2 of the intensity passed in a perpendicular plane. Next exclude α_1 and α_2 from the equations obtained.

5.141. See the hint to Problem 5.139.

5.147. See Appendix 8.

5.153. See Problem 5.24.

5.154. Calculate the polarization P of the plasma, next the electric susceptibility χ, and, finally, the permittivity ε.

6.12. $E = \frac{I_{vis}}{A(\lambda_2 - \lambda_1)} \int_{\lambda_1}^{\lambda_2} V(\lambda)\, d\lambda$. Determine the value of the integral by counting the number of squares of the coordinate network under the curve in Fig. 5.9.

6.104. The levels are very dense. Therefore, when calculating dn/dE, we may consider that the variable n in the expression for E_n varies continuously.

6.105. Representing the psi-function in the form $\psi(x, y) = A \sin k_1x \cdot \sin k_2y$, find the values of k_1 and k_2 satisfying the boundary conditions. To find the values of the energy, introduce the expression for $\psi(x, y)$ satisfying the boundary conditions into the Schrödinger equation.

6.108. Use the Schrödinger equation in spherical coordinates and take into account that $\partial\psi/\partial\theta$ and $\partial\psi/\partial\varphi$ equal zero.

6.135. Use Hund's rules.

6.136. Compile a table similar to Table 5.4 in I. V. Savelyev, *Physics. A General Course*, Vol. III, p. 134 (Mir Publishers, Moscow, 1981). Use Hund's rule.

6.137. See the hint to Problem 6.136.

6.145. In case (b), $g = 0$.

6.169. Determine the interatomic distance d and assume that $\lambda_{min} \lesssim 2d$.

APPENDICES

1. Fundamental Physical Constants

Atomic mass unit	$1\ \text{amu} = \begin{cases} 1.6604\times10^{-27}\ \text{kg} \\ 931.42\ \text{MeV} \end{cases}$
Avogadro constant	$N_A = 6.022\times10^{23}\ \text{mol}^{-1}$
Bohr magneton	$\mu_B = \begin{cases} 0.927\,41\times10^{-23}\ \text{J/T} \\ 0.927\,41\times10^{-20}\ \text{erg/Gs} \end{cases}$
Bohr radius	$r_0 = 0.529\times10^{-10}\ \text{m}$
Boltzmann constant	$k = \begin{cases} 1.380\,622\times10^{-23}\ \text{J/K} \\ 0.861\,708\,2\times10^{-4}\ \text{eV/K} \end{cases}$
Electron mass	$m_e = \begin{cases} 0.910\,96\times10^{-30}\ \text{kg} \\ 0.511\,00\ \text{MeV} \end{cases}$
Elementary charge	$e = \begin{cases} 1.6022\times10^{-19}\ \text{C} \\ 4.803\times10^{-10}\ \text{cgse} \end{cases}$
Gas constant	$R = 8.314\,34\ \text{J/(mol}\cdot\text{K)}$
Gravitational constant	$G = 6.6720\times10^{-11}\ \text{m}^3/(\text{kg}\cdot\text{s}^2)$
Neutron mass	$m_n = \begin{cases} 1.674\,95\times10^{-27}\ \text{kg} \\ 939.57\ \text{MeV} \end{cases}$
Planck's constant	$\hbar = \begin{cases} 1.054\,591\,5\times10^{-34}\ \text{J}\cdot\text{s} \\ 0.658\,217\,6\times10^{-15}\ \text{eV}\cdot\text{s} \end{cases}$
Proton mass	$m_p = \begin{cases} 1.672\,65\times10^{-27}\ \text{kg} \\ 938.28\ \text{MeV} \end{cases}$
Rydberg constant	$R = 2.067\,068\,7\times10^{16}\ \text{s}^{-1}$
Speed of light in vacuum	$c = 2.997\,925\times10^8\ \text{m/s}$
Standard acceleration of free fall	$g = 9.806\,65\ \text{m/s}^2$
Standard atmospheric pressure	$p = 1013.25\ \text{hPa}$
Stefan-Boltzmann constant	$\sigma = 5.670\times10^{-8}\ \text{W/(m}^2\cdot\text{K}^4)$
Wien's displacement law constant	$b = 2.898\times10^{-3}\ \text{m}\cdot\text{K}$
	$\dfrac{1}{4\pi\varepsilon_0} = 8.9875\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$
	$\dfrac{\mu_0}{4\pi} = 10^{-7}\ \text{N/A}^2$

2. Astronomical Quantities

Quantity	Value
Mass (in kg):	
Sun	1.97×10^{30}
Earth	5.96×10^{24}
Mean radius (in m):	
Sun	6.96×10^{8}
Earth	6.37×10^{6}
Mean distance (in m):	
Sun-Earth	1.496×10^{11}
Sun-Jupiter	7.778×10^{11}
Earth-Moon	3.884×10^{8}

3. Density ρ of Selected Substances (in 10^3 kg/m^3)

Substance	ρ	Substance	ρ
Aluminium	2.70	Mercury	13.6
Beryllium	1.84	Potassium	0.87
Copper	8.93	Salt NaCl	2.17

4. Gas Constants (at S.T.P.)

Gas	Relative molecular mass	Diameter of molecule d, nm	Self-diffusion coefficient D, 10^{-4} m^2/s	Viscosity coefficient η, μPa·s	Thermal conductivity $\varkappa$, W/(m·K)
Ar	40	0.35	0.16	21.0	0.017
He	4	0.20	1.62	20.7	0.143
H_2	2	0.27	1.28	8.6	0.168
N_2	28	0.37	0.17	16.7	0.024
O_2	32	0.35	0.18	19.9	0.025
Air	29	—	—	17.2	0.024

5. Van der Waals Constants

Gas	a, Pa·m^6/mol^2	b, 10^{-5} m^3/mol
Hydrogen (H_2)	0.024	2.7
Nitrogen (N_2)	0.135	3.9
Oxygen (O_2)	0.136	3.2

6. Constants of Water and Ice

Quantity	Value
Specific heat capacity [in kJ/(kg·K)] of:	
water	4.18
ice	2.10
Heat (in kJ/kg) of:	
melting of ice	333
vaporization of water	2250

7. Coefficient of Surface Tension α (in N/m)

Substance	α	Substance	α
Water	0.073	Mercury	0.470

8. Wavelength Intervals Corresponding to Different Spectrum Colours

Spectrum colour	Wavelength interval, nm	Spectrum colour	Wavelength interval, nm
Violet	400-450	Yellow	560-590
Blue	450-480	Orange	590-620
Light blue	480-500	Red	620-760
Green	500-560		

9. Work Function A (in eV) of Selected Metals

Metal	A	Metal	A
Aluminium	3.74	Sodium	2.27
Nickel	4.84	Zinc	3.74

10. Atomic Number Z and Mass m (in amu) of Selected Elementary Particles and Isotopes

Z	Particle or isotope	Symbol	m
—	Electron	e	0.000 548 5
—	Neutron	n	1.008 665 0
1	Proton	p	1.007 276 5
1	Hydrogen	^{1}H	1.007 825
5	Boron	^{11}B	11.0093
10	Neon	^{20}Ne	19.992 44
14	Silicon	^{28}Si	27.9769
26	Iron	^{56}Fe	55.9349
30	Zinc	^{68}Zn	67.9248
56	Barium	^{137}Ba	136.9058
82	Lead	^{207}Pb	206.9759
92	Uranium	^{235}U	235.043 93

11. Formulas for Approximate Calculations

(The inequalities indicate the values of x at which calculation by the approximate formulas leads to errors not exceeding 0.1%)

$$\frac{1}{1 \pm x} \approx 1 \mp x, \qquad x < 0.031$$

$$\sqrt{1+x} \approx 1 + \frac{1}{2} x, \qquad x < 0.093$$

$$\sqrt{1-x} \approx 1 - \frac{1}{2} x, \qquad x < 0.085$$

$$e^{\pm x} \approx 1 \pm x, \qquad x < 0.045$$

$$\ln (1 \pm x) \approx \pm x, \qquad x < 0.045$$

$$\sin x \approx x, \qquad x < 0.077 \text{ rad } (4.4^\circ)$$

$$\cos x \approx 1 - \frac{1}{2} x^2, \qquad x < 0.387 \text{ rad } (22.2^\circ)$$

12. Calculation of Sums with the Aid of Integrals

Assume that we have a sum of the kind

$$\sum_{n=0}^{N} f(a + nh) = f(a) + f(a + h) + f(a + 2h) + \ldots + f(b)$$

where $f(x)$ is a function that does not vary too rapidly, h is a small quantity, and $b = a + Nh$,

It is a simple matter to comprehend that the required sum equals the area S of the bars shown in Fig. A.27 divided by h. Replacing

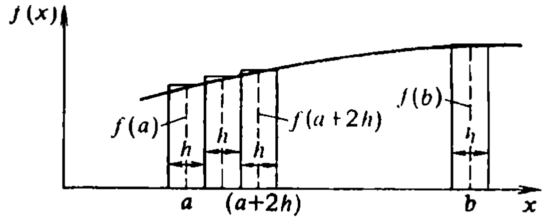

Fig. A.27

the area of the bars with the area confined by the curve $f(x)$, we can write that

$$S \approx \int_a^b f(x)\,dx + \frac{1}{2} hf(a) + \frac{1}{2} hf(b)$$

Hence follows the approximate relation

$$f(a) + f(a+h) + f(a+2h) + \ldots + f(b) \approx \frac{1}{h}\int_a^b f(x)\,dx + \frac{1}{2} f(a) + \frac{1}{2} f(b)$$

We must note that the smaller the value of h, the greater is the accuracy of the formula.

13. Selected Mathematical Formulas

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

The area of an ellipse is $S = \pi ab$, where a and b are the semi-axes of the ellipse.

$$\mathbf{a}[\mathbf{bc}] = \mathbf{b}[\mathbf{ca}] = \mathbf{c}[\mathbf{ab}]$$

$$[\mathbf{a}[\mathbf{bc}]] = \mathbf{b}(\mathbf{ac}) - \mathbf{c}(\mathbf{ab})$$

$$(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$$

$$(\tan^{-1} x)' = \frac{1}{1+x^2}$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

$$\int \frac{dx}{\sqrt{a^2+x^2}} = \ln (x+\sqrt{a^2+x^2})$$

$$\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{1}{a^2} \frac{x}{\sqrt{a^2+x^2}}$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a}$$

$$\int \frac{dx}{\sqrt{x^2-a^2}} = \ln (x+\sqrt{x^2-a^2})$$

$$\int_0^\infty e^{-\beta x}\, dx = \frac{1}{\beta}$$

$$\int_0^\infty e^{-\beta x}\, x\, dx = \frac{1}{\beta^2}$$

$$\int_0^\infty e^{-\beta x^2}\, dx = \frac{1}{2} \sqrt{\frac{\pi}{\beta}}$$

$$\int_0^\infty e^{-\beta x^2} x\, dx = \frac{1}{2\beta}$$

$$\int_0^\infty e^{-\beta x^2} x^2\, dx = \frac{1}{4} \sqrt{\frac{\pi}{\beta^3}}$$

$$\int_0^\infty e^{-\beta x^2} x^3\, dx = \frac{1}{2\beta^2}$$

14. Selected Numbers

$e = 2.718\,281$	$\ln x = 2.3026 \log x$
$\log e = 0.434\,294$	$\pi = 3.141\,592\,6$
$\ln 10 = 2.302\,585$	$\pi^2 = 9.869\,624$
$\log x = 0.4343 \ln x$	$\sqrt{\pi} = 1.772\,453\,8$

15. SI Prefixes to Names of Units

Prefix	Multiple of basic unit	Symbol	Prefix	Multiple of basic unit	Symbol
Giga	10^9	G	Milli	10^{-3}	m
Mega	10^6	M	Micro	10^{-6}	μ
Kilo	10^3	k	Nano	10^{-9}	n
Hecto	10^2	h	Pico	10^{-12}	p
Centi	10^{-2}	c			

16. Table of Sines

The number at the intersection of a row and a column gives the value of the sine of the angle equal to the sum of the angles corresponding to the given row and column. The values of sines for intermediate angles can be obtained with an error not exceeding two units of the fourth digit by interpolation.

	0°	2°	4°	6°	8°
0°	0.0000	0.0349	0.0698	0.1045	0.1392
10°	0.1736	0.2079	0.2419	0.2756	0.3090
20°	0.3420	0.3746	0.4067	0.4384	0.4695
30°	0.5000	0.5299	0.5592	0.5878	0.6157
40°	0.6428	0.6691	0.6947	0.7193	0.7431
50°	0.7660	0.7880	0.8090	0.8290	0.8480
60°	0.8660	0.8829	0.8988	0.9135	0.9272
70°	0.9397	0.9511	0.9613	0.9703	0.9781
80°	0.9848	0.9903	0.9945	0.9976	0.9994

TO THE READER

Mir Publishers would be grateful for your comments on the content, translation and design of this book.

We would also be pleased to receive any other suggestions you may wish to make.

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Printed in the Union of Soviet Socialist Republics

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